REPORT OF THE SUPERINTENDENT

OF THE

U. S. COAST AND GEODETIC SURVEY

SHOWING

THE PROGRESS OF THE WORK

DURING THE

FISCAL YEAR ENDING WITH

JUNE, 1878.

WASHINGTON:
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LETTER

FROM

THE SECRETARY OF THE TREASURY,

COMMUNICATING,

In obedience to law, a report of the Superintendent of the Coast and Geodetic Survey, showing the progress made in the survey of the Atlantic, Gulf, and Pacific coasts for the year ending June 30, 1878.

DECEMBER 17, 1878.—Ordered to lie on the table and be printed.

TREASURY DEPARTMENT, *December* 10, 1878.

SIR: In compliance with section 4690, United States Revised Statutes, I have the honor to transmit herewith, for the information of the Senate, a report addressed to this department by Carlile P. Patterson, Superintendent of the Coast and Geodetic Survey, showing the progress made in the survey of the Atlantic, Gulf, and Pacific coasts, during the year ending June 30, 1878, and accompanied by a map illustrating the general progress made in that work.

Very respectfully,

JOHN SHERMAN,
Secretary.

Hon. WILLIAM A. WHEELER,
Vice-President of the United States, President of the Senate.

ABSTRACT OF CONTENTS OF REPORT.

CONTENTS OF APPENDICES.

ALPHABETICAL INDEX.

C.

D.

H.

I.

J.

K.

N.

Q.

R.

S.

T.

U.

V.

W.

REPORT.

OFFICE OF THE COAST AND GEODETIC SURVEY,
Washington, December 10, 1878.

SIR: I have the honor to transmit this report, showing in detail the progress made in the survey of the coasts of the United States and geodetic operations prosecuted in connection with that work in the course of the fiscal year ending June 30, 1878.

The field parties and the vessels used for transportation are of necessity exposed to all varieties of climate and to all extremes of weather. It is therefore a subject of great satisfaction to report that the vicissitudes of the year, so sadly marked by unusual mortality in one section of the Union and by the record of many storms along the coast, have entailed no disaster to the service. Of several parties on the shores of the Lower Mississippi one remained at work until late in August. Yellow fever was then raging above and below the working ground, but the operations were in no instance delayed by sickness.*

On the Atlantic, Gulf of Mexico, and Pacific coast of the United States, an aggregate of twenty-nine vessels has been in employ. The usual risks have been met by the parties when afloat, but no loss has been incurred beyond the outlay for repairing the slightly injured vessels. In January last one of the steamers of the survey was in great peril during several days in the Gulf of Mexico, but was finally relieved by the exertions of her officers and crew. Of this incident further mention will be made in connection with a notice of the work in which the vessel was at that time engaged.

As heretofore the aim has been steadily maintained of limiting the outlay for each party to the least sum consistent with its efficiency. All items in estimates for outfit and for monthly expenditures in field work and hydrography have been closely scanned, and it is gratifying to add that the assistants have cheerfully sustained all arrangements for promoting economy.

A few years ago the hydrography, for want of suitable vessels, was left behind the field work, but recently the advance made by the sounding parties has nearly equalized progress as between those branches of the service. In the course of the fiscal year ending June 30, 1878, the hydrography was prosecuted by fifteen parties working in twenty-four localities. Each of the Navy officers who have conducted work under my direction, and the officers associated with them, will be named in separate notices of the work done by the several parties. It gives me pleasure to record that the officers detailed from the Navy Department have worked with energy and intelligence. Their efficiency in the discharge of duty is a gratifying evidence that the present naval standard befits the honor of the nation, and secures the well-being of one of its most important interests.

The distribution of all the parties in the course of the year is given in tabular form in Appendix No. 1, which shows in geographical order the localities of the work.

It will be seen that progress has been made in topographical development on parts of the coast not heretofore included within the limits of operations. That the work should be advanced yearly over new or unsurveyed ground has been regarded as an imperative duty, but while sustaining that policy, some regard, of necessity has been given to calls for the revision of hydrography at important localities when the expense to be incurred was comparatively light.

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*The sailing master in the party working near Donaldsonville, La., having passed several summers in the Lower Mississippi, preferred to remain in the section, and volunteered to take charge of the vessel after the close of field work. He continued in good health until the 25th of October, but was then seized with yellow fever and died on the 30th of that month.

At several of our principal seaports the local authorities have appropriated means for revising the shore line and soundings of early surveys, so as to develop changes that have occurred and afford data for the stay of encroachments. These in most cases have caused alterations hurtful to the channels, but the injury done can be ascertained only by careful resurvey. When the facts are ascertained and proper restraining limits are pointed out, further encroachment on the water space is prohibited by municipal or by legislative action.

With means furnished by the State of Maryland a complete resurvey has been made of the outlines and water space of the harbor of Baltimore, and similar work is now in progress at Philadelphia, for which means were provided by the city authorities. These operations keep mainly in view the data requisite for answering inquiries as to the causes of alteration for the worse; hence the shore-line survey and soundings are conjoined with elaborate observations of the currents, and comparison of their courses and velocities with those of earlier periods. Numerous observations are recorded, and the issue of their discussion is to point out restrictions needful for the preservation of the harbor. As already said, each of the principal ports of the Atlantic seaboard has been for such purposes under consideration by the local authorities, and more recently the harbor of Norfolk.

In all the instances here alluded to, appeal has been made for the services of trained observers, and, as already intimated, such calls have been favorably met in cases in which officers of the government have been associated as advisory boards with harbor commissioners. Pay of hands and all expenses incidental to the resurveys of harbors have been borne by the local authorities. And if we regard the projected improvements in such cases as purely local, and leave them out of view, it is nevertheless desirable that the appropriations for the Coast and Geodetic Survey should be sufficient for maintaining due progress in the regular work, and for providing at the same time the means for revising hydrography in which national interests are involved, so far as related to general navigation, and in which appeal cannot be made to municipalities. Cases of this kind, beyond the range of riparian interests or ownership, will be cited in the further mention which will be made presently under the head of estimates.

The geodetic work of the survey, resulting in finally-determined positions on land, of course suffers no change under causes that affect here and there the shore-line surveys and the hydrography, nor so far as yet known any change whatever. Nevertheless, geographical points, whether determined by astronomical observations or by triangulation, depend for exactness of relation upon previous knowledge respecting the figure of the earth.

During the last hundred years the interest manifested in geodetic operations has been steady, but through all the earlier parts of that period the interest was necessarily prospective. The question at issue could not be solved until data had been accumulated by measurements made on the surface of the earth over wide ranges of latitude and longitude. Europe, Asia, Africa, and America have contributed material, so far as geodetic work has been extended, for determining the figure of the earth, and at no preceding time has the interest in regard to that question reached the degree of earnestness now generally manifested. In order to further in all ways possible such means as might be taken to secure the object sought, the leading scientific men of Europe have instituted an International Geodetic Association. The annual meetings of that body have greatly enhanced the spirit of research, and will probably lead to renewed exertions and to the adoption of all expedients that the science of the present day may be able to present for an early and satisfactory settlement of the question. With the sanction of the Treasury Department our own country is fitly represented in that association by Assistant J. E. Hilgard. The methods used in the Coast and Geodetic Survey having been kept in accord with all improvements, the geodetic work of the United States is properly regarded in Europe and in the Geodetic Association as being fully equal in grade to any work of the kind. Our interest therefore in the question concerning the figure of the earth may be considered as commensurate with the extent of our territory.

The geodetic operations now in progress, and of which further mention will be made in the body of this report, are so arranged as to furnish points for correcting maps of the interior, and at the same time to add largely to the data needful for determining the magnitude and figure of the earth in general, and especially in regard to the figure most nearly in accord with the surface or the United States. Our first contribution in results such as have been eagerly desired by successive

masters of geodetic science in times past will be the computed length of the arc of the meridian extending from Nantucket Island to Mount Blue, in Maine; the length of the arc from Ocracoke Inlet, N. C., to the head of Chesapeake Bay, in Maryland; and the oblique arc upwards of twelve hundred miles long, beginning at Calais, on the northeastern boundary of the United States, and extending southward and westward to Atlanta, in Georgia. These measures are all derived from the main triangulation of the Atlantic coast.

In the course of a few years, if means are available, data will be at hand for assigning the true measure of one of the longest parallels of latitude traversing the area of the United States. When complete the chain of great triangles for the geodetic connection between the survey of the Atlantic coast and that of the Pacific coast, will stretch east and west nearly twenty-six hundred miles, and the work will bind together in true geographical and geodetic relation all intervening States along or near the 39th parallel, and indirectly all the other States and Territories of the Union.

The hydrography of the Gulf of Mexico will be a subject of mention under the head of Section VIII, in Part II of this report. When the gulf soundings were so far advanced as to give certainty, and therefore special value to researches in regard to living forms that inhabit deep-sea bottoms, Prof. Alexander Agassiz, of Harvard, was notified that trawls and dredges would be sent with the vessel, and was invited to accompany her with an assistant to superintend the dredging collections, take charge of and report upon such specimens of deep-sea and other ocean fauna as might be brought up by the hydrographic party of Lieut. Commander C. D. Sigsbee, U. S. N., Assistant Coast and Geodetic Survey, in the steamer Blake.

Professor Agassiz promptly accepted the invitation and joined the party, with his assistant, Mr. S. W. Garman, expecting to return after a short stay in the Gulf to his duties at Cambridge. But, early in the cruise, the vessel, while under charge of a Spanish pilot, was stranded on a reef off the coast of Cuba, and was relieved from imminent peril only by the steady courage of her officers. Although inconvenienced by the unexpected delay, Professor Agassiz remained near by and effectively aided in obtaining assistance, and when the steamer was again afloat pursued the intended investigations. The results were incidentally gathered, but they are ample, and will be recognized by naturalists as adding greatly to the means for investigation in courses pursued elsewhere by expeditions fitted out at great cost in Europe.

The specimens procured by deep-sea dredging in the Gulf of Mexico were assorted by Professor Agassiz, and committed for final investigation to naturalists severally the most eminent in the branches of study to which the different specimens pertain.

In pursuance of arrangements to which reference was made in my last annual report, a number of stations were occupied, in the course of the fiscal year, for gaining needful knowledge in regard to the variation of the compass, or declination of the magnetic needle. Along the Atlantic coast of the United States the declination is well known, observations having been made incidentally at many stations, as shown on a sketch appended to this volume. But whatever the declination may be at any one position near the coast, a line along which the magnetic needle has the same variation can be well traced out to sea only by starting from points in the interior, at which points the same variation has been actually found. Hence the necessity of distributing observations over a large area. The stations included in the operations of the present year, and which have not been previously occupied, will be mentioned in the body of this report, and also in Appendix No. 1.

The discussion of the secular change of the magnetic declination has been continued by Assistant Charles A. Schott, all recent observations being taken into account for bringing the results up to the present time. In June last he repeated the determinations, which have been heretofore made annually, of magnetic measures at the reference station in Washington, D. C.

At the observatory in Madison, which was erected by the authorities of the University of Wisconsin, magnetic observations have been kept up during the year. The instruments, furnished from this Office, are self-registering, and have yielded continuous photographic traces that indicate even the least magnetic perturbation. The first year's records from this observatory have been discussed, and the results will soon be arranged for publication.

From the tidal division of the Office printed tables are issued annually, predicting all the tides of the coming year for ports on the Atlantic and Pacific coasts of the United States.

The times are given with a near approach to precision at special places where long series of observations were recorded in past years, but with somewhat less exactness for intermediate positions. For some years the tables have been regarded as necessary aids in the navigation of our coasts. These tidal predictions result from the application of systems of analysis and discussion that have been from time to time improved, but chiefly by modifications of methods formerly in use. The last improvement in analysis, and known as the "*harmonic method*," was devised by Sir William Thomson, and has been adopted by the tidal committee of the British Association for the Advancement of Science. By this treatment the tide is followed hourly, and is scanned in strict accordance with time through the entire series of observations.

The tidal records of Pulpit Harbor, North Haven, in the Fox Island group, Penobscot Bay, Maine, for a period of six years, have been analyzed by the harmonic method and discussed by Mr. William Ferrel of the Coast and Geodetic Survey. His report on the subject (Appendix No. 11) explains the principles of this kind of analysis which, in application, is not restricted to tidal observations, but may be used for any series of numerous observations made at equal intervals of time. Then follows, in the paper, the application of these principles for the determination of tidal constants of all the principal tide components, derived from the Pulpit Harbor observations. The results are thoroughly discussed and compared with theory, and with results obtained by the same method in other parts of the world. Means are thus afforded for a careful examination of the tidal theory. In this connection, also, the paper shows that much study has been given to the character of the tides in shallow waters. Mr. Ferrel, in concluding, applies results for computing and predicting tides along the whole coast of New England, the type of the tide appearing to be the same throughout, as shown by comparing results from the Pulpit Harbor series with those derived from the series recorded in Boston Harbor.

To illustrate the nature of the work proposed in the detailed estimates which were submitted to the department in October last, a synopsis was given showing the field and office operations of the past fiscal year. The synopsis is here inserted as usual in previous annual reports. It will be noticed that the work has been advanced in upwards of one hundred localities on the Atlantic coast, Gulf of Mexico, and Pacific coast of the United States, including geodetic stations intermediate between the eastern and western coasts. For convenience, the work of the land and hydrographic parties will be recapitulated in the usual geographical order. The work afloat and the geodetic and other operations of the fiscal year ending June 30, 1878, include—soundings in the seaward approaches of Mount Desert Island, off the coast of Maine; and in that vicinity topographical surveys at the head of Frenchman's Bay and Blue Hill Bay; hydrography of the vicinity of Deer Isle and Isle au Haut, and tidal observations in Penobscot entrance; geodetic work at Gunstock Mountain and at Gilford, for determining geographical points in New Hampshire; triangulation for the harbor commissioner's survey of Boston upper harbor; determination of the positions of light-houses on the coast of Massachusetts; development of the action of sea currents as affecting navigation across the Gulf of Maine; tidal observations at Providence, R. I.; shore-line survey and soundings in Duck Island Harbor, Conn.; detailed topographical survey of the north and west approaches to New Haven, Conn., and of the western shore of Jamaica Bay, including Rockaway Inlet; survey of Coney Island, and of the shores of Sheepshead Bay and Gravesend Bay, Long Island, N. Y.; hydrography of Rockaway Inlet and of the adjacent parts of Jamaica Bay; pendulum observations at New York City; tidal observations at Governor's Island and at Sandy Hook; topography of the shores of Hudson River near Peeksville, N. Y.; tidal bench-marks at Stuyvesant and Albany, established by lines of level; primary triangulation connected with geodetic stations in New Hampshire, Vermont, and Massachusetts; latitudes and longitudes for State commissioners in adjustment of the boundary line between New York and Pennsylvania; geodetic points determined in the northern part of New Jersey and in Eastern Pennsylvania; latitude, longitude, and the magnetic elements at Harrisburg; points determined and special observations of tides and currents in the Delaware River at and near Philadelphia; triangulation for light-house positions in Delaware Bay; topographical survey continued eastward of Norfolk, Va.; tidal observations at Fort Monroe; special observations in the waters of Chesapeake Bay in regard to salinity and density; bench-marks along the Potomac near Washington, D. C., for comparing flood levels; magnetic declination, dip, and intensity determined at Washington; lines with spirit level run between

Hagerstown and Cumberland, Md.; primary triangulation at stations on the Blue Ridge, Va.; marine notes derived from examinations between Cape Henry and Cape Fear, for the Atlantic Coast Pilot; positions of life-saving stations between Cape Henry and Cape Hatteras, determined and marked on sheets of the survey; topography of the shores of Cape Fear River below Wilmington, N. C.; primary triangulation between Kent Island base line, Maryland, and the base near Atlanta, Ga., closed at stations in North Carolina; hydrography of the coast of South Carolina above Murrell's Inlet, including Little River; coast examinations between Fernandina, Fla., and the Dry Tortugas, and marine notes compiled for publication; tidal observations at Fernandina; hydrography of the eastern coast of Florida from Mosquito Inlet southward to Cape Cañaveral; survey of the Saint John's River, Fla., extended southward to the vicinity of Tocoi, and of Indian River to the neighborhood of Cape Malabar; hydrography of Charlotte Harbor, and triangulation of Sarasota Bay on the Gulf coast of Florida; survey of Crooked River, adjacent to Saint George's Sound, and supplementary soundings in Duer's Channel and off Light House Point; deep-sea soundings with observations for temperature and density in the waters of the Gulf of Mexico; triangulation, topography, and hydrography of Barataria Bay, La., nearly completed; continuous record of the water level of the Mississippi at New Orleans; survey of the river continued at Donaldsonville, Natchez, Vicksburg, Greenville, and Helena; latitude and longitude determined at the same places; triangulation continued in Northern Alabama, and stations selected in Northern Mississippi; reconnaissance along the coast of Louisiana and Texas for triangulation between Vermilion Bay and Galveston Bay; triangulation of Laguna Madre, Tex., extended southward of Baffin's Bay; magnetic declination, dip, and intensity determined at Dollar Point, San Antonio, Hempstead, and Groesbeck, Tex., and at Vinita, in Indian Territory. On the Pacific coast of the United States, and at points intermediate between the eastern coast and western coast, the operations of the fiscal year include—hydrography of the bar, and of parts of the harbor of San Diego, Cal.; triangulation for determining the geographical positions of San Clemente Island and Santa Barbara Island; soundings in approaches to the Pacific coast from Point Dume westward to Santa Cruz, inland; topographical survey continued on Catalina Island; soundings in its western approaches, and between Santa Rosa Island and Point Concepcion; topography of the coast in the vicinity of Point Arguello, and from Ynez River northward towards Point Sal, and of the coast north and south of Point Sur; tidal observations at Saucelito in San Francisco Bay; supplementary observations on the horizontal angles recorded at the primary stations on Mount Helena and Mount Diablo, Cal.; selection of points for primary triangulation along the coast between Point Arenas and Cape Blanco; hydrography of the approaches to Columbia River, Oreg., and detailed survey of the shores and channel between Mount Coffin and Kalama; selection of points for primary triangulation in Washington Sound, and across the Strait of Fuca; sites for base lines examined in the Willamette Valley, Oreg., and on Whidbey Island, W. T.; hydrography of Admiralty Inlet; survey of the shores of Puget Sound from Commencement Bay to Budd's Inlet, and of the northern part of Hood's Canal; inspection of topography in the field, in this section, compilation of the titles of charts, &c., illustrating the coast features and hydrography of Alaska.

At stations of the geodetic connection between the Atlantic coast triangulation and that of the Pacific coast, and at others occupied for the determinations of latitude, longitude, and the magnetic declination, or variation of the compass, the operations include—observations for latitude and longitude at Memphis and Nashville, Tenn.; at Cairo, Ill.; at Hickman and Paducah, Ky.; and telegraphic exchanges by signals at Nashville for the longitude of places along the Mississippi River, as already recited; determination of the magnetic elements at Nashville, and of latitude and azimuth near Lebanon, Tenn.; stations selected in Kentucky for the triangulation between Cumberland Gap and the Ohio River, and in Ohio for geographical positions between Athens and Columbus; also in Southern Illinois, and near Madison in Wisconsin; the magnetic elements determined at the last-named place, and also at La Crosse; at Minneapolis in Minnesota; at Sibley, Des Moines, Davenport, and Keokuk, in Iowa; at Omaha in Nebraska; and at Lawrence in Kansas; latitude and longitude determinations at Summit Station, Central Pacific Railroad, Cal.

Progress commensurate with the field-work has been made in the Coast and Geodetic Survey Office, in which the work comprises the reduction and discussion of all observations; the preparation for publication of the records and results; the drawing of hydrographic charts from the original

note-books; and of topographical and hydrographic sheets for publication; the engraving, electrotyping, printing, and issue of the same, and the repairs of instruments used in the survey.

Tide-tables for the principal ports of the United States for the year 1879 have been published; the drawing of seventy-eight charts has been in progress, forty-two have been completed, and of that number twenty-one have been issued by photolithography.

Seven copper-plate engraved charts have been completed, and six others are in hand, exclusive of one hundred and thirty-five plates on which additions have been engraved. Twenty-one thousand six hundred and twenty-three copies of charts have been issued within the year, and upwards of one thousand copies of the annual reports from the office. The second volume of the Atlantic Coast Pilot is now in course of publication.

ESTIMATES.

The estimates for continuing the work of the Coast and Geodetic Survey during the fiscal year ending June 30, 1880, were submitted to the department in October last. The aggregate of estimates then presented, although greater than the appropriation of the current fiscal year, is much less than the amounts first submitted in October of the preceding year. In consequence of reduced appropriations during several years, it has been impossible to revise the surveys of several important localities with due regard to the progress to be made at the same time over new ground. For several years the interests of commerce have required that the hydrography should be revised in Long Island Sound; in New York Bay; in Delaware Bay and River; at the Mississippi River Delta; in San Francisco Bay; at the Columbia River entrance; and at other localities where changes have taken place in the course or direction of the channels. The amounts stated in the estimates barely suffice for making such advances in the work as would keep pace with the rate of development. Along the great inland waters of Washington Territory, all points available for commercial purposes have already attracted the attention of settlers.

The estimates for continuing the survey of the Atlantic and Gulf coast of the United States during the year ending June 30, 1880, are intended to provide for the following progress:

FIELD-WORK.—To continue the topography of the western shore and islands of Passamaquoddy Bay and its estuaries; to complete the topography between Penobscot Bay and Narraguagus Bay, and that of the shores of the Penobscot near Bangor; for the determination of heights at some of the principal trigonometrical points between Boston and the Saint Croix, and of coefficients of refraction; to complete the hydrography between Penobscot Bay and Narraguagus Bay, and continue soundings in the coast approaches eastward of Petit Manan Island; to continue a topographical and hydrographic survey of Portsmouth Harbor; to make such additional triangulation as may be requisite for that and other surveys on the eastern coast, and determine the position of new light-houses between Eastport, Me., and New York; to continue soundings along the coast of Maine, and other off-shore hydrography between Cape Cod and Manan, and make special examination for the sailing lines for charts; to continue the observations of sea and tidal currents in the Gulf of Maine; to continue tidal observations, and to make such astronomical and magnetic observations as may be required; to continue such topographical and hydrographic resurveys of the coast between Cape Cod and New York as may be found necessary; to continue the survey of the Connecticut River from its mouth up to Hartford; to make such examination as may be required in New York Harbor, and such surveys in its vicinity as may be found necessary, including the continuation of the topographical and hydrographic resurvey of the south coast of Long Island; to make along this part of the coast observations on tides and currents; to extend the plane-table survey of the Hudson River above Peekskill; to continue the triangulation between the Hudson River and Lake Champlain; to make the requisite astronomical observations; to continue the topographical and hydrographic resurveys of the coast of New Jersey, and of Delaware Bay and River; to connect the Atlantic coast triangulation with that of Chesapeake Bay, near the boundary line between Maryland and Virginia; to complete the detailed survey of James River, Va., including the hydrography, and continue the plane-table survey of the Potomac River; to continue southward the main triangulation from the Atlanta base to the Mississippi River at or near Memphis, including astronomical and magnetic observations; to continue the supplementary hydrography between Cape Henlopen, Del., and Cape Henry, Va., and in Chesapeake Bay, and

also the tidal observations; to measure base lines of verification and determine azimuths for the coast triangulation south of Cape Lookout; to make the astronomical and magnetic observations requisite; to continue the off-shore hydrography between Cape Henry and Cape Fear; to complete the hydrography of Pamplico Sound and its rivers, and that of Bogue Sound, and sound the entrance to Cape Fear River, and continue the adjacent topography; to continue the topographical and hydrographic survey of rivers near the coast of South Carolina and Georgia; to determine azimuths for the triangulation of the coast of South Carolina and Georgia; to continue the detailed survey of the sea islands and water passages between Charleston and Savannah, and to make tidal observations; to continue the off-shore hydrography between Cape Fear, N. C., and the Saint John's River, Fla.; to continue southward from Cape Malabar, near latitude 28° north, the triangulation, topography, and hydrography of the eastern coast of Florida, including Indian River; to continue the triangulation, topography, and hydrography of the Saint John's River; to make the requisite astronomical observations; to continue hydrography off the eastern coast of Florida from Cape Cañaveral to the southward; to continue soundings and observations for deep-sea temperatures, currents, and dredgings in such parts of the Gulf Stream northward of the latitude and eastward of the meridian of Cape Florida as may be deemed advisable, and also in the Caribbean Sea, and within the same limits such as may be considered advantageous in conjunction with the United States Commission on Fish and Fisheries; to continue the astronomical and magnetic observations requisite between Cape Florida and Pensacola; to complete the hydrography of Charlotte Harbor, and continue the triangulation, topography, and hydrography of the western coast of Florida between Cedar Keys and Tampa Bay, and between Tampa Bay and Charlotte Harbor; to continue the same classes of work to the southward of Charlotte Harbor; to run lines of soundings and dredging, and make observations of sea-temperatures in the Gulf of Mexico, and develop the hydrography of the Gulf coast included in field operations; to connect the trigonometrical survey of the Mississippi River at New Orleans with that of Lake Borgne and Lakes Pontchartrain and Maurepas, and of the Mississippi River above New Orleans to the head of ship navigation; to determine geographical positions, and make the astronomical and magnetic observations requisite; to extend the triangulation, topography, and hydrography of the coast of Louisiana westward of the Mississippi Delta, and continue the hydrography of the Gulf of Mexico between the mouth of the Mississippi and Galveston, Tex.; to continue the triangulation, topography, and hydrography of the coast of Texas westward between Sabine Pass and Galveston, and between Corpus Christi and the Rio Grande; to measure a base line of verification, and make the astronomical and magnetic observations requisite between Sabine Pass and the Rio Grande; to continue the hydrography of the approaches to the coast of Texas; to continue the triangulation connecting the surveys of the Atlantic and Pacific coasts and to furnish points for State surveys; to continue the determination of the positions of new light-houses and life-saving stations along the coast between New York and the Rio Grande; to continue the field work for the verification of data for the Coast Pilot; to continue the organized system of magnetic observations required for a complete magnetic survey, and to run lines of levels connecting points in the main triangulations with the sea level.

OFFICE WORK.—To continue the deduction of results by computation from the field operations along the Atlantic and Gulf coasts, and in connection with interior geodetic surveys, including astronomical, geographical, magnetic, spirit-leveling, and tidal work; to advance the publication of the Coast Pilot for the Atlantic and Gulf coasts; to prepare the predictions of tides for 1880 for the same; to continue the publication of original topographic maps and reductions thereof, and to plot the hydrographic surveys; to make additions to the drawings of sailing charts A from Cape Sable to Cape Hatteras, and of Nos. I and II, Cape Sable to Sandy Hook, and Nantucket to Cape Hatteras; to continue the drawing and engraving of the general chart of the coast from Quoddy Head to Cape Cod, and to complete the engraving of the western part of this chart; to begin the drawing of coast charts Nos. 1 and 2 from Saint Croix River to Petit Manan Light-House; to continue the drawing and engraving of coast charts Nos. 3 and 4, including Frenchman's Bay and Penobscot Bay, and to finish the drawing of the first named of these charts; to begin the engraving of the harbor charts of Passamaquoddy Bay and of Machias and Englishman's Bays; to continue that of Eastport Harbor and Cobscook Bay; to complete the drawing and continue the engraving of the

harbor charts of Frenchman's Bay and Somes Sound, and of the charts of Blue Hill and Union River Bays; to complete the drawing of the harbor charts of Eggemoggin Reach, Head Harbor, and Swan Island Harbor; to begin the drawing and engraving of the chart of Newburyport Harbor (new issue), and the drawings of the charts of Thames River, New London, and Norwich; to begin the drawing and engraving of the charts of the Hudson River (new issue) from New York to Troy; to complete the drawings and engravings of the charts of Rockaway Inlet, Jamaica Bay, and Fire Island Inlet, N. Y., and of Barnegat Inlet, N. J., and the drawing and engraving of the chart of Delaware River, showing the hydrographic resurveys; to continue the drawing and engraving of coast chart No. 23 from Absecon Inlet to Cape May; to complete the engraving of the chart of James River from the entrance up to City Point (in three plates), and the drawing and engraving of the chart of the same river from City Point to Richmond; to continue the drawing and engraving of sailing chart B from Cape Hatteras to Key West; to continue the drawing and to begin the engraving of coast charts Nos. 45 and 46, from Cape Hatteras to Cape Lookout; to continue the drawing of coast charts Nos. 49 and 50 of Cape Fear River and approaches; to begin the engraving of coast charts No. 47 to 53, inclusive, showing the coast from Cape Lookout to Charleston; to begin the engraving of the general coast charts Nos. 8 and 9 from Saint Mary's River to Cape Florida; to continue the engraving of the chart of Saint John's River (in three plates); to complete the drawing and begin the engraving of coast charts Nos. 60 and 61 from Halifax River to Cape Cañaveral; to finish the drawing of the preliminary charts of the inside passage (Indian River) to Indian River Inlet; to begin the engraving of the general coast charts Nos. 11, 12, 14, 15, embracing parts of the coast between Key West and Galveston; to complete the drawing and engraving of the chart of Tortugas Harbor and to continue those of coast chart No. 75, Charlotte Harbor; to complete the drawing and engraving of coast chart No. 77, Tampa Bay and approaches, and the engraving of the harbor chart of Tampa Bay; to make additions of hydrography to the drawings of the sailing chart of the Gulf of Mexico; to complete the engraving of coast chart No. 79, Cedar Keys and vicinity; to begin the drawing and engraving of coast chart No. 80, Cedar Keys to Appalachee Bay; to continue the drawing and engraving of coast charts Nos. 81 and 82, Appalachee Bay to Appalachicola Bay; to begin the drawing of the harbor chart of approaches and entrance to Suwanee River, and to complete the chart of Saint Mark's River (new issue); to make additions to the drawing of coast chart No. 83, Saint George's Sound to Saint Joseph's Bay, and to continue the engraving of the chart of Saint George's Sound (new issue); to complete the drawing of the chart of Saint Andrew's Bay (new issue); to begin the engraving of coast charts Nos. 84 and 85, Cape San Blas to Santa Rosa Island; to complete the engraving of coast chart No. 91, Lakes Borgne and Pontchartrain, and to begin that of coast charts Nos. 92 and 93, Mississippi Sound to entrance to Mississippi River; to complete the drawing and engraving of the charts of Barataria Bay entrance and of Barataria Bay and approaches; to make such additions of hydrography and topography as may be required to the drawings of the general charts of the Gulf coast; to begin the engraving of coast chart 96, Mississippi River to Barataria Bay, and to complete that of coast chart No. 99, Point au Fer to Atchafalaya Bay; to continue the engraving of the general coast chart No. 16 from Galveston to the Rio Grande, and of coast chart No. 104, Galveston Bay, and coast charts Nos. 108, 109, 110, Matagorda Bay to Corpus Christi Bay; for material for drawing, engraving, map printing, for electrotyping, and photographing, and for instruments and apparatus.

Total for the Atlantic and Gulf coasts, involving work in thirty-two States and four Territories, will require $395,000.

The estimates for continuing the survey of the Pacific coast of the United States are intended to provide for the following progress:

FIELD WORK.—To make the requisite observations for latitude, longitude, and azimuth, and the magnetic elements at stations along the Pacific coast of the United States; to continue off-shore soundings along the coast of California, Oregon, and Washington Territory, and tidal observations at San Francisco, Port Townshend, and at such other localities as may be necessary; to continue the main coast triangulation from Monterey Bay to the southward, or from Point Concepcion to the northward, and from San Pedro towards San Diego, including the islands off that part of the coast; to continue reconnaissance for the main triangulation of the coast from San Pedro to Point Concepcion, from Russian River to the northward, from Columbia River north to Puget

Sound, and south up the Willamette Valley; to continue the primary triangulation through the Sacramento and San Joaquin Valleys and measure a base line; to continue the coast triangulation and topography from Newport, Los Angeles County, towards San Diego, and that of the islands off the coast of California; to measure a base line and continue the tertiary triangulation and topography of the coast north of Point Arguello, towards Point Sal, and the tertiary triangulation and topography from Point Buchon towards San Simeon; to continue the hydrography between San Diego and Monterey Bay; to develop the hydrographic changes in San Francisco Bay and its approaches; continue the triangulation and topography of the coast between Bodega Bay and Point Arena; complete hydrography between Cape Mendocino and the Klamath River, and continue that between Cape Sebastian and Point Orford; to observe currents along the coast and take soundings and temperature observations in the California branch of the Kuro-Siwo current, and execute such other hydrographic work as local demands may require; to continue tidal and current observations at the Golden Gate and observations on the ocean currents along the coast of California; to continue the triangulation, topography, and hydrography of the Columbia River; to complete the detailed survey between Cape Sebastian and Crescent City and off-shore hydrography at Crescent City Reef; to measure a base line and continue the triangulation of the Strait of Fuca and the topography and hydrography of Puget Sound and adjacent waters; to continue the triangulation eastward to connect the surveys of the Pacific and Atlantic coasts; to furnish points for State surveys and measure a base line; to continue the reconnaissance of the coasts and islands of Alaska, with observations for tides and currents, and to make the requisite astronomical and magnetic observations; to continue the field work for the description of the coast and verification of the Coast Pilot of the coasts of California, Oregon, and Washington Territory; to continue the organized system of magnetic observations required for a complete magnetic survey; and to run lines of levels connecting points in the main triangulations with the sea level.

OFFICE WORK.—To make the computations of the field observations, including astronomical, geodetic, geographical, magnetic, and tidal work; to continue the compilation of the Coast Pilot of the coasts of California, Oregon, and Washington Territory, and continue the compilation and publication of the Coast Pilot of Alaska; to prepare tidal predictions for 1880; to continue the publication of the original topographic maps and the reductions thereof, and to plot the hydrographic surveys; to continue the drawing and engraving of the general coast chart No. 1 from San Diego to Santa Monica, and to continue the drawing and engraving of No. 2 from Santa Monica to Point Concepcion; to complete the drawings of the local harbor charts within these limits as the material becomes available; to continue the drawing and engraving of the general coast chart No. 7, Point Arenas to Cape Mendocino; to continue the engraving of No. 8, Point Arenas to Saint George's Reef, and to begin that of No. 9, Saint George's Reef to Koos Bay; to begin the drawing of general coast chart No. 11, Cascade Head to Shoalwater Bay; to continue the drawing and engraving of the charts of Columbia River, of Puget and Washington Sounds, with their adjacent waters; to continue the drawing and engraving of the chart of the northwestern coast from the 49th parallel to, and including, the Alaska coast and islands in Behring Sea; for materials for drawing, engraving, and map printing; for electrotyping and photography; and for instruments and apparatus.

Total for the Pacific coast, involving work in five States and one Territory, will require $255,000.

For repairs and maintenance of the complement of vessels used in the Coast Survey will require $50,000.

For continuing the publication of the observations made in the progress of the Coast Survey will require $10,000.

For general expenses of all the work, rent, fuel; for transportation of instruments, maps, and charts, miscellaneous office expenses; and for the purchase of new instruments, books, maps, and charts, will require $38,600.

OBITUARY.

One of the most useful members of the Coast Survey Office, Mr. John T. Hoover, died in the forty-fourth year of his age of rapid pulmonary decline on the 25th of May, 1878. Within the fortnight preceding that date he was at the office as usual, efficiently discharging duties to which he had been long accustomed.

Mr. Hoover entered the service in boyhood, and, in addition to native industry and patience, brought a decided aptitude for method and dispatch in office details. These qualities were happily united with an amiable disposition and constant readiness to further the interests of the work. He was throughout life greatly esteemed by a large circle of friends—in the church where he was active, in leading charitable societies, of which he was an officer, and in the office to the duties of which his energies were devoted. In each of these his tact and facility in the discharge of business were cheerfully made effective for the benefit of his associates.

PART II.

The distribution of the field and hydrographic parties in the course of the fiscal year ending June 30, 1878, is shown in tabular form in Appendix No. 1. Following the order in which the parties are therein mentioned, abstracts will now be given of reports on the work done in each locality.

Much of the efficiency in the service depends upon means of transportation and accommodation. Eleven steamers, of from three hundred and fifty down to sixty tons burthen; eighteen schooners, of from one hundred and sixty-four to thirty tons burthen, and four small steam launches have been employed in the work of the year. In the oversight needful for keeping all the vessels in good condition, and careful supervision of the estimates for outfit and repairs, I have been ably assisted by Commander Edward P. Lull, U. S. N., hydrographic inspector of the Coast and Geodetic Survey. The patient application of that accomplished officer, and his readiness in giving personal attention to the requirements of the service at points however distant, have greatly furthered the interests and success of the survey.

Lieut. William H. Parker, U. S. N., assisted in the office of the hydrographic inspector from August 13, 1877, until the 19th of June of the present year.

On the 28th of June, 1878, Lieut. Commander Philip H. Cooper, U. S. N., was assigned to duty in the Coast and Geodetic Survey. This report was then closing, but arrangements were in progress for the periodical examination of depth on the bars of harbors along the Atlantic coast. The details of this service are in charge of Lieutenant-Commander Cooper, and will be the subject of mention in my next annual report.

W. H. Mapes, inspecting engineer, as in past years, renders, under the direction of the hydrographic inspector, faithful service in the supervision of repairs and of alterations when requisite on the vessels of the survey. His intelligence and care in the execution of details committed to his charge evince high regard for the best interests of the work.

The abstracts which follow are illustrated by the several progress sketches, as numbered in the heading of each of the sections. The general progress made in the survey is shown in Sketch No. 1.

SECTION I.

MAINE, NEW HAMPSHIRE, VERMONT, MASSACHUSETTS, AND RHODE ISLAND.—(SKETCHES NOS. 2 AND 3.)

Hydrography of the coast of Maine.—The approaches of the coast of Maine in the vicinity of Mount Desert Island have been sounded by a party in charge of Lieut. J. F. Moser, U. S. N., Assistant Coast and Geodetic Survey, working with the steamer Endeavor. For the erection and location of signals the vessel left Rockland on the 26th of June, 1877, and was employed in that preliminary work until the 5th of July, when soundings were begun. Tidal observations were at the same time commenced at Bass Harbor.

The hydrographic work was frequently interrupted by heavy fogs, but unusually good weather prevailed during September and until the middle of October. A few days of remarkably clear weather were improved by running from Mount Desert Rock three radial lines northward and westward to connect with the inshore hydrography, which last-mentioned work was extended by Lieutenant Moser from the vicinity of Schoodic Point to the limit of previous operations near Isle au Haut. A few radial lines of soundings were also run to seaward from Mount Desert Rock. In that vicinity the steamer Endeavor was repeatedly run in different directions over the Columbia Ledge, but no depth of less than four fathoms was found.

Near the shore the bottom consists of rock, but further out the specimens brought up in sounding showed stiff, dark clay, with small broken pieces of hornblendic rock.

The work done by the party of Lieutenant Moser developed the hydrography coastwise for a

distance of thirty miles, and extending ten miles to seaward outside of Mount Desert Island, and outside of others to the eastward of Penobscot entrance. The general statistics are:

Miles run in sounding	1,058
Angles measured	4,884
Number of soundings	6,410

On the 1st of November, 1877, the steamer left the coast of Maine, and after refitting at Norfolk was employed in hydrographic work in Section IV, as will be further mentioned under that head.

Lieutenant Moser was assisted during the season by Master J. B. Murdock, U. S. N., and Master F. E. Greene, U. S. N.

While working southward of Mount Desert, Lieutenant Moser's attention was called to the site of a rock between Sutton's Island and Great Cranberry Island. After due search the danger was found about midway in the beating channel between the two islands. The rock is small at the top, and has on it only seven and a half feet at mean low water. Being little known it was marked on the chart as Sperlin Rock, after the name of the resident of Cranberry Island who made known particulars which aided in the development.

Late in June a ledge was developed between Pemaquid Point and Monhegan Island. The least water found was seventeen feet on what seemed to be the pinnacle of a rock.

Topography of Skilling River, Me.—North of the site of work mentioned under the preceding head, the topographical survey of the main coast has been advanced in the vicinity of Skilling River, one of the estuaries of Frenchman's Bay. For this work Assistant Charles Hosmer took the field in the middle of July, 1877. The weather was very unfavorable, but in the course of six weeks the usual margin of topographical details had been mapped to represent the shores of Partridge Cove and Raccoon Cove. When the survey was well in hand with a prospect of joining to the westward with plane-table work, which will be noticed presently, Mr. Hosmer was prostrated by a sudden attack of illness, and in consequence the field party was discharged at the end of August. Assistant Hosmer, as soon as possible, proceeded to his home, and by careful treatment was in due time restored to his accustomed health. The statistics of the work done by him in this section are:

Miles of shore line surveyed	28½
Roads, miles	6
Area of topography, square miles	3

In the spring of the present year Assistant Hosmer again took the field for service, which will be noticed in this report under the head of Section VIII.

Topography of Blue Hill Bay, Me.—For continuing the survey of the coast of Maine to the northward and westward of Mount Desert Island, a party was organized to work under the charge of Assistant A. W. Longfellow, and took the field on the 22d of June, 1877. Joining at the western limit of the sheet projected for the survey of this season with plane-table details which had been mapped in a previous year as far eastward as Blue Hill primary station, Mr. Longfellow surveyed the north shore of Blue Hill Bay, and extended his work eastward to include Newbury Neck and the opposite shore of Union River Bay. The north end of Long Island also appears on the topographical sheet. "The general character of the topography upon the north shore of Blue Hill Bay is a rough, wooded, granite hillside, the quarries of which furnish the principal industry of Blue Hill Village. These quarries supply to the government, and to many cities of the Union, an exceptionally fine quality of dressed stone for public buildings." The small islands and ledges in Blue Hill Bay are represented on the plane-table sheet. Field work was closed on the 22d of September. The following are the general statistics:

Miles of shore line surveyed	30
Roads, miles	22
Area of topography, square miles	13

Hydrography of Jericho Bay and Head Harbor, Me.—After the completion of repairs on the schooner Earnest, Lieut. J. M. Hawley, U. S. N., Assistant Coast and Geodetic Survey, sailed with

his party from Belfast on the 22d of May, 1877, to continue the hydrographic work southward and eastward of Eggemoggin Reach, and beyond Naskeag Point, where soundings were closed last year. In the course of a week the requisite signals were all in place, and tidal observations were in progress. Soundings were then begun in Jericho Bay, and the weather being favorable, the projection was filled by the end of June. The outer limit of the hydrographic sheet includes the shore of Swan Island.

Special attention was given by Lieutenant Hawley to the development of rocks and shoals in Jericho Bay. "The tides in the bay between Deer Isle and the mainland on the north, and Swan Island on the south, set east and west, the flood setting to the westward. At the western end of Swan Island it sweeps around towards the sea, and the general direction of the set being northwest and southeast, the tidal current is strong in the narrow channels, but its force is influenced by the wind." Among the dangers determined in position is Thornbush Ledge, at the eastern entrance of Naskeag Harbor.

The work here under notice was done with the steam launch Sagadahoc, the schooner being used for quarters at her anchorage in Naskeag Harbor.

On the 1st of July the schooner Earnest left her anchorage and proceeded to Isle au Haut, but on the way a tide gauge was put up at Green's Landing. The vessel was then brought to anchor in the Thoroughfare, and a party was organized, in charge of Master G. C. Hanus, U. S. N., to make a hydrographic survey of Head Harbor. In this place, at the south end of Isle au Haut, the schooner Earnest had been caught by a heavy southwest gale, and cast ashore, as mentioned in my report of last year.

The party detailed for the survey left the Thoroughfare in the steam launch and cutter on the 5th of July, and completed the sounding of Head Harbor in the course of the next nine days. Tidal observations were continued at Head Harbor, Isle au Haut Thoroughfare, and Green's Landing while the soundings were in progress. The work, as proved by the experience of the party in the preceding season, was dangerous, and could be advanced at favorable times only by sounding in the cutter, the place being too much exposed to admit of working with the steam launch. While prosecuting the soundings, the officers and men were accommodated in houses along the shore of Head Harbor.

After completing the work last mentioned, Lieutenant Hawley commenced on a third projection which had been made to include the waters between Isle au Haut and Deer Isle. This work was vigorously prosecuted in sections by two parties detailed from the Earnest, the anchorage of which was not changed until the 7th of August, when the vessel was moved to Green's Landing. Dense fogs were frequent in August, and the work during that month was much delayed. The southern half of the hydrographic sheet was however filled with soundings when it became necessary to recall the schooner in order to make ready for her transfer to the Pacific coast under previous arrangements.

Deer Isle Thoroughfare and the adjacent waters were sounded by the 11th of September, when the work was closed. The party then proceeded to Belfast, and, after laying up the steam launch, sailed for Baltimore, at which port the schooner was refitted for her voyage to San Francisco. Lieutenant Hawley transferred the command of the Earnest to Sailing Master P. G. Letournau, and himself took charge of the schooner Silliman for the prosecution of work which will be mentioned under the head of Section VI in this report. He was assisted on the coast of Maine by Masters G. C. Hanus and A. H. Cobb, U. S. N., and by Ensign A. Mertz, U. S. N. The statistics of the work done by the party in Section I are:

Miles run in soundings	491
Angles measured	4,544
Number of soundings	36,667

Tidal observations.—The series of tidal and meteorological observations commenced at North Haven, in Penobscot Bay, in January, 1870, has been steadily maintained during the last fiscal year by Mr. J. G. Spaulding. The self-registering gauge is furnished with heating apparatus. Every precaution has been taken to avoid breaks, and so far as the record extends, this series is one of the most perfect among the tidal records. Occasionally, when of necessity the apparatus is

stopped for cleaning or repairs, the continuity of the record is secured by frequent staff readings, as at other stations.

Since the year 1872 a good self-registering tide gauge has been used at Providence, R. I., by the city engineer in making local improvements. The records, up to the close of the year 1876, have been received at the Office, and in the preliminary discussion there is promise of good results. Excepting for the record paper and blank forms furnished to the observer at Providence, from time to time, the series of observations at that port is maintained without cost to the government. In due time the records will prove valuable for investigating the tides of Narragansett Bay.

Triangulation in New Hampshire.—On the 13th of May Prof. E. T. Quimby again took the field, and occupied the remainder of that month in resetting the signals needful for the measurement of horizontal angles from the summit of Gunstock Mountain. Early in June the theodolite was in adjustment at that station, and as soon as practicable the measurement of horizontal angles was completed by observing on signals at stations Cardigan, Bean Hill, Prospect, Whiteface, Trask, Catamount, and Rattlesnake. Near the middle of the month the instruments were transferred to Gilford, and from that summit most of the signals above mentioned were also observed on, previous to the 20th of June, at which date field operations were closed for the month.

Professor Quimby will again take up the triangulation in July, and prosecute work as heretofore during the summer. Further mention of the progress made by his party will be made in my next annual report.

Triangulation of Boston Upper Harbor.—The legislature of Massachusetts having provided for a resurvey, the board of harbor commissioners of Boston made request for the detail of a Coast Survey observer to make the requisite triangulation for establishing points. Assistant Francis Blake, jr., was assigned for the service, and took the field in September, 1877.

In the coast triangulation of a former year, one of the stations occupied was in South Boston. Mr. Blake found the ground marks of that station, and established a second point on Powderhorn Hill, by observing with the theodolite from two of the early stations in the vicinity of Waltham.

After connecting the line from South Boston to Powderhorn with the general coast triangulation, other stations were chosen for the special survey, and were occupied in succession. Forty-one points in all were determined in position for the topographical work contemplated by the board of harbor commissioners. Eight other positions beyond the city limits were established and marked with granite posts. From these points future changes in the harbor lines can be readily traced for comparison with the results of the local survey of this year. At several of the stations occupied, Mr. Blake observed the direction of subsidiary objects, for the use of the city engineers.

The results of the work done by the triangulation party were checked by a base line measured by Mr. Edward S. Pilbrock, engineer in charge of the filling of part of South Boston Flats. Assistant Blake's determination of the length of the line, by computing through the triangulation, agrees within less than half an inch with the measurement of the engineer.

The statistics of the triangulation are:

Signals erected	23
Stations occupied	23
Points determined	49
Angles measured	225
Number of observations	5,868

After completing the computation of his field observations and furnishing the results to the harbor commissioners of Boston, Mr. Blake, under a necessity which then arose for close personal attention to private affairs, to my great regret resigned his place in the survey. He had rendered able and very acceptable service during a period of twelve years.

Light-houses determined in position.—With the requisite instruments, Assistant G. A. Fairfield reached Provincetown, Mass., on the 20th of August, 1877, and made arrangements for determining in position all the light-houses between Wellfleet and Marblehead. The adjacent stations of the primary triangulation were visited and identified, and for the five lights on Cape Cod Peninsula, angular measurements were subsequently made from Mill Hill, Scargo, Shootflying, and Manomet Hill.

At Plymouth, after determining in position the light-house, Mr. Fairfield measured horizontal angles for marking on charts the Standish Monument and the monument in the town, both of which serve as leading marks in approaching the harbor. Passing on to the vicinity of Cohasset, three stations were occupied with the theodolite for ascertaining the exact relative position of the light-house on Minot's Ledge. The marks at the station "*Nantasket*" which had been occupied in the survey of Boston Harbor, having been displaced, were renewed by Assistant Fairfield.

Adjacent to the six lights in the neighborhood of Marblehead three station points were identified, and were occupied for the measurement of horizontal angles by means of which five of the aids to navigation were determined. Marblehead light-house being visible from only one of the stations was of necessity occupied, and was fixed in position by the measurement of angles on a signal on Coddin's Hill and Baker's Island light-house.

Mr. Fairfield concluded field work on the 24th of November. The statistics are:

Signals erected	8
Stations occupied	14
Angles measured	53
Points determined	17
Number of observations	1,010

In the latter part of the fiscal year Assistant Fairfield was assigned to field service for the determination of points in the State of Illinois, as will be stated in further detail under Section XIV in this report.

SEA CURRENTS.

The general scope of the work intended off the coast of New England, and for which provision had been made in the construction of the schooner Drift, was mentioned in my last annual report. The work had then been commenced, and was in progress at the end of the fiscal year. Master Robert Platt, U. S. N., successfully recorded current observations at one station in June, 1877, the depth being sixty fathoms at his anchorage.

The second position chosen for the vessel was in thirty-five fathoms to the southward and westward of George's Shoal and Cultivator Shoal. Good observations were recorded, though the weather was bad, with high winds and sea and frequent fogs. At this station the flood-tide swell was particularly noticed, making up at each flood tide and running down on the ebb. Northeast of the station last mentioned, Master Platt found a spot on which the soundings gave only eight fathoms. The shoal spot was indicated by discolored water. Between George's Shoal and Cultivator Shoal the Drift was brought to anchor in sixteen fathoms, and current observations were recorded. The tidal swell was perceptible at this station, and at the strength of the flood and ebb tide the schooner was surrounded with strong and well-marked tide rips. These rips disappeared on the turning of the tide.

The fourth current station was occupied about fifteen miles to the eastward of George's Shoal. In good weather the schooner was anchored in thirty-two fathoms, and an excellent series of observations was recorded. While at this station a brig was noticed to be trying to work to the northward, but remained in sight during thirty-six hours, having unfortunately tacked again and again to be set back by the action of the tidal current. The direction and force of the current being known on board the Drift, by the observations then in progress, the schooner was moved without delay a distance of twenty-four miles to this station, although the wind was directly ahead.

Near the northeast edge of George's Bank current observations were recorded successfully, and also at a sixth station ten miles south of Brown Bank. The Drift was anchored in one hundred and thirty-five fathoms, and the movement of the water at the surface and as low down as seventy-five fathoms was noted, but after working sixteen hours with success the vessel was driven off by bad weather. Very heavy tide rips were crossed in standing to the westward, and gales were frequent in the latter part of July.

After needful repairs at Boston the Drift resumed work, and occupied a station in sixty-five fathoms off Cape Sable. The operations there gave good results, and the weather was favorable. The last position in which the vessel anchored was on the north side of Brown Bank in fifty-two fathoms.

A dense fog prevailed while the observations were in progress, and but little wind until the record was almost complete, when the work was closed by a sudden southeast gale, in which by the parting of her hawser the anchor of the Drift was lost.

In returning toward Boston, breakers were frequently noticed where the depth was found to be upwards of one hundred fathoms. These proved to be very heavy tide rips, and so prevail where they were observed as to warrant their insertion on charts of the approaches to Massachusetts Bay. Some of the rips are twenty and others as much as forty miles long and at least ten miles wide. Sometimes they resemble shoal water, or breakers ahead, and the report of Master Platt states that strangers coming on them without notice at night would be greatly perplexed. His own experience is thus expressed: "We drifted along with the current, where the depth was one hundred and seventeen fathoms, until we came into these breakers, and found only a very heavy tide rip; the sea so high and combing that we had to reduce all sail; three reef the mainsail and haul the boom well off to keep from rolling our mast out."

At intervals across the entrance of Massachusetts Bay five current stations were occupied between Cape Ann and Cape Cod, and three others to the eastward and along the south coast of Cape Cod. In six positions while the schooner was at anchor the force and direction of the currents were determined for a depth of thirty fathoms; at fifteen stations the observations were made for a depth of ten fathoms, and at nine stations the surface currents were determined. Temperature observations were recorded at the same time, and the density of the water was tested by about three hundred observations. The general statistics of the work are:

Sextant observations for position	344
Temperature observations	843
Current observations	7,032

This work was concluded late in October. The vessel then passed southward by way of New York, and reached Norfolk on the 6th of November, and was laid up during the winter. Master Platt was assisted in the work in this section by Ensign J. P. Underwood, U. S. N.

From the observations mentioned under this head, Assistant Henry Mitchell, under whose immediate direction the details of the work were prosecuted, deduced general rules for the guidance of mariners in crossing the Gulf of Maine. These rules will enable navigators who observe them to avoid George's Shoal, and will greatly assist them in beating against a head wind between Cape Cod and Cape Sable. The rules have been issued as a printed notice to mariners, together with a sketch showing the positions of the extended tide rips to which reference has already been made.

SECTION II.

CONNECTICUT, NEW YORK, NEW JERSEY, PENNSYLVANIA, AND DELAWARE.—(SKETCHES NOS. 4 AND 5.)

Survey of Duck Island Harbor, Conn.—This survey was made by Subassistant Joseph Hergesheimer, who measured a suitable base line on the beach and traced the shore lines in the latter part of June, 1877. The soundings were made in the following month, and show the character of bottom and depths in the harbor between Menunketesuck Point and Hammond Point. The statistics are:

Shore line traced, miles	12
Low-water line, miles	8
Roads, miles	5
Miles run in sounding	93
Angles measured	1,203
Number of soundings	8,722

Under the head of Sections VI and VII further mention will be made of field work executed by Mr. Hergesheimer.

Topography near New Haven, Conn.—In order to close with proper limits the plane-table work near New Haven, Assistant R. M. Bache took the field in the summer of 1877 and extended the detailed survey north and west of the points which had been previously occupied. To the usual

variety of detail found adjacent to the city the sheet of this year adds features much more rugged than any represented by either of the preceding sheets. The statistics are:

Shore line of streams, miles	14
Roads, miles	23
Area of topography, square miles	5

The subsequent field work of Assistant Bache will be mentioned under the head of Section VI in this report.

Survey of Rockaway Inlet and vicinity, N. Y.—A resurvey being desirable to determine the nature and extent of changes in shore line at Rockaway Beach, on the south shore of Long Island, Assistant J. W. Donn was detailed for service and took the field early in July, 1877. Along the beach all the triangulation points used for the early survey had been washed away. From points to the eastward of the inlet, determined in 1875, triangulation was therefore extended by Mr. Donn to include the western side of Jamaica Bay and the ground between the bay and Coney Island.

As soon as practicable the shore lines were traced by means of the plane table. Assistant Donn, after comparing his results with the outline shown by the earlier survey, thus remarks in his report from the field: "In the bay marshes only slight changes were observable, but on Rockaway Beach and at the south end of Barren Island and the entrance of Dead Horse Inlet there was only a slight resemblance to the outlines shown by the original work. Great changes are noticeable after every heavy easterly storm. The extension of Rockaway Beach to the westward is at present rapid and unceasing. A reconnaissance of the whole beach from the inlet to Far Rockaway showed no encroachment of the sea that might, by effecting an opening to the eastward, check the westerly growth. Eastward of Far Rockaway, at the distance of two miles, an inlet has been formed, which is rapidly enlarging, and forms an additional outlet for the waters in the western extremity of Great South Bay. The whole configuration of the shore in the vicinity of Far Rockaway appears greatly changed from that shown by the early survey. West of Rockaway Inlet remarkable changes have occurred in the line of the islands and bays, the alterations extending up to Gravesend Bay. From Dead Horse Inlet an unbroken beach extends to the western extremity of Coney Island, and the whole shore has been thrown back towards the main land half the width of Sheepshead Bay, which must soon be obliterated if the extension of Rockaway Beach continues."

The statistics of the field work are:

Stations occupied	16
Horizontal angular measurements	1,037
Marsh line surveyed, miles	40
Shore line, miles	12
Area of topography, square miles	15

Field work by the party of Assistant Donn was completed on the 18th of October. Further operations by the same party in this quarter will be mentioned presently.

Within the shore lines traced by Assistant Donn the hydrography was carefully executed by Lieut. Washburn Maynard, U. S. N., Assistant Coast and Geodetic Survey, in August, September, and October, 1877. The work includes Rockaway Inlet and its approach from the southward, and the western part of Jamaica Bay to a short distance above Canarsie. Big Fishkill channel was taken as the limit of soundings to the eastward. The several channels between it and the western shore of the bay were also developed, and are shown on the chart which was issued by photolithography as soon as practicable after the completion of the field work.

In the hydrographic survey, Lieutenant Maynard was assisted by Masters W. F. Low and S. H. May. The work was done in the steamer Fathomer. In my next annual report mention will be made of the progress of the survey to the eastward of Rockaway Inlet. The statistics of the present season are:

Miles run in sounding	244
Angles measured	4,576
Number of soundings	36,604

The work done by the party of Lieutenant Maynard is comprised in two hydrographic sheet

Survey of Coney Island, N. Y.—In the field operations of last year the triangulation of the south coast of Long Island had been brought as far eastward as Manhattan Beach. One of the stations was at the east end of Coney Island, and another was occupied in the village of Gravesend. For continuing work in the present season, Assistant Donn found and reoccupied two of the stations, and from them observed with the theodolite on a number of cupolas along the line of Coney Island. Three additional stations were occupied, including Fort Tompkins, and from the angular measurements points were provided for the plane-table survey. The topography was commenced on the 1st of May of the present year. Interruptions were frequent by wind, rain, and fog.

Each feature, natural or artificial, is shown in its proper position on the plane-table sheet of Mr. Donn, and for this part of the coast of Long Island the sheet is an excellent standard for comparison with future surveys. Of the changes now going on in the vicinity, he remarks: "Sheepshead Bay and the creek connecting it with Gravesend Bay are rapidly filling up, and it is altogether probable that Coney Island will be an island only in name at no distant day."

Field work in the vicinity of Coney Island was completed early in June. The following are statistics:

Coast and bay shore surveyed, miles	12
Creeks, miles	10
Roads, miles	18

During the summer, Assistant Donn was engaged in Jamaica Bay in the prosecution of a survey which was commenced in the summer of 1877. Of this work mention will be made in my next report.

Pendulum experiments.—After careful preparation, Assistant C. S. Peirce arranged the pendulum, which he had previously used under other conditions, to swing in vacuo, and during the month of September, 1877, it was swung at various pressures with the heavy end down. For these operations time was determined before and after experiments with the pendulum.

While abroad as a delegate to the International Geodetic Association, Mr. Peirce at intervals made comparisons of the length of our pendulum standard with that of the Prussian Geodetical Institute.

In November tests were made of certain scales used for the measurement to hundredths of a second of the traces on chronograph sheets, the aim being to measure the chronograph record without estimations.

The pendulum was swung in December with the heavy end up, and in the course of the winter Assistant Peirce compared all the micrometers which he had used in previous experiments. Subsequently an elaborate series of measures was made for comparing our pendulum meter with a German meter.

In April and May of the present year the pendulum was swung at New York in air of which the temperature was about 100 degrees Fahrenheit.

Among other operations conducted this season were experiments for the coefficients of expansion of two meter bars and micrometer tests with reference to the spectrum meter. Additional observations were made to compare the statical and dynamical flexures of the stand which supports the pendulum. The records of the various experiments mentioned in this abstract are contained in twenty-six volumes.

Tidal observations.—At Governor's Island, in New York Harbor, the series of observations with a self-registering tide gauge has been kept up by Mr. R. T. Bassett. In freezing weather the difficulty of maintaining a continuous record has been met by applying heated water to keep the working parts of the apparatus free from ice. A nineteen-year cycle, including continuous observations during winter, will soon be completed at this station. The record, as heretofore, is frequently resorted to by local engineers for adjusting the levels proper for bridges, wharves, sewers, dikes, and harbor improvements generally. For comparison with the results at Governor's Island the same observer occasionally records day observations at Hamilton Avenue Ferry wharf in Brooklyn.

In October, 1875, a well-furnished self-registering tide gauge was established at Sandy Hook on one of the wharves of the New Jersey Southern Railroad, and was put in charge of Mr. J. W. Banford, who, having charge of the depot and the direction of the laborers employed about the

place, commands all the facilities requisite for keeping the apparatus in good working order. The chief officers of the railroad company also have been very obliging, and by their favor the operations have been successfully maintained. Sometimes the water is quite rough, but good results have been secured by the selection of a screened position for the tide gauge and by carefully attending to its condition. The movable parts of the apparatus wear rapidly at this station, but it is doubtless the only point at which, without great expense, it would be practicable to record the tides in the vicinity of Sandy Hook. The registers show that the rise and fall are largely affected by winds, especially when great storms are prevailing at sea off the coast of New Jersey and Long Island.

Topography of Hudson River, N. Y.—The party of Assistant H. L. Whiting was organized on the 13th of July and was at work in the field until the 21st of November, 1877. In that interval the detailed survey of the eastern side of Hudson River was advanced from Croton Landing upwards to Peekskill. The resulting map represents about nine miles of the river side as measured by its general course, with the usual margin of characteristic topography, the numerous intersecting roads, and the surface features of the ground above and below Verplanck's Point. Between the limits of the work of this season the scenery on the west side of the river is among the most striking that the topographer is called upon to delineate. Above the somewhat abrupt terminations of the highlands the river valley again assumes the softer features which are seen about Newburg and Fishkill. Mr. W. C. Hodgkins aided in the field work on the Hudson. The statistics are:

Shore line surveyed, miles	26
Roads, miles	59
Creeks and ponds, miles	26
Area of topography, square miles	8½

On the sheet of this work now in the office the topographical details are elaborated with great care.

In the latter part of the fiscal year the survey of the banks of the river was resumed by this party.

Hudson River levels.—The destruction of the initial bench-mark established near Albany, in 1858, in connection with a line of levels for referring the tide planes of the river at Albany to those of New York Harbor, made it necessary to refer the Albany station to a bench-mark below, the identity of which was not in doubt. This involved the necessity of running a line of levels from the miter sill of canal lock No. 1, at Albany, down the river to a bench-mark established in 1858 at Stuyvesant. The distance is about twenty miles.

The leveling was intrusted to Assistant O. H. Tittmann, who began operations on the 15th of December, 1877, with instruments of greater nicety of construction than have been usually employed for such purposes. Subassistant Andrew Braid and Mr. J. B. Baylor assisted in the work. To insure accuracy, two lines were run, and both were closed on the bench-mark at Stuyvesant on the 23d of December.

In the work here noticed, Lieut. J. H. Willard, U. S. Corps of Engineers, effectively co-operated with and assisted the party of Assistant Tittmann.

Primary triangulation.—For connecting the triangulation of the valley of the Hudson with the primary work of the coast, two additional stations have been occupied by Assistant Richard D. Cutts. Early in June, 1877, the instruments which had been used in the preceding season were transferred from South Adams, and were set up on the summit of Mount Tom, in Massachusetts. Heliotropes were at the same time stationed on Greylock Mountain, and on Mount Monadnock. By the 5th of July the angular measurements needful at Mount Tom were completed, and as soon as practicable the party and instruments were in readiness for observing from the summit of Mount Equinox, in Vermont. That station is thirty-eight hundred and forty-three feet above mean tide. In consequence of the decreased temperature, resulting from its height and latitude, the summit, when occupied, was enveloped in fog for nearly half the number of days when the air was clear a thousand feet below the summit. Assistant Cutts, however, succeeded in observing on the heliotropes which he had previously stationed on five outlying primary stations. Operations

on Mount Equinox were completed by the 5th of September. The camp equipage was then stored at Factory Point. Accompanied by Mr. C. H. Sinclair as aid, Assistant Cutts next visited Mount Washington, to ascertain what point on that summit had been used as a signal while angular measurements were in progress in the triangulation of New Hampshire. As the summit had not been occupied with the theodolite by any observer of the Coast Survey, the question at issue was to determine whether the point observed on in 1851 and 1853, from distant stations to the eastward, was identical with the signal used in the operations of 1876. After careful examination and due measurements with the theodolite, Assistant Cutts found that the object observed on in the last-mentioned year was the northeastern chimney of the new hotel, and that the signal formerly observed on, the place of which is marked by a copper bolt, was fifty-four yards west by south from the chimney. Exclusive of measurements on the summit of Mount Washington, the statistics of the triangulation work are:

Stations occupied	2
Angles measured	9
Number of observations	620

By an aggregate of two hundred observations the vertical heights of eight stations were determined in the course of the season by angular measurements.

Pennsylvania and New York boundary line.—A joint commission of three members from each of the two States met at New York City on the 29th of May, 1877, to make arrangements for correcting and finally marking the boundary line, which, to be exact, should, at all points between the Delaware River and a point near Lake Erie, correspond with the 42d parallel of north latitude. At the conference it was agreed that the line should be traced out by a party of surveyors in the employ of the joint commission, and that request should be made for the services of a Coast Survey observer to determine the latitude at each of four points, the extremes being two hundred and fifty-three miles apart. Subassistant Edwin Smith was detailed for the work desired, and, as previously arranged, he reported in person to the joint commission on the 10th of July, at Hale's Eddy, which is very near the eastern end of the boundary line. Mr. J. B. Baylor, of the Coast Survey, accompanied Mr. Smith to aid in the observations for latitude, which were to be made only at points specified by the commission.

At the eastern end the initial monuments set by the surveyors in 1786 were carefully sought for, but without success. The commissioners therefore went some miles westward, traced back to Hale's Eddy, from known points of the old line, and erected a monument, the geographical position of which in latitude and longitude was determined by Subassistant Smith and Mr. Baylor by observations continued through ten days and nights. The subsequent computations for the latitude of that monument place it two hundred and seventy-four feet north of the 42d parallel. Similar observations for latitude were made at Firm's station, near Great Bend, about twenty miles west of the eastern end of the line. The results for latitude show that the old boundary there passes three hundred and forty-three feet south of the 42d parallel. At this station the marked stone was found where it had been placed in 1786.

Fifty miles farther westward Burt station was occupied near Wellsburg. Latitude was determined at a position indicated by the commission as on the old boundary line, and the point occupied by the instrument was found, by subsequent computation, to be seven hundred and ninety-one feet north of the 42d parallel.

The fourth station (Clark's) is within a mile of the western end of the boundary line. Observations for latitude were made in the usual way, and from the computations it appeared that the boundary, as traced there in 1786, is only one hundred and forty-nine feet north of the 42d parallel. The near agreement between determinations made with the best instruments in the summer and autumn of 1877, and results found in 1786 with instruments less accurate in construction, is remarkable.

The four stations occupied by Subassistant Smith were all well marked by brick piers, capped with stones that may be easily lettered. At each of the piers a meridian line was established.

In the interval of two months preceding the 10th of September, when the field work was closed, observations had been successfully recorded on twenty-five nights. The records of the four

stations contain two hundred and twenty-three observations on sixty-seven pairs of stars for latitude. Time was determined on seven nights by fifty observations on twenty-three stars; and observations were made in the usual way on Polaris for value of the micrometer.

At Travis, Burt, and Firm the longitude was determined by exchanging signals with the Naval Observatory at Washington. Due acknowledgment of the service rendered for the purposes of the joint boundary commission was made in the report of the chairman, Col. James Worrall.

Geodetic operations in New Jersey.—At the outset of the fiscal year Prof. Edward A. Bowser was yet engaged at Mount Rose in the measurement of horizontal angles with the theodolite. After completing the record at that station the instrument was transferred to Pickles station, where the measurements were resumed, and at intervals continued until the close of the calendar year.

Signals were observed on at Mount Horeb, Goat Hill, Haycock, Montana, and Mount Olive; and observations were at the same time recorded for determining the positions of subsidiary stations. Field work was discontinued during the latter part of the winter, but was resumed in April, and was prosecuted at intervals until the close of the fiscal year. The statistics will be given in my next annual report.

Much of the work conducted this season by Professor Bowser involved the erection of signals and their adjustment, and the posting of heliotropes at stations too distant to admit of the use of ordinary signal poles.

Geodetic survey.—In previous seasons the scheme for triangulation in Eastern Pennsylvania was brought into connection with stations of the primary work north of Philadelphia. To the southward and westward of that city reconnaissance had been made with a view of joining also with stations of the primary triangulation which crosses the head of Chesapeake Bay, but for the present season means were not available for resuming field operations until near the close of the fiscal year.

Prof. L. M. Haupt, of the University of Pennsylvania, reorganized his party early in June. After due arrangements a station was selected in Chester County, and by the use of night signals directions were obtained for clearing lines of sight to Principio station, near the head of Chesapeake Bay, and to Meeting House Hill, a point to the westward of Wilmington, in Delaware. At the first-named station a structure sixty feet high was found requisite for the measurement of horizontal angles. That station and those adjacent to it will be occupied in the course of the summer. At the close of the fiscal year the party was engaged at a station on the south side of the Lehigh, where the needful angular measurements were completed for joining the northern part of the scheme with the triangulation of New Jersey.

Latitude and longitude of Harrisburg, Pa.—After completing work which has been noticed under a preceding head, and before returning to the office, Subassistant Smith stopped at Harrisburg, under directions to make observations there for latitude and longitude. Colonel Worrall immediately conferred with the governor of Pennsylvania and other officials at the State capitol, and a point for observations was selected near the southeastern gate of the capitol grounds.

Mr. Smith, aided by Mr. Baylor, commenced observations for latitude on the 14th of September, and in the course of six nights seventy-two entries were made in the record on sixteen pairs of stars. On five nights sixty-four observations were made on thirty-three pairs of stars for chronometer correction, and on each night telegraphic signals were exchanged with the Naval Observatory at Washington to determine the difference of longitude between that observatory and the point of observation in the capitol grounds. At the same place observations were made on separate days by Mr. Baylor for magnetic declination and intensity.

The point at which observations were made for latitude and longitude was marked by a fine sandstone block, and another was set as a meridian mark two hundred and fifty-six feet north of the astronomical station. Determinations for geographical position were referred to the capitol dome by measuring a base line with steel tape and the requisite angles with a theodolite. Mr. Smith completed the computations at Harrisburg, and returned to the office on the 1st of October to engage in field work in another section.

Special survey of Philadelphia Harbor.—With means provided by the city authorities this work was commenced in the latter part of October, 1877, by Assistant S. C. McCorkle. In the course of a fortnight a scheme for triangulation was developed, signals were erected without delay, and

observations with the theodolite were begun at Girard College on the 16th of November. From this station, lines to the Commissioner's Hall and the United States arsenal had been previously determined, and upon them the work was based by the introduction of other points. These stations were occupied in succession for the measurement of horizontal angles, and the weather being favorable, field work was prosecuted until the end of the year. Office work was prosecuted during unfavorable weather, the party being in that interval disbanded.

In March Mr. McCorkle again took the field, and continued the determination of points until the 22d of June. The provision made by his work for the special survey includes the following statistics:

Signals erected	35
Stations occupied	35
Positions determined	90
Angles measured	603
Number of observations	8, 010

The stations were marked by sections of iron pipe upward of two feet long and four and a half inches in diameter. After the pipe was firmly set upright the signal pole was inserted, and on that the angular measurements were made. In the course of the summer the wharf lines of the city will be mapped on a large scale by Assistant R. M. Bache, his survey being based on points determined by the triangulation. Further notice of that work will find place in my next annual report, to which it properly belongs. The triangulation includes points on both sides of the Delaware at Philadelphia, and also stations on Windmill Island and Petty's Island in the river.

The physical survey needful at Philadelphia was placed in the charge of Assistant Henry Mitchell, and under his immediate direction the requisite observations were made by Assistant H. L. Marindin. These were so arranged as to develop particular information in regard to the tides and currents of the river, especially for the western channel between Five Mile Point and the south end of Petty's Island, and until the end of December, 1877, the operations of this party were confined within the limits just named. Six tide-gauges were erected and an aggregate of upwards of four thousand observations was recorded. Twelve bench-marks were made, and these were referred to the tide-stations by levelling. At five stations across the river transverse velocity-curves were determined, and eight hundred and two observations on the currents were recorded. On the same sections one thousand six hundred and twenty soundings were made.

In work connected with the physical survey Assistant Marindin occupied eighteen stations with the theodolite and erected as many range marks. For ascertaining the position of current floats six hundred and forty-four angles were measured and three hundred and thirty-nine others for the location of eight measured base lines, which were used in deducing transverse curves of current velocities. From the observations here recapitulated, result twenty-seven transverse curves and nine vertical curves, illustrating the velocity of the water at and below the surface in the section of the Delaware included in the survey. The work was resumed in May last, and was yet in progress at the end of the fiscal year when this report was closed.

Light-houses, Delaware Bay.—For determining the exact positions of the light-houses in Delaware Bay, Assistant J. A. Sullivan reached Cape May in the middle of September, 1877, and soon identified the station "Higbee" by the ground marks which had been placed when that point was occupied in the triangulation of the coast some years ago. The tower at Cape Henlopen having been occupied with the theodolite at the same period, Mr. Sullivan adopted as a base for his work the line which crosses the entrance of the bay.

The observations made at "Higbee," Cape Henlopen light-house, and other stations, previous to the end of December, afford data for the positions of the lights at Hereford Inlet, Cape May, Maurice River, Egg Island, Cross Ledge, Brandywine Shoal, and Mispillion Creek. The distance and direction from the old light-house of the new light-house at Cape May were determined. Cape Henlopen beacon-light and Delaware Breakwater light-house were observed from Cape May and Cape Henlopen. The new stations "Week's Landing" and "Two Mile Beach" were determined in order to check the position of Hereford Inlet light-house as derived from observations made there and at Cape May. While making this test the opportunity was taken to determine the positions

of spires in Cape May City and at Sea Grove and the life-saving station-houses No. 36 to No. 40, inclusive.

Between Hereford Inlet and Cape May the additional points established by the field work of this season give ready means for revising the shore-line survey of that quarter, which as elsewhere on the coast is subject to change by the action of the sea.

While determining angles for computing the positions of the light-houses mentioned under this head, Mr. Sullivan recorded eleven hundred and forty-eight observations with the theodolite. In the summer of the present year he was occupied in the determination of points west of the Mississippi, of which further mention will be made in my next annual report.

SECTION III.

MARYLAND, VIRGINIA, AND WEST VIRGINIA.—(SKETCHES NOS. 6 AND 8.)

Topography eastward of Norfolk, Va.—In continuation of the survey in this quarter, Assistant C. M. Bache resumed work on the 21st of November, 1877, and prosecuted the detailed plane-table survey until the 1st of June of the present year. After joining properly with the work of the previous season, Assistant Bache mapped the ground intervening between it and Lynnhaven Bay, and also extended the detailed survey towards the coast in the vicinity of Cape Henry. Care was taken to connect properly with the several topographical sheets made of the adjacent features in former years. The statistics are:

Shore line surveyed, miles	60
Roads, miles	145
Area of topography, square miles	35

At the end of the fiscal year Assistant Bache made preparation for resuming plane-table work in Section II.

Tidal observations.—By the voluntary service of Mr. W. J. Bodell, who has other employment at Fortress Monroe, the series of tidal observations has been continued through the year at that station. The instrument used combines the latest improvements, and the clock is furnished with a balance and lever escapement, that form having been found less liable to derangement from occasional jarring at the wharf than the pendulum clock which was at first used.

The float boxes and other wooden fixtures at this station were rapidly destroyed by worms, and when copper or zinc was substituted the corrosion was such as to render their use expensive and inconvenient. During the present year tubes of wrought iron covered with enamel have been on trial at this station.

Special observations in Chesapeake Bay.—During the months of August, September, and October, 1877, Lieut. Frederick Collins, U. S. N., Assistant Coast and Geodetic Survey, with the schooner Palinurus, was engaged in a series of observations on the density of the waters of Chesapeake Bay and its more important estuaries. Twenty-six sections were made between the head of the bay and the capes, on which an aggregate of one hundred and sixty-four stations were occupied. At each station specimens of the water were obtained at intervals of two fathoms, from the surface downward to the bottom. These were tested with delicate hydrometers for density, and the same specimens were sent to the office for chemical analysis. In all five hundred and seventy specimens were thus obtained and tested.

This interesting work is the initial step towards the sytematic examination of the waters of all the estuaries that penetrate the coast of the Atlantic within the limits of the United States. The service of the party in the Palinurus has been of a preliminary nature, as it was necessary to devise apparatus suited for the purpose, and to determine practically the best methods for carrying on the work. The report of Lieutenant Collins (see Appendix No. 14, Coast Survey Report for 1877) gives full information concerning the apparatus, which was devised by Prof. J. E. Hilgard, for obtaining specimens of water, and describes the methods of conducting the experiments. It is intended hereafter to connect observations of temperature and currents with determinations of density, so that full data may be at hand for the solution of all questions that may arise when a study of the results is undertaken.

The work of the season is satisfactory, as showing that the data desired can be obtained readily.

Lieutenant Collins was ably assisted in the operations of the season by Master Francis Winslow, U. S. N. Under other heads mention will be made of the subsequent work of the party in the Palinurus.

Potomac River freshet.—In order to place on record for convenient reference or future comparisons the condition of the Potomac in the vicinity of Washington during the great freshet of November 25 and 26, 1877, Mr. Charles Junken, of the Coast and Geodetic Survey Office, was detailed to determine the height of the flood above the plane of mean high tide at points above and below Georgetown, D. C. At the Chain Bridge the water reached fully up to the level of the top of the old piers, and Mr. Junken found that the rise amounted to thirty-six feet above the level of mean high tide. At Lockmills the rise was twenty-seven feet; at the Aqueduct, thirteen feet nine inches; at the Philadelphia Steamboat Company's wharf, Georgetown, the rise was ten feet seven inches; at the Long Bridge it was six feet two inches; at the southwest corner of the Arsenal wharf the rise was five feet six inches; and at Alexandria the water rose three feet above the level of mean high tide.

During a storm which raged in Chesapeake Bay in September, 1876, the water at Alexandria rose four feet two inches above mean high-tide level. This, the greatest rise known at that port, was due to the accumulation of water in the estuary of the Potomac when there was no freshet in the river.

Magnetic observations.—At the magnetic station on Capitol Hill, Washington, D. C., observations for declination, dip, and intensity were recorded by Assistant Charles A. Schott on the 14th, 15th, and 17th of June, 1878. Similar observations have been recorded on Capitol Hill in June of each year since 1856.

The results found this year confirm what had been noticed in regard to the great annual change in declination at Washington and at places along the Atlantic coast. During the same period the change in the magnetic dip and intensity has been small.

Lines of level.—Arrangements have been completed for running with the utmost precision a line of levels from the Atlantic coast westward, to follow as nearly as practicable along the thirty-ninth parallel of latitude. Instruments for the purpose were devised in the office by Assistant Hilgard, and under his direction several observers have been trained for field work. Of these Subassistant Edwin Smith was detailed to prosecute work on the line here under notice. In the middle of October, 1877, he started on a reconnaissance from Hagerstown, Md., but finding the line of the National Road not well suited for the intended work, ultimately decided that the line should be run on the tow-path of the Chesapeake and Ohio Canal, which for that purpose he examined as far west as Cumberland.

From a bench-mark at Hagerstown the work was started on the 22d of October towards Williamsport, Md., following of necessity the turnpike which runs between those towns. The country passed over by the levelling party is hilly, and owing to bad weather the bench-mark at the last-named town was not reached until the 9th of November, but under more favorable conditions the work was advanced to Cumberland in the course of the next twelve days. On the 22d of that month the party was transferred to Hancock, but several days of rain followed, and caused the almost unprecedented freshet which has been mentioned under a preceding head. The destruction of property was great in the vicinity of Hancock, and the damage to the canal was such as to make it impracticable to keep on with the level operations. Mr. Smith, in consequence, disbanded his party, and returned to the office on the 1st of December, 1877. The distance run, as shown by records of the party, in levelling in both directions was upwards of twenty-three miles. Throughout the section, which is about ninety miles between Hagerstown and Cumberland, it is proposed to establish primary bench-marks at distances averaging six miles. Five such benches were established by Subassistant Smith, the first being on the foundation of the court-house in Hagerstown. The second mark is on the aqueduct of the Chesapeake and Ohio Canal over Conococheague Creek; the third, on the abutment of dam No. 5, in Potomac River; the fourth, on the sixth lock above dam No. 5; and the fifth, on the aqueduct over Licking Creek.

In May of the present year Subassistant Andrew Braid was detailed to continue work with the level in going westward toward Cumberland. His operations, extending through the summer and beyond the close of the fiscal year, will be stated at length in my next annual report.

Primary triangulation.—Near the close of the last fiscal year Assistant A. T. Mosman was at work with his party on Cahas Mountain, about fifty miles southwest of Lynchburg, Va. The measurements needful at that station were completed on the 18th of July, 1877. As usual, subsidiary objects were observed on in addition to the signals at primary points. The height of the station on Cahas Mountain was found to be thirty-five hundred and seventy feet. After carefully marking the point which had been occupied by the theodolite, Mr. Mosman transferred the party to Smith's Mountain, on the south side of Staunton River, where observations were commenced on the 6th of August. The weather following that date was rainy or foggy for weeks at a time, with rare intervals in which the signals could be seen through a hazy atmosphere. In consequence, the requisite observations at this station could not be completed until the middle of September. The scheme of field work, including also subsidiary points, is such that thirteen triangles converge at Smith's Mountain, and of these the third angle in each was determined by the observations. Assistant Mosman had previously occupied the southern stations of the quadrilateral, which was completed by the operations at Cahas and Smith's Mountain. Further on in the direction towards the Atlanta base, work of the present year, which will be referred to in detail presently, closes the chain of quadrilaterals that passes southward and westward from stations on the Potomac River, near Washington, D. C. After completing the observations at Smith's Mountain, Assistant Mosman stored his camp fixtures at Afton, Va., to be in preparation for developing a chain of quadrilaterals westward from the line joining Humpback and Fork stations. His party closed operations at Smith's Mountain on the 21st of September. The following is a summary of statistics:

Primary stations occupied	2
Angles measured	13
Number of observations	884

For ascertaining the value of the level and of the micrometer of the twenty-inch theodolite No. 114, which was used in the work of this season, the usual observations were made.

Mr. W. B. Fairfield aided in the field and in computing from the records of the triangulation.

In October and November Mr. Mosman visited twelve primary stations in Virginia and West Virginia, going westward from Staunton, and noted in the vicinity of each the requisites for transportation and means of access to the points which are yet to be occupied. His observations on that reconnaissance include preliminary measures of the angles from a series of intervisible stations, the most westward of which are within forty miles of the Ohio River. In the latter part of April, 1878, the party was again organized for occupying station Humpback. Signals were erected for starting westward with a chain of quadrilaterals from a line of the series which has been completed along the Blue Ridge. Humpback Mountain was occupied in May, and for a few days the prospect was fair for securing horizontal measurements at another primary station previous to the close of the fiscal year. But snow fell on three days of that month, and this period was followed by ten days, during which the sun could not be seen. The observations needful at Humpback were, however, completed on the 12th of June. Assistant Mosman immediately after transferred his party to Elliott's Knob, which has an elevation of forty-four hundred and fifty feet above the sea, and at the end of June the theodolite and astronomical instruments were in position and in complete readiness for commencing observations.

SECTION IV.

NORTH CAROLINA.—(SKETCHES NOS. 7, 8, AND 9.)

Coast Pilot.—Early in June, and after the completion of similar duty off the coast of Florida, as will be stated presently under the head of Section VI, Lieut. Frederick Collins, U. S. N., Assistant Coast and Geodetic Survey, with his party in the schooner Palinurus, commenced an examination of the entrances, inlets, coast features, and dangers to navigation between Cape Henry and Cape Fear. Each place was visited in succession, and notes were recorded for use in the compilation of the Coast Pilot of this section. In addition, views were drawn by Mr. J. R. Barker, of the following:

—of Cape Fear River and entrances (four views); of Beaufort, N. C., and the entrance to Core Sound (two views); of Morehead City and the entrance to Bogue Sound; of Cape Lookout and of Ocracoke Inlet (two views each); of Hatteras Inlet; of Cape Hatteras (three views); of Bodie's Island Light-House; of Nag's Head, and of Currituck Beach and Light.

Lieutenant Collins was assisted in this work by Masters Francis Winslow and H. H. Barroll, U. S. N., and also in the previous service of the party, of which notice will be made under the head of Section VI.

After completing the examinations on the coast of Virginia and North Carolina, the Palinurus sailed for Norfolk, and reached that port on the 25th of June.

Life-saving stations.—In previous years the life-saving stations of the coast of New England and of the Middle States have been determined in position, and marked in their proper places on the charts. The field work pertaining to this service has been performed by Assistant F. H. Gerdes, who resumed operations at Cape Henry in the summer of 1877. Preliminary information in regard to the stations, and means of passing from one to another, was furnished as heretofore by Captain Merryman. Above Hatteras, Mr. Gerdes had transportation in the United States revenue sloop Saville, and was otherwise aided in the work by Lieut. Walter Walton. By continuous storms in September, the party in that vessel was detained seven days in a small creek on Roanoke Island. Above Currituck Light-House, Assistant Gerdes reached the several stations in succession by passing along the beach in carts. Owing to restriction in the means allotted for the operations of the year, the work of this party was closed above Hatteras Inlet, but as soon as practicable the measurements will be resumed at Chickamicomico. As heretofore, the positions of the life-saving stations, as they have been from time to time ascertained, are marked on the original topographical sheets.

Assistant Gerdes was temporarily aided in the field by Mr. L. C. Kerr.

Topography of Cape Fear River, N. C.—The topographical survey of the banks of the Cape Fear River was resumed by Assistant C. T. Iardella, in the middle of January of the present year. Previous work had brought the survey southward to Hawkins Point and Saunders Point, and from those limits the plane-table work of this season was extended along both shores in the direction of the river entrance. On the western side the detailed survey was continued back to the main road, which, in some places, is upwards of four miles from the river shore. All the details between the east bank and Masonborough and Myrtle Sounds were mapped. To the southward the field work done by Assistant Iardella includes the Cape Fear entrances and Smith's Island. All the branches of the river were followed a few miles from the main stream, and their courses are shown on the plane-table sheet.

The outline of Snow's Marshes, as shown by this survey, reveals considerable alterations, due probably to the effect of the sea at New Inlet, where the stone dike, which was designed to close that entrance, had sunk in the quicksand. The marshes, in consequence, became exposed to the action of the tide and the swell of the ocean.

As reported by Mr. Iardella, Zeek's Island has increased to the southward and eastward. The strip of beach, on which a base line was measured some years ago for the triangulation of Cape Fear River, has been nearly washed away. At the north end of the base, the shore line is now two hundred meters west of the point occupied, and that point is entirely covered by water. As might have been expected, changes equally marked were found in tracing the shores of Smith's Island. The work was concluded at the end of June. A synopsis from the field report shows in statistics:

Signals erected	38
Shore line surveyed, miles	30
Roads, miles	167
Marsh and streams, &c., miles	178
Area, in square miles	102

Triangulation in North Carolina.—Early in July, 1877, the party of Assistant C. O. Boutelle was transferred from King's Mountain to Benn's Mountain, in Burke County, North Carolina.

That primary station is twenty-nine hundred and thirteen feet above the sea level. It lies at the southeastern extremity of the South Mountain Range and commands the horizon in every direction. The Black Mountains, about thirty-five miles distant, are the highest summits of the Appalachian system, the most elevated being sixty-seven hundred feet above the sea.

At Benn's Mountain the instruments and camp fixtures were taken to a position below the observing station, and the summit was reached by constructing a narrow-gauge road, over which the baggage was transported in a mountain cart. Observations were commenced on the 20th of July, and were continued until the 25th of August. Six primary and three secondary signals were observed on in the usual manner. The nearest signal was at a distance of upwards of thirty-one miles; the most distant being sixty miles from the observing station. Sixty-seven prominent objects, mostly mountain peaks, were observed for position and elevation. Prof. W. C. Kerr, State geologist of North Carolina, visited the party on Benn's Mountain, and assisted Mr. Boutelle in identifying the mountains which were observed upon as subsidiary stations.

Mr. C. A. Ives joined the party at this station and served as aid until the close of the season.

Early in September Assistant Boutelle moved the party to Poore's Mountain, in Wilkes County, North Carolina. At this station, which is twenty-six hundred and eighty-five feet above the sea, the triangulation from the Atlanta base line meets that which had been extended in an opposite direction from the Kent Island base in Chesapeake Bay. The work done at Poore's Mountain closes a chain of continuous quadrilaterals that stretch about six hundred miles between the two bases.

As usual in autumn in this region, observations at Poore's were delayed by smoke; but the requisite measurements were concluded by the 12th of October. Five primary and two secondary stations were observed in series as usual with the twenty-inch theodolite. The most distant signal observed on was sixty-five miles distant from the summit of Poore's Mountain. Exclusive of stations at which signals were placed, seventy-nine prominent objects were observed for position and height. These were chiefly mountain peaks, in the identification of which Professor Kerr assisted, as he had done at Benn's Mountain. Many had been observed on from the station last named, and thus they were well determined in position.

At Poore's Mountain experiments were made with magnesium lights as signals for the measurements of horizontal angles at night. Two of the lights were shown from stations, one of which was thirty-five miles and the other fifty-eight miles distant. Observations upon them were recorded as in the day-time, during four hours on the night of October 9, and for equal periods on the two nights following. The magnesium tape was delivered at the rate of fifteen inches per minute, and burned in the focus of an eight-inch paraboloid reflector. Both of the night signals were visible to the naked eye of the observer on Poore's Mountain throughout the night, showing as stars of the first magnitude.

After completing the needful observations, Mr. Boutelle discharged his party and stored the instruments at Statesville, N. C., to await the arrival of the season for resuming field work, of which mention will be made in this report under the head of Section VIII.

The triangulation which was closed at Poore's Mountain has occupied the party of Assistant Boutelle for an average of seven months in each year during its progress. Forty primary stations have been occupied, twenty-nine by Mr. Boutelle, and eleven by Assistant Mosman.

In the course of the winter, computations of the work were made and revised, and these, with the field records of the triangulation, have been deposited in the office.

Messrs. J. B. Boutelle and W. B. French were in service with the party while the field work was in progress in this section. The statistics are:

Stations occupied	2
Angles measured	9
Vertical angles	16
Number of observations	2,850

SECTION V.

SOUTH CAROLINA AND GEORGIA.—(SKETCH No. 10.)

Coast hydrography of South Carolina.—For extending inshore soundings southward from the limits of previous work, Lieut. J. F. Moser, U. S. N., Assistant Coast and Geodetic Survey, left Norfolk on the 14th of January last with his party in the steamer Endeavor, and, after some delay in consequence of bad weather, crossed the bar of Little River, on the northern boundary of South Carolina. In previous seasons the coast hydrography had been carried as far south as Tubbs Inlet. By the work of this season it has been extended upwards of forty miles southward and westward to Murrell's Inlet, along a stretch of coast so little traversed that one of the tidal observers on duty saw no person pass near his station in the course of nineteen days. Two stations were occupied while soundings were in progress, and, for comparison, observations were recorded also on temporary tide gauges set as near the outer beach as possible.

The soundings were adjusted by means of large signals set about four miles apart on the highest sand hills. For inshore work two small signals were erected in the spaces between large ones.

"The inlets between Little River and Georgetown entrance are all practically closed even for the smallest vessels. Between Little River Bar and Singleton Swash the depth increases seaward with great regularity, but from Singleton Swash to Murrell's Inlet successive ridges, with two to six feet less water on them than on either side, appear to run nearly parallel to the coast."

Lieutenant Moser's work of the season is contained on three hydrographic sheets, two of which show the coast soundings to a distance of about nine miles off shore. Little River, including the bar and approaches, was surveyed and represented on a separate sheet.

Masters A. C. Dillingham, J. B. Murdock, and F. E. Greene, U. S. N., assisted in the hydrographic work. The statistics are:

Miles run in sounding	460
Angles measured	1,749
Number of soundings	11,375

The work was continued until the 20th of April. Lieutenant Moser then proceeded with the steamer to New York, where the vessel was refitted for service on the northern coast. Work previously done by the party within the past fiscal year has been stated under the head of Section I in this report.

Tidal observations.—For the use of the engineer officers of the Army, and in regard to projected improvements in the channel of Savannah River, a self-registering tide gauge was refitted at the office and forwarded to Savannah in April. Record paper and forms as usual were supplied for the observer. The results will of course be available for the tidal notes on the river chart.

SECTION VI.

EAST FLORIDA, SAINT MARY'S RIVER TO ANCLOTE KEYS ON WEST COAST.—(SKETCHES Nos. 11, 12, AND 13.)

Coast Pilot.—After the prosecution of hydrographic service, which has been mentioned under the head of Section III, Lieut. Frederick Collins, U. S. N., Assistant Coast and Geodetic Survey, with his party in the schooner Palinurus, sailed from Hampton Roads on the 4th of January to resume duty on the coast of Florida. The vessel passed southward without incident, and after touching at Saint Augustine left that port on the 2d of March with a westerly breeze. Descriptions were made of the inlets north of Cape Cañaveral and other notes as material for the Atlantic Coast Pilot. After taking views of the vicinity of the cape, the Palinurus stood out on the evening of the 3d in order to pass northward of the Bahamas. The wind was then fresh, but by midnight it increased to a gale. Next morning the schooner was in a heavy, confused sea, but being well managed suffered no damage to the hull or spars, although the sea frequently came aboard and drenched everything below. In the course of the day all the sails in succession were split by the force of the gale, but

were repaired on the instant as far as practicable. Lieutenant Collins made for a lee under the reefs of Abaco Island, and there weathered the gale during the night. Unfortunately the morning opened with a storm from the northeast. The position of the schooner was dangerous, and to avoid further risk with damaged sails the course was laid for Nassau, where the mainsail, foresail, and jib were sent ashore for repairs.

On presentation by the United States consul, the governor-general of the Bahamas very cordially received Lieutenant Collins, and offered all assistance that might be needful in refitting the schooner.

The Palinurus sailed from Nassau on the 12th of March and stopped at Elbow Key (Abaco Island) to secure a view of the light-house, and then returned and came through the Providence Channel, where views were made of Great Stirrup Key, Great Isaac's Key, the Bemini Islands, and Gun Key.

The party arrived off Cape Florida on the 16th, and a view was taken of that vicinity. Passing southward through the Hawk Channel, which had been previously examined by Lieutenant Bradbury, some views of important landmarks along the keys were added, after which the vessel proceeded to Key West. During a westerly blow, which made it impracticable to start as intended for the Tortugas, the Palinurus was injured by the collision of a large steamer, the owners of which promptly defrayed the expenditures for repairs.

The work of the season comprises full notes and descriptions by Lieutenant Collins of the coast and harbors from Fernandina southward to the Dry Tortugas, and includes also the results of a careful examination of the Hawk Channel, from Cape Florida to Key West, inside of the reefs. Between the same limits views have been drawn of every prominent landmark, and of the entrances of each important harbor and inlet, including the light-houses in the Providence Channel through the Bahama Islands, a passage much used by our coasting vessels. Views were also made of Great Isaac's Light-House from the westward, of the Bemini Islands, of Gun Key, and of Double Headed Shot Keys, all of which will be of importance to vessels when working through the Strait of Florida. The sketches, numbering one hundred and three in all, were drawn by Mr. J. R. Barker, and are vouched for by Lieutenant Collins as being faithful representations.

In the course of the season the party of Lieutenant Collins sailed nearly five thousand miles. The weather during the progress of the work was unusually bad, and was the cause of vexatious delays.

Masters Francis Winslow and H. H. Barroll, U. S. N., were attached to the party, and are warmly commended in the report of Lieutenant Collins for their cheerful and efficient aid in the operations.

As opportunities offered, observations were made and recorded for the temperature of the water at the surface, and also for its density. An aggregate of two hundred and sixty-two specimens were tested for specific gravity and temperature, and these include four sections across the Gulf Stream off the coast of Florida.

Having successfully accomplished all the work laid out for the party, Lieutenant Collins sailed northward in the Palinurus, and arrived at Norfolk, Va., on the 25th of June. The observations recorded on the coast between Cape Henry and Cape Fear have been mentioned under the head of Section IV in this report.

Tidal observations—With a self-registering gauge of the best construction, the series which was commenced at Fernandina, Fla., in March, 1877, by Mr. H. W. Bache, was continued at that station until September. The observer had been some time ill, and had not recovered when yellow fever became prevalent at Fernandina. He was consequently relieved, but returned in December and resumed observations. The meteorological registers are kept up in connection with records of the tides.

Hydrography eastern coast of Florida.—For continuing the inshore hydrography from Mosquito Inlet southward and eastward towards Cape Cañaveral, the steamer Bache left Hampton Roads on the 26th of April, and, after touching at Charleston, S. C., passed on without incident to the intended working ground. The party on board was in charge of Lieut. Commander C. M. Chester, U. S. N., Assistant Coast and Geodetic Survey. At Mosquito Inlet a boat party was sent in to establish a tidal station and to put up signals along the beach to the southward. While this work was in progress

some needful repairs were made to the machinery of the vessel. The vessel then steamed along, carrying soundings to her anchorage inside of Cape Cañaveral Shoals, and the opportunity was taken in passing to inspect the condition of the signals which had been erected. Some of these had gone down, and, owing to the growth of palms near the cape, many of the signals could not be distinguished in the offing. To overcome the difficulty thus presented, Lieutenant-Commander Chester had a few signals set up for boat work, and made arrangements for occupying the light-house and an adjacent station with the theodolite.

Soundings were commenced on the 6th of May. The steamer ran lines east, northeast, and on the opposite course in the bay, between Southeast Shoal and the mainland, the lines being at an average about a mile and a half apart. At the same time a boat party made soundings on traverse lines over Southeast Shoal early in the day when practicable, and later along the coast, the last connecting with the lines of soundings run by the steamer. In this way, and by additional lines run at right angles to the parallel lines first mentioned, the bay was thoroughly sounded.

On a second projection for developing the outer shoals, normals to the coast were run a mile or rather more apart to a general depth of fifteen fathoms, and inside of the outer shoals the number of lines was doubled, and these were crossed by others run parallel to the coast. Between the light-house and each of the shoals, and between the separate shoals, lines of soundings were run in addition to such as were recorded in the course of the general survey. Each of the shoals was also carefully developed by radial lines run from a buoy set previously over the spot on which the least depth was found. But a shoal mentioned as being a mile and a half to the northward of the "Bull" was not found, though it was carefully sought for. In regard to it, Lieutenant-Commander Chester remarks: "As we ran over the place when water was breaking heavily on the "Hetzel," and when heavy rollers were almost breaking on the "Bull" in three fathoms, the conclusion is that the shoal in question does not exist."

In general outline the shoals were found by the party as represented by the reconnaissance of 1850; but there is now a dangerous shoal bearing about north six miles from the light. On the way down the steamer passed over that shoal, and the subsequent development proved that the least depth on it is ten feet. When the vessel was at anchor on parts of the shoals current observations were recorded.

After completing the hydrographic survey of the "Hetzel Shoal," Lieutenant-Commander Chester took up the inshore soundings along the coast and filled the projection which had been made to include the sea approaches to Eastern Florida, between Mosquito Inlet and the bay under Cape Cañaveral. This work occupied the party until the middle of June. Lieuts. Uriel Sebree and A. V. Wadhams, U. S. N., Master T. G. C. Salter, U. S. N., and Ensign C. H. Amsden, U. S. N., were attached to the hydrographic party.

Late in June the steamer Bache was detailed to meet the steamer Hitchcock at the mouth of Saint John's River and take that vessel in tow and insure her safety at sea as far as Cape Florida, the intention being to transfer the Hitchcock for service in the Mississippi River. After supplying the vessel with coal for the passage, Lieutenant-Commander Chester started for New York, and reached that port on the 10th of July.

Survey of Saint John's River, Fla.—Arrangements for resuming this work were made early in November, 1877, but as yellow fever soon after prevailed at Jacksonville, it was not deemed expedient to organize a party for field work until late in December. Without delay the steamer Hitchcock was repaired at Jacksonville, and moved to the working ground in the middle of January of the present year. At Patricio Point, Assistant F. W. Perkins resumed the survey which had been previously conducted by Assistant Ogden. Provision was made for carrying forward the triangulation and topography jointly. The plane-table survey was advanced by Mr. C. A. Ives, aid in the party of Assistant Perkins, and was closed for the season at Racey's Point. The triangulation was extended southward as far as Tocoi.

Ensign J. P. Underwood, U. S. N., joined the party in March, and, though unwell, he rendered acceptable service by his judicious care of the vessel and crew. Early in April, Assistant Perkins was assigned to the field service in Section VIII. The vessel was then transferred to the charge of Subassistant Vinal.

The statistics of work done by the party of Assistant Perkins on the Saint John's are:

Stations occupied	10
Angles measured	178
Number of observations	3,667
Shore line surveyed, miles	46
Roads, miles	72
Number of soundings	7,528

When Assistant Perkins was assigned to duty on the Mississippi River, Subassistant W. I. Vinal was transferred from the party of Assistant R. M. Bache, with directions for continuing the survey of the Saint John's. The work was prosecuted until the 9th of June, when the steamer Hitchcock was sent to Jacksonville for repairs. Subsequently the vessel was transferred for service in the Mississippi River as already mentioned. Mr. Vinal took up the hydrography at the limit to which it had been extended by Assistant Ogden in 1877, and going southward sounded the river to a point about three miles beyond Tocoi. Before leaving the section, the triangulation was carried several miles to the southward of the hydrographic limits. The statistics are:

Signals erected	5
Stations occupied	6
Angles measured	66
Number of observations	792
Miles run in sounding	318
Angles observed	2,045
Number of soundings	22,491

Ensign J. P. Underwood, U. S. N., was attached to the party in the steamer Hitchcock, but in consequence of feeble health, due to pulmonary disorder, his attention was of necessity limited to the care of the vessel and crew.

Survey of Indian River, Fla.—The party detailed for continuing the survey of the eastern coast of Florida below Cape Cañaveral resumed operations, under the charge of Assistant R. M. Bache, on the 12th of January, 1878. As heretofore, the sloop Steadfast was used for transportation.

Until the 7th of February the plane-table survey and hydrography were advanced evenly from the south end of Merrill's Island, in Indian River, and were then suspended, as the prevailing winds favored for moving into Banana River, of which the projection made last year had not been filled with soundings. But, soon after, a succession of gales followed, and the duty thus became tedious and difficult. By the end of April, however, that work was completed. The party then returned to resume the survey of Indian River, but found that the tide gauge had been swept away from the sheltered place in which it had been left. Mr. Bache at once took means for the purpose, and was successful in regaining the plane of reference. Bench-marks were then established along the shores of the river, and the marks were connected by simultaneous observations and by lines of levelling. This course was made especially requisite by the unusually high water maintained in Indian River during the winter.

In May the work advanced without hindrance, and by the middle of that month the triangulation, topography, and hydrography had been advanced to stations twelve miles southward of the mouth of Banana River. The work was there closed for the season, and the Steadfast returned to her anchorage in the upper part of the river, where the vessel was properly secured against injury during the summer.

Lieut. Thomas N. Lee, U. S. N., was attached to this party, but in consequence of failing health he was constrained to leave the section before the close of operations. After his return from Florida, Lieutenant Lee reported in person at the office, but his constitution was broken by rapid consumption. He died at Ellengowan, Md., in July.

The part of Indian River included in the survey here under notice is separated from the ocean by a strip of land which at one point has a breadth of only about two hundred yards.

The statistics of the season are:

Shore line surveyed, miles	41
Creeks and ponds, miles	32
Roads, miles	$5\frac{1}{2}$
Area of topography, square miles	18
Miles run in sounding	562
Angles	616
Casts of the lead	22, 014

Mr. C. A. Ives aided in the field work of the party in this section. During the summer, Assistant Bache was employed in a special survey in Section II, of which further mention will be made in my next annual report.

Hydrography of Charlotte Harbor, Fla.—After the completion of supplementary soundings, of which mention will be made in the next section of this report, Lieut. J. M. Hawley, U. S. N., Assistant Coast and Geodetic Survey, with his party, in the schooner Silliman, made arrangements for the hydrographic survey of Charlotte Harbor. With some difficulty nineteen stations, which had been occupied for the triangulation, were identified. Additional to these, upward of seventy signals were erected for the adjustment of soundings. The details of construction were entrusted to Master G. C. Hanus, U. S. N., who was attached to the party, and credit is due for economy in the requisite outlay. By direction of Lieutenant Hawley the signals which served for soundings in one part of the bay were removed and used in other places. In regard to the station points, Master Hanus reports: "The grass and even mangrove trees had overgrown many of the triangulation points, but, owing to the general accuracy of the topographical survey, the points sought were nearly always found. The cedar stakes set as marks some years ago by the triangulation party were well preserved, but of the pine stakes set only one remained perfect."

While soundings were in progress in Charlotte Harbor tidal observations were recorded at a station near Hickory Bluff, and also at Cape Haze. The supervision of the two tide gauges was committed to Master A. H. Cobb, U. S. N., and by his attention a record of the rise and fall at intervals of only ten minutes was secured for a calendar month at one of the stations, and for six weeks at the other. It was noticed, as resulting from high winds, that the water level was raised by southerly or southwesterly breezes, and correspondingly depressed when the wind was from the north. Owing to the unusual rainfall, the waters of Shell Creek were found to be seven feet above the ordinary level, and this condition, conjoined with high winds prevalent during the latter part of the winter, made it a matter of some difficulty to determine planes of reference for the two resulting hydrographic sheets. On one of them, the soundings are referred to the gauge at Hickory Bluff; on the other they are referred to the gauge at Cape Haze. Bench-marks were established at both localities, and these are described in the tidal record.

Soundings were begun on the 6th of February, and the hydrography was advanced daily while the weather permitted. Frequent winds in February and March made it impracticable to observe the currents, but later in the season two stations were occupied, one in the channel between Pine Island Shoal and Cape Haze Shoal, and the other in the channel to the northward of Cape Haze. By the 10th of May the hydrography of Charlotte Harbor was completed from Pease Creek through Boca Grande to the Gulf. Lieutenant Hawley reports that the channel as far in as Hickory Bluff has nowhere less than nine feet at mean low water. Between Hickory Bluff and Pease Creek there are several hard lumps with only five feet of water on them.

"The holding ground in some parts of Charlotte Harbor is the best which we have found at any place in Florida; it is either very sticky mud, or a mixture of mud, sand, and shells. In bad weather, fishing smacks make a harbor just inside of Boca Grande to the southward, but the bottom there is sandy, and the place not as secure as the anchorage between Pine Island and the shoal to the northward."

The statistics of the work are:

Miles run in sounding	722
Angles measured	2, 523
Number of soundings	29, 949

Eighty-two signals were erected by the party in the course of the season. At Manatee some indispensable repairs were made, after which the Silliman sailed for New York, but off the coast of Virginia was struck with heavy weather and constrained to put into Hampton Roads. Subsequently the injury to her mast was repaired at Baltimore, and the vessel proceeded to the coast of Maine for service, which will be mentioned in my next annual report.

Master Albert Mertz, U. S. N., has been on duty with this party during the season.

Triangulation of Sarasota Bay, Fla.—This work was taken up by Subassistant Joseph Hergesheimer on the 9th of March, after the completion of a survey to be mentioned under the next head. As the result of operations, which were closed on the 15th of June, the triangulation proceeding southward from Tampa has been carried through Sarasota Bay, which, for the greater part of its length, is separated from the Gulf of Mexico only by the narrow strip known as Long Key. For defining the shore line, points were determined at intervals on both sides of the bay quite down to the narrow passage which connects with the shallow inland waters leading towards Charlotte Harbor. Much rain fell during the time employed by the party, and the measurement of angles was in consequence retarded. The statistics of the work are:

Signals erected	30
Stations occupied	17
Angles measured	169
Number of observations	3,270

The schooner Quick was used for transportation by the party in Sarasota Bay.

SECTION VII.

WEST FLORIDA, ANCLOTE KEYS TO PERDIDO BAY.—(SKETCH No. 14.)

Survey of Crooked River, Fla.—This water passage bounds the north side of Saint James's Island, which lies at the eastern entrance of Saint George's Sound. Its development was provided for in the operations of a party detailed in December, 1877, to work under the direction of Subassistant Hergesheimer, as already noticed under the preceding head.

After making needful arrangements at Apalachicola the party reached Crooked River in the schooner Quick early in January of the present year, and as soon as practicable traced the shore lines of the river from its junction with the Ocklockony quite through to the western mouth of the river which passes into Saint George's Sound. The bar off the western entrance was sounded, and also the water passage throughout its entire course. This work was completed on the 23d of February. The statistics are:

Shore line traced, miles	58
Miles run in sounding	66

Under the head of Section II notice has been taken of field work done by Mr. Hergesheimer in the summer of 1877.

Hydrography, Saint George's Sound, Fla.—Before taking up the work of which mention has been made under a preceding head, Lieutenant Hawley, with his party in the schooner Silliman, made supplementary soundings in Duer's channel, at the eastern entrance to Saint George's Sound, and also off Light-House Point. The aggregate statistics are:

Miles run in sounding	49
Angles measured	312
Number of soundings	2,715

Masters Hanus, Cobb, and Mertz, U. S. N., assisted in this work, and also in the hydrographic survey of Charlotte Harbor. The party in the Silliman was occupied in Saint George's Sound during the latter part of December, 1877.

SECTION VIII.

ALABAMA, MISSISSIPPI, LOUISIANA, AND ARKANSAS.—(SKETCH No. 15.)

Hydrography of the Gulf of Mexico.—In December, 1877, full preparation was made for the return of the hydrographic party in charge of Lieut. Commander C. D. Sigsbee, U. S. N., Assistant Coast and Geodetic Survey, with the steamer Blake, to complete deep-sea soundings in the Gulf of Mexico. At my request, Prof. Alexander Agassiz consented to accompany the party, and to direct in such operations as might yield desirable information in regard to the conditions and forms of life that inhabit the Gulf bottom. The steamer left New York with all needful appliances for dredging and sounding, and was at Havana at the close of the year. In the plan of work, the necessity for touching at Spanish ports was kept in view; and permission to do so was cordially granted by the Governor-General, who directed, also, that facilities should be furnished by the authorities of any port at which the Blake might have occasion to anchor. On the 2d of January the steamer stood off and on, dredging and sounding in the vicinity of the entrance to Havana, and the next two days in the direction toward Sand Key. Excepting on the coral mud bottom, the dredge-tangles brought up many living forms. While thus engaged near Sand Key, a norther began without any sign of approach, and bad weather prevailed during several days. To avoid its effects Lieutenant Commander Sigsbee took shelter at Key West, but as soon as practicable resumed operations with the vessel, and in the middle of January the party was engaged in dredging off Bahia Honda, on the north coast of Cuba, about sixty miles to the westward of Havana. On the 19th the Blake stood in toward the entrance of the harbor merely to inquire for a pilot for the Colorado Reefs, and then resumed work in the offing. The port official shortly afterwards visited the Blake, and took the opportunity to state that if the steamer had occasion to enter, her signal for a pilot would be promptly answered from the fort, and it so fell out that thick weather came on a few hours after. That change, but more especially the wish for information concerning the intended cruising-ground of the steamer to the westward of Bahia Honda, inclined Lieutenant-Commander Sigsbee to enter the port at least for the night, and accordingly the signal was set at five o'clock in the evening for a pilot. A boat sent by the commander of the fort was soon alongside of the Blake, and the man, who professed to be a pilot, was taken on board. He was closely questioned as to bearings and landmarks, and both Lieutenant-Commander Sigsbee and Lieutenant Ackley checked his replies as the marks came into view. He met all inquiries with confidence, and declared that the ship was going well. This was very shortly before the leadsman reported only three and a half fathoms of water. When sharply asked if that was sufficient, the pilot asserted that there was plenty of water, "and more beyond," and had scarcely done so when the steamer struck. The vessel had, in fact, through misdirection, been run by the force of her engines up an inclined plane, and was lying helpless on hard coral rock bottom with only seven feet of water under the bow and nine feet astern, whereas the mean draught of the steamer when afloat was over ten feet. To keep the vessel from being set further on the reef, Lieutenant-Commander Sigsbee immediately sent out the anchors to seaward and their chains were hove taut, one from the stern and the other from the starboard bow. Means were at the same time taken to discharge the coal overboard at a safe distance from the port side. Master McCrea had been sent to the fort and with the transportation there furnished reached the nearest telegraph station. The consul-general of the United States at Havana, Henry C. Hall, esq., acted promptly on the request of Lieutenant-Commander Sigsbee for assistance, and dispatched vessels already fitted, in hope that the Blake might be at once hauled off, as the steamer would suffer from the effect of any swell of the sea. Everything possible was got out and sent on shore by the Blake's company; and the engines were backed, but all expedients failed, as did also the attempt to haul the vessel off by the tow of another steamer. Soon after, on the evening of the 21st, a norther came up; the sea became so rough that all hands on board were in peril. Lieutenant-Commander Sigsbee felt constrained to send the crew ashore, retaining on board, besides himself, only Lieutenant Ackley, Passed Assistant Engineer W. S. Moore, two machinists, a fireman, and five seamen. Professor Agassiz had taken active part in all the means employed and proposed to share the danger of the situation with the commander, as did also the officers, who requested permission to take the place of the seamen retained on board. Lieutenant-Com-

mander Sigsbee declined to make any change, all the persons selected at first being equally willing to remain. As the ship began to pound with the rising sea, the small company on board busied themselves about the deck and below in securing what was left, and to be in readiness for the worst; the joint of the Kingston valve was broken to admit of flooding the ship if necessary. At half past eight o'clock at night the seas were breaking along both sides, and the vessel pounded so that those on board found it difficult to stand without holding fast. The engine frames raised at every blow, and the stern swung one point in shore. All hopes of a favorable shift of wind were given up, and at this juncture, all on board feeling that the vessel could not hold together for another half hour, Lieutenant-Commander Sigsbee gave order for hauling the fires and opening the Kingston valve. The vessel filled rapidly, and in a few minutes rested quietly on the bottom. A whaleboat had been kept under the lee of the port bow, and after a half hour spent in securing hatches, it was felt that nothing more could be done for the vessel at that time, so the men were sent into the boat and were followed by Mr. Moore, Lieutenant Ackley, and Lieutenant-Commander Sigsbee. With little trouble the party reached the shore shortly before ten p. m. Toward morning the wind and sea went down, and the opportunity was taken to discharge the coal that yet remained on board. The condition of the vessel soon after striking on the reef had been made known by Thomas Elven, seaman, an expert swimmer and diver, who went down and passed along her whole length under both sides. Four days later he made a second examination, and his reports were of great value in directing the exertions that finally restored the steamer to the service.

On the morning of the 23d of January the light-house tender Dandelion arrived off Bahia Honda from Key West, dispatched by Commander Smith W. Nichols, U. S. N., who, being light-house inspector there, had earnestly co-operated with Consul-General Hall in sending means for the relief of the Blake. A pilot from the port came and remained on board the tender, but as Lieutenant-Commander Sigsbee soon followed, his services were accepted by Captain Cosgrove to pilot in the Dandelion. That vessel was anchored bow and stern near by, and the two engineers succeeded in connecting the boilers of the tender with the whistle-pipe of the Blake. Steam was thus applied from the Dandelion to work the donkey pump for clearing the grounded vessel of the water which had been admitted for her safety. At four p. m. of the 24th the water was so much reduced as to admit of lighting fires under the port boiler of the Blake. The main engine was started two hours afterward, and by nine p. m. the steamer was free of water, but still remained aground. Soon after there arrived the tug La Gitana towing a lighter containing sixty-six wine-pipes, which vessels and appliances had been forwarded from Havana by Consul-General Hall, on the application of Lieutenant-Commander Sigsbee, through telegrams, and by Professor Agassiz in person. The Spanish tug co-operated with the Dandelion in an attempt to haul off the Blake, but without success, though the vessel was then divested of nearly all that could be moved. Finally La Gitana was directed to take a hawser from the starboard bow, while the Dandelion held hard as before to the starboard quarter, and when the Spanish tug made a surge, the Blake responded by swinging rapidly around, until her head was northwest instead of south, as previously, though left hard aground aft for about ten feet forward from the stern post. The hawser of the Dandelion was then cast off from the quarter, and made fast to the foremast head of the Blake, and the tender hauled straight ahead to depress the bows of the Blake, which were in deep water, while La Gitana surged again with a twelve-inch hawser, but the Blake still remained fast astern. As the tide was then beginning to fall the bow was held in position by the tug while the tender planted the starboard bow anchor of the Blake, the other being already out in a position right ahead or northwest. Two heavy anchors being thus well set for the final effort, the assisting vessels were released for the night, with the understanding that all the force possible would be applied at high tide, which would not occur until the forenoon of the following day. On the morning of the 26th, Thomas Elven, seaman, again went under the Blake, and reported that the vessel held only near the stern post. As the bow was then floating in two and a half fathoms, the sixty-six empty wine-pipes brought by La Gitana were arranged on the decks of the Blake, as far forward as possible, in two tiers, and were filled with sea-water, as were also the forward water-tanks. Some additianal weight at the bows of the Blake was added by reshipping articles which had been previously transferred, and the effect of this expedient, as expected, was to relieve the pressure of the stern on the reef. When everything was in readiness to apply force if necessary, Lieutenant-Commander Sigsbee and Lieu-

tenant Sharrer went over the side, into the dinghy, and sounded around the vessel. They perceived that the starboard chain seemed to slacken, and in a few moments were gladdened by the certainty that the Blake was afloat. The windlass was at once manned, the engines started, and the ship commenced swinging to the wind. Although the vessel had been slung with chains, and girdled by her crew, and wine-pipes provided to be used as camels or caissons, no necessity arose to employ them in the way which was at the outset thought requisite. The spirit, intelligence, and energy of the commander and officers of the steamer Blake, and the steady devotion of the crew well deserved the success that crowned their efforts. The ship struck at high water, spring tide, when the rise was twenty-nine inches, and when got off the rise was only eleven inches. By immense labor, in which all on board assisted at the outset, the draught of the vessel was speedily lessened by twenty-seven inches. In general reference to the exertions of his associates, Lieutenant-Commander Sigsbee says:

"During all our troubles Professor Agassiz did everything possible to render assistance, and though disappointed in the break which had occurred in the marine investigations, he made our affairs paramount in his endeavors.

"To Lieut. S. M. Ackley and Passed Assistant Engineer W. S. Moore I am indebted for suggestions of a number of useful expedients, for their unremitting personal labors, and for cheerfully sharing in the risk of remaining on board on the night of the 21st. All belonging to the vessel worked hard, and voluntarily would have remained during that night, if permitted. To the officers I owe expressions of sympathy and confidence. Machinist Peterson, who worked very hard, was the only one on board able to say, from previous observation, how the ship would be likely to behave when filled with water after grounding. His intelligent description confirmed the belief that the measure which I had under consideration tended to the ultimate safety of the Blake. Among the crew there was no word of complaint; all applied themselves manfully, and in consequence of severe labor the double ration served out was all consumed."

The report of Lieutenant-Commander Sigsbee makes special mention of the interest manifested for the success of the cruise of the Blake, in many courtesies by Commander Smith W. Nichols, U. S. N., light-house inspector at Key West, and by Lieut. Eugene B. Thomas, U. S. N., commanding the naval depot at that port. Commander Nichols dispatched the Dandelion as being best fitted to render effective assistance. The commander of that vessel, Capt. Philip J. Cosgrove, reached Bahia Honda at a most opportune moment, and though so sick as to require treatment from the surgeon of the Blake, the energetic direction of the Dandelion at once proved to be a gratifying relief. To this zealous and capable officer my thanks have been expressed, as also to others whose activity tended to further the movements needful for the relief of the stranded vessel.

As soon as practicable, after being floated at Bahia Honda, the steamer Blake proceeded to Havana, and in the course of a few days was repaired at that port.

Early on the 12th of February Lieutenant-Commander Sigsbee started from Key West with fine weather to resume work. When clear of the reef a course was shaped to the westward of the Tortugas, and in that vicinity a few casts were made with the dredge, but nothing of note came up from moderate depths. The bottom was generally soft gray mud. At eight hundred and sixty-three fathoms the dredge came up full of mud, and attached to the tangles brought also one star-fish. This was at a position sixty miles west of Tortugas. Bad weather followed for some hours, but on the 15th the trawl was worked at depths of nine hundred and fifty-five and eleven hundred fathoms in soft yellow mud. Of eight kinds of deep-sea fishes brought up, some were blind, and others remarkable in structure. A large variety of rare living forms was there obtained, besides five kinds of sponges, and coral branches of great delicacy. Further westward another haul in nineteen hundred and twenty fathoms was made with equal success. Very large carnation-colored shrimps, and a blind but well-shaped fish, a foot in length, were among the numerous specimens held by the trawl.

Passing on southward and westward the trawl was again got overboard in fifteen hundred fathoms on the slope of the Bank of Campeche, where the bottom is generally soft coral mud, but in reeling in, it was found that the trawl had fouled on something at the bottom, and could not be cleared. This loss left only one trawl on board. The steamer passed on westward, sounding along the north side of the bank. As the dredge brought up few specimens, a modification was

made with a view of retaining less of the mud, which seemed to be scooped up from the bottom and held to the exclusion of any specimens that might be near the spot where the apparatus was dropped. Lieutenant-Commander Sigsbee and Master H. M. Jacoby, U. S. N., soon completed the changes deemed requisite for success, and the first trial of the new dredge brought up neither mud nor sand, but many specimens of value; among others, five entirely new sea-urchins, each closely resembling, in form, color, and size, the half of a large fresh lemon. North of the bank, and in fifteen hundred and sixty-eight fathoms, the trawl was kept an hour on the bottom and came up with specimens among which was an exceedingly rare kind of stemmed polyp. Along the west side of the bank the dredge was cast at intervals, and generally with success.

Passing southward to Alacran Reef on the 18th of February, the Blake was anchored in six fathoms on the northwest edge of the reef, and Professor Agassiz examined the shoal water about the reef; after which the vessel got under way, passed along the west and south sides of the Alacran, and steamed for the edge of the Yucatan Bank, near the northeast point of the Yucatan Peninsula. The intention of Lieutenant-Commander Sigsbee, by the advice of Professor Agassiz, was to run a line from the one-hundred-fathom curve at that place toward Cape San Antonio; but during the night of the 19th of February a southeast gale made it necessary to defer the purpose. When the weather cleared, the Blake was put on her course, and the trawl was used in fifteen and twenty fathoms, bringing up the profusion usually found near shore. Keeping on, under easy steam, in the direction towards Cape San Antonio, rough weather was again met, and the Blake put back and took shelter under the lee of a shore that makes out northward of Jolbos Island, where the vessel was detained until noon of the 22d of February. The steamer again started eastward, although the weather was yet unsettled, but dredging was impracticable, and no stop was made short of the Colorado Reefs, in the vicinity of Cape San Antonio. Soundings were recorded in the vicinity of the Reefs, and the dredge was about to be started when a squall came on. Standing again back towards the reefs, the Blake approached to within a mile and a half, and found three hundred and forty-six fathoms. Of the appearance of the reefs, Lieutenant Commander Sigsbee says: "A more dangerous-looking locality I have never seen. The white water was wholly within the breakers, the latter being in blue water. No land was in sight."

Preparation was made on board the Blake for casting the dredge near the Colorado Reefs, but at the instant of backing into the sea one of the rudder chains broke adrift and got foul of the propeller. The sails were quickly adjusted, but the vessel did not answer, nor could the chain be relieved by a boat's crew sent astern. Lieutenant Commander Sigsbee went over the stern in a bow line, and seeing that the chain could be got away only by breaking, gave order for turning the engine at full speed. As a result the chain snapped from the rudder, but probably remaining in the propeller it was not deemed expedient to stay longer at the reef. Standing broad off into deep water a cast was made in twelve hundred and twenty-two fathoms. The course was then changed to northward and eastward, and the trawl was used and brought up specimens from a depth of thirteen hundred and twenty-three fathoms. Further on in the direction toward Key West a very successful haul was made in eight hundred and sixty fathoms. Two other equally fortunate dredgings were made at positions south of the Tortugas. Some of the forms secured in these were entirely new. The steamer reached Key West on the 25th of February in time to escape very heavy weather that prevailed during several days. On the 1st of March the vessel again started, and stopped off the Marquesas for examinations deemed desirable by Professor Agassiz. The coral there was found to be dead. Continuing westward inside of the reef a heavy sea was met, and in consequence Lieutenant-Commander Sigsbee went into Tortugas Harbor. Next day successful hauls were made with a dredge which had been constructed on board, but the sea remained heavy. After a cast in fifteen hundred fathoms the course was laid for the Mississippi Delta, off which the dredge was used in three hundred and twenty-one and five hundred and thirty-three fathoms, and finally at the one-hundred-fathom curve opposite to the entrance of South Pass. Near the eastern outset of this line the Blake encountered a tremendous sea, in which the seaworthiness of the vessel after her pounding on the reef near Bahia Honda was fully proved. The sea continuing heavy during the entire run, and also off the mouths of the Mississippi, the Blake was passed between the jetties and anchored at New Orleans on the 9th of March. From that port Professor Agassiz proceeded directly to Boston. Lieutenant-Commander Sigsbee took in a supply of coal and refitted the vessel.

The only continuously good weather during the season in the Gulf happened while the steamer Blake was fast on the reef off Bahia Honda. This fortunately availed for the safety of the vessel, but the results of the work, done for the most part during rough weather, are nevertheless satisfactory.

Soundings and bottom temperatures were carefully recorded in connection with the dredging operations.

The Blake left the Delta on the 24th of March, and in the course of two days was in one hundred fathoms on the Florida Bank. Hauls were made with the dredge in cruising towards water of less depth, and after touching at Key West at positions off the coast of Cuba in the vicinity of Havana. Much bad weather was experienced in April, but the latter part of that month was spent in the endeavor to repair a defect in the submarine cable between Havana and Key West. This service was undertaken at the request of the late president, Hon. William Orton, of the Western Union Telegraph Company, to which, from its organization, the survey has been continuously under obligations for many facilities that favored progress in the work. By the party in the Blake the cable was at first underrun from the south beach at Key West, and was buoyed at the edge of the reef. When the weather served a further attempt was made to underrun to the vicinity of the defect, which was judged to be about twenty-seven miles from Sand Key, but the cable was found to be so slack that the appliances on board the steamer became ineffective. The agent of the International Ocean Telegraph Company directed and witnessed all the operations, and at last decided that further efforts were useless in the absence of special means for the service desired.

Having refitted at Key West the Blake again left that port on the 2d of May, and started in sixteen fathoms a line of soundings just outside of Sand Key Light-House, and with good weather extended work towards the Yucatan Bank, but the wind and sea next day constrained a return to Tortugas after a run of seventy miles. On the way the Blake was struck by a sea which broke seven dead-light panes in the starboard main-deck ports, though the glass was a quarter of an inch thick. The storm prevailed during several days, but on the 8th the vessel was got under way for Key West, several of the company being ill. A few days after, Lieutenant-Commander Sigsbee was again off the Tortugas with the intention of sounding quite across to the coast of Cuba. This line was closed a short distance east of the entrance to Mariel, where a depth of three hundred and eighty-one fathoms was recorded quite close inshore. As the weather was unsettled the Blake laid off near the entrance to Havana on the night of the 12th, and then steaming westward started a line off Bahia, and carried soundings northward and westward to a point about forty miles west of the Tortugas, where a depth of one hundred and fifty-six fathoms was recorded. The interval of calm that favored work on this line closed, but the vessel was moved to a position about thirty miles further west, and there found seventeen hundred and fifty-three fathoms. From that station soundings were run to the coast of Cuba in a direction generally parallel to the line last mentioned and at nearly equal distances four other lines, the last starting near Cape San Antonio were run radiating from the coast of Cuba. These developed an eastern prolongation or spur of the Yucatan Bank, and that development was further verified by a line of soundings which crossed the radial lines. This final line of soundings was run northward and eastward and on the course towards Key West, where the Blake arrived on the 26th of May.

At proper intervals on all the lines of soundings, serial temperature stations were made, at each of which the temperature of the water was recorded at the surface and at varying depth to the bottom of the Gulf. Full series were registered at fifty-nine stations, the records of which will bear directly in the investigation of the currents that pass between Cuba and Yucatan. Of the ten lines of soundings here mentioned the greatest depth found was two thousand and sixty-four fathoms at a station about midway between Yucatan Bank and the Tortugas. At the place in which each of the soundings was recorded the density of the Gulf water was tested.

On the passage of the steamer Blake northward Lieutenant-Commander Sigsbee stopped off Cape Florida, and ran a line of soundings with numerous temperature observations quite across the Gulf stream, terminating the line at Gun Key. The depths found are greater than those heretofore reported for that vicinity. Passing on northward from Gun Key the vessel reached New York on the 5th of June.

In the work of the season Lieutenant-Commander Sigsbee was ably seconded by Lieuts. S. M. Ackley and W. O. Sharrer, U. S. N.; by Masters H. M. Jacoby and Henry McCrea, U. S. N., and by Ensign G. H. Peters, U. S. N. Assist. Surgeon C. J. Nourse and L. P. Sigsbee served as recorders in the hydrographic party.

The general statistics of the work are:

Miles run in sounding	1,316
Number of deep-sea soundings	196
Temperatures recorded	1,008
Water densities	207
Deep-sea dredgings	102
Specimens of bottom	147

Triangulation of Barataria Bay, La.—This work was essentially complete at the close of the preceding season. The shore line had also been traced for the use of the hydrographic party. It remained merely to extend from the head of the bay a short triangulation so as to connect with stations used in the survey of the Mississippi River. For this purpose Assistant W. H. Dennis resumed operations on the 10th of December, 1877, with a party in the steamer Barataria. A reconnaissance was made and signals were erected for a set of quadrilaterals north of the work of last year. At Woodland a tripod sixty feet high was required in order to obtain a line of sights above the buildings by which the station was surrounded. Between the bay and the river, at the points of nearest approach, a ridge intervenes covered by a growth of live oak, through which four avenues, averaging nearly half a mile in length, were cleared by cutting to admit of observing with the theodolite.

While Mr. Dennis was engaged in measuring angles for connecting his work with the river survey, the shore lines of the upper branches of Barataria Bay were mapped by Mr. F. C. Donn, the aid in the party.

The statistics of the supplementary work of this season are:

Shore line traced, miles	88
Angular measurements	4,952

The shallow waters at the head of the bay were sounded while the topography was in progress. This work was completed early in April. Under another head mention will be made of the subsequent operations of the party under the direction of Assistant Dennis.

Hydrography of Barataria Bay, La.—The party assigned for this work commenced operations on the 24th of January, 1878, under the charge of Lieut. W. I. Moore, U. S. N., Assistant Coast and Geodetic Survey, with the schooner Ready. After establishing a tide gauge, the soundings were prosecuted at all favorable intervals inside of the bay, and when weather permitted in the approaches. The resulting sheets take in the Gulf hydrography for about four miles east and as far west of the entrance, and to a distance of rather more than three miles to seaward. Inside, the bay was sounded to a limit about six miles from the Gulf shore.

During March and April the work was delayed by prevalent fogs. These were constant except for about one hour near midday.

The following remarks are taken from the report of Lieutenant Moore:

"On approaching the entrance to Barataria Bay the water shoals gradually from seven fathoms (at three miles from the light-house) to seven feet on the crest of the bar. Inside the bar the water deepens to seven fathoms when abreast of Fort Livingston.

"Inside the entrance no noticeable changes have taken place within the last twenty-five years. Thirteen feet may be carried until near the "Quartelles," with shell heaps on either side. Above this the bottom seems to be flat, with six and a half feet of water up to the entrance of the bayous, which are invariably guarded by bars having from two to five feet of water on them.

"There is good anchorage in three fathoms, Barataria light bearing south-southeast, bottom black mud."

As might be expected the tides were found to be much affected by the direction of the wind. It was noticed also as a peculiarity that the tide would begin to rise at the entrance while the current was running out, and fall while the current was setting in.

The general statistics of this survey are:

Miles run in sounding	585
Angles measured	3,634
Number of soundings	31,458

Lieutenant Moore was assisted in this work by Lieuts. W. F. Low and Sidney H. May, U. S. N. Soundings were continued in Barataria Bay until the 1st of June.

Tidal observations.—The record made by means of a staff-gauge by Mr. G. Faust, at New Orleans, during regular intervals of six hours has been continued. The daily or tidal fluctuation in the Mississippi is very small, but the annual changes, amounting to eleven or twelve feet, produced by floods, are of much interest. The observations are kept up as heretofore, because these large changes show a tendency towards recurrence at regular periods, and hence the accumulated records may be expected to yield a fair approximation in advance in regard to the state of the river throughout the year.

Base lines near Mississippi River.—Full provision was made for the advance of the triangulation parties on the Mississippi River, by the assignment, early in April, of Assistant H. G. Ogden, for the measurement of base lines at Donaldsonville, at Natchez, and at Vicksburg.

At Donaldsonville the line was located about a mile and a half below the city, and is thirty-three hundred and thirty meters in length. The measurement was made over a plantation road, and levels were run to reduce the mean height of the base to the top of the levee or railroad. The azimuth was observed for the base line by fifty-six measures, resulting from nine sets of observations in five positions of the instrument near elongation on four nights.

For use in the triangulation approximate computations of the base length and azimuth were furnished to Assistant Perkins, who was then conducting the triangulation above Donaldsonville.

Opposite to Natchez a base line was laid out and measured of twenty-nine hundred and seventy meters along the Concordia road at Vidalia. Azimuth was measured at an adjacent point of the triangulation. The angle between a mark set about two miles to the northward and the line leading from "Lookout" to "Giles" was ascertained by sixty measures, which resulted from ten sets of observations with the instrument in five positions. Four nights were employed for determining the azimuth.

Under circumstances which constrained Assistant Ogden to leave the field early in June, the charge of the party was devolved on Mr. C. H. Sinclair, who had efficiently aided in the operations at Donaldsonville and Natchez. Mr. W. B. French was attached to the party at the opening of the season, and remained in the field until the close of operations.

The base line measured by Mr. Sinclair was laid out by Assistant Hosmer, and properly connected with points of his triangulation of the Mississippi River, in the vicinity of Vicksburg. For more than half its length the line passes through the principal street of the town of Delta, La., the remainder of the line ranging through adjacent cotton fields. The length, as found by Mr. Sinclair's measurement, is 2715.3 meters. Inclement weather delayed the completion of the measurement until the 29th of June.

The azimuth of a line of the triangulation, leading from a station in Vicksburg across to a station on the island opposite to the town, was determined by twenty-one sets of observations in five positions of the theodolite. Four nights were occupied in that work. The latitude of the azimuth station was determined by Subassistant Smith, of whose work mention will be made under another head.

After his return to the office and completion of the computations, Mr. Sinclair was assigned to field duty in Section II.

Triangulation of the Mississippi River near Donaldsonville, La.—As stated under a preceding head, Assistant F. W. Perkins remained until the middle of April at work in Section VI. After transferring his party to the charge of Subassistant Vinal, for continuing the survey of the Saint John's River, Fla., Mr. Perkins started for New Orleans, and reached that port on the 27th of April, 1878. The schooner Research was at once fitted out for extending the survey of the Mssissippi above Donaldsonville, to which point the triangulation had been carried in previous seasons. As already mentioned under another head, a base-line was laid out on the east side of Donaldsonville.

This was properly connected with the preceding triangulation, and to the westward additional stations were selected for continuing the work up the river. Mr. Perkins will remain in the field beyond the close of the fiscal year.

Triangulation of the Mississippi River near Natchez, Miss.—After closing work which has been already mentioned, Assistant W. H. Dennis transferred his party to the vicinity of Natchez, Miss. Opposite to the city and near the west bank a short base was measured and connected with stations above and below Vidalia. Proceeding up the river additional points were selected and signals were erected for including about thirty miles of the course of the Mississippi. This work was begun on the 22d of April, and was continued until the close of the fiscal year.

In the immediate vicinity of Vidalia six stations were occupied and twenty-nine hundred and eighty-two observations were recorded in the measurement of horizontal angles.

Mr. F. C. Donn served as aid in the party on the Mississippi as well as in Barataria Bay.

Mississippi River survey at Vicksburg.—Early in April last, Assistant Charles Hosmer reached Vicksburg, under instructions for determining points by triangulation for the accurate survey of the Mississippi in the vicinity of that city. A base line was laid out near the terminus of the Vicksburg and Shreveport Railroad, and signals were erected at the ends of the line. These two stations were occupied with the theodolite, and points were selected in connection with them for defining the bend of the river above and below the city.

By the 12th of May the work at Vicksburg had been pushed beyond the limits for which means of transportation for this party were available. Hence, when a number of points had been determined for use in the detailed survey near Vicksburg, the party was transferred for similar service, which will be mentioned under the next head.

Mississippi River survey at Greenville.—For defining the bend of the Mississippi in the vicinity of Greenville, Assistant Hosmer marked out a base line on the east side of the river below the town. Signals were erected to mark the ends of the line and to serve also as stations for the theodolite. Other points were selected in connection with the base, but reconnaissance for that purpose was greatly impeded by the extreme high water. The work was however continued until the 24th of May. The synopsis of statistics below includes the work done in the vicinity of Vicksburg by the same party:

Signals erected	17
Stations occupied	16
Angles measured	151
Number of observations	793

The observations recorded at Vicksburg and Greenville determined twenty-seven points in position.

After the close of the fiscal year, Assistant Hosmer made arrangements for resuming field work on the coast of Maine, where he had been previously employed. Of that work mention has been made under the head of Section I in this report.

Latitude and longitude observations.—Under the general charge of Assistant G. W. Dean, who directed the requisite observations at Nashville, Tenn., Subassistant Edwin Smith was detailed in May last to determine geographical positions at points along the Mississippi River, the survey being then in progress. The operations conducted in person by Assistant Dean will be referred to under the head of Section XIII in this report.

At Nashville, Mr. Dean observed during four nights in May jointly with Mr. Smith for determining personal equation. The last-named observer then proceeded to Helena, Ark., and occupied a station in the court-house grounds. The observations there made were however referred to the spire of the court-house. In the last week of May, longitude signals by telegraph were exchanged with Assistant Dean.

Five favorable nights were employed by Subassistant Smith in recording observations for the latitude of the court-house at Helena.

After determining azimuth for the triangulation mentioned under the preceding head, Mr. Smith moved to Natchez, Miss., where he successfully exchanged longitude signals with Mr. Dean, who remained at Nashville. Mr. Charles Tappan, who had been previously in the service, effi-

ciently assisted in the work at Natchez. Cloudy and sometimes rainy weather delayed the observations, but by the 21st of June the latitude observations were completed.

At Vicksburg a point selected by the triangulation party for the determination of azimuth was occupied by Subassistant Smith as an astronomical station. Signals were exchanged on three nights preceding the 11th of July for longitude. The latitude of the same station was determined by observations during four nights.

Mr. Smith, accompanied by Mr. Tappan, reached Greenville, Miss., on the 13th of July, and after due arrangements observed for latitude at a station near the Episcopal church, referring the results for geographical position to the spire as at stations already mentioned. The latitude of the station was determined by observations continued through three nights.

Late in July, Mr. Smith returned to Nashville, and the month closed while observations for personal equation were in progress.

The original and duplicate records of the work of this party, together with the field computations, have been received at the office.

Field work previously conducted by Subassistant Smith has been mentioned under Section II and Section III in this report.

Triangulation, Mississippi River.—About fifty-two miles of the course of the Mississippi, including the stretch between the Helena base line and Bennett's Landing, have been included in the operations of the party conducted by Assistant C. H. Boyd.

After preparations made in November, 1877, the steamer Baton Rouge was moved from New Orleans up the river to the vicinity of Helena, and without delay an examination was made for the site of a base line. The ground finally selected ranges along the west bank of the river below Helena. A base upwards of two miles in length was measured by Assistant Boyd, and the ends were securely marked in the usual way. Signals were erected at the adjacent stations, and others in succession at points best suited for defining the course of the river in passing upwards.

At Helena the shore line of the Mississippi was traced and a topographical survey was made of the town.

Some of the lines of sight required for the triangulation entailed much labor and occasional sickness in the party, the selected points being accessible only by clearing avenues through swamp land, all of which is densely wooded.

At a position near the middle of the base line observations were recorded for latitude, and the azimuth of the line, which in that part of the work was of the average length of the triangle sides, was determined.

Master W. Kilburn, U. S. N., was attached to this party, and had charge of the vessel. He also assisted efficiently in the field operations. The statistics are:

Stations occupied	61
Angles measured	137
Number of observations	12,204
Shore lines traced, miles	21
Creeks and bayous, miles	34
Roads, miles	69
Area of topography, square miles	18½

Sixty-six points were determined by the triangulation.

Field work was discontinued at the end of June. Mr. C. H. Van Orden, who co-operated in the work on the Mississippi, was then assigned to service in Section I. Mr. Bion Bradbury served as pay yeoman and recorder.

Triangulation.—Soon after the opening of the present fiscal year, and while the field work was in progress at Gunter's Mountain, in Northern Alabama, Assistant F. P. Webber died in camp, of bilious remittent fever, as mentioned in my last annual report. By this sad event the charge of the triangulation party was devolved on Assistant F. D. Granger. Prof. A. H. Buchanan was at the same time temporarily attached to the party, and engaged in the measurement of horizontal angles while means were in progress for bringing into view the signal which had been set at "Summit," a station southwest of Guntersville. The work at Gunter's was completed on the 18th of

August, and without delay the instruments were transferred to a station near Huntsville. Professor Buchanan soon after resumed his field work in Tennessee, leaving Mr. Granger at Rowe's Mountain, where the angular measurements requisite were not completed until the 9th of October, 1877. Frequent rains and strong winds much delayed operations at that station. Of necessity, the tripod for the theodolite was sixty-seven feet above the summit of the mountain, and hence reliable observations could be recorded only during good weather.

Summit Station was occupied in the middle of October, and the requisite observations were completed there near the end of the month.

To the westward of Decatur a station has been selected and permanently marked, and will be occupied in the further progress of the work.

At the three stations occupied by the party in the course of the season, horizontal and vertical angles were measured. The statistics are:

Stations occupied	3
Number of pointings for horizontal angles	1,042
Number of pointings for vertical angles	1,349

During the winter Assistant Granger completed the computations and other office work pertaining to his field operations.

In May of the present year a general reconnaissance was begun by Mr. Granger of the region between the Mississippi River and Huntsville. Difficulties like those heretofore reported oppose the rapid prosecution of work. The summits are mostly about equal in height and are heavily timbered, both conditions being unfavorable for the selection of stations. Some in consequence will require elevated platforms for the theodolite; and at several, the requisite lines of sight must be opened through timber. Notwithstanding many impediments, the field report gives promise of the ultimate development of a scheme of triangulation with lines of the length required in primary work.

Assistant Granger was temporarily aided in the field by Mr. A. P. Barnard.

When the fiscal year closed, Mr. Granger was engaged in selecting stations in the neighborhood of Ripley, Miss.

Primary triangulation.—As already mentioned under the head of Section IV in this report, Assistant C. O. Boutelle, in the course of the fiscal year, closed at a station in North Carolina the chain of quadrilaterals that stretches from the Kent Island base line in Maryland southward and westward to the base line near Atlanta, Ga. The winter was employed in computations pertaining to that work. On the 1st of April Mr. Boutelle resumed field operations. After completing the arrangements of his party for work in Northern Alabama, he proceeded to the westward of the Atlanta base line and resumed work for extending the triangulation which had been in charge of the late Assistant Webber.

Assistant Boutelle, after making a reconnaissance in hope of finding stations that would be intervisible from the ground, was constrained to erect a platform seventy feet high for the theodolite. This structure is on Wilson's Mountain, in Northern Alabama. At the close of the fiscal year the twenty-inch theodolite was in position and the measurement of horizontal angles was in progress.

SECTION IX.

TEXAS AND INDIAN TERRITORY.—(SKETCH No. 16.)

Reconnaissance, coast of Texas.—For extending the survey by triangulation between Sabine Pass and the head of Galveston Bay a careful examination has been made by a party under the charge of Assistant H. G. Ogden. The party left Galveston on the 27th of December, 1877, and going northward and eastward traversed the Gulf coast for about fifty miles, noting in the journey the features of country that might avail for the purposes of the survey. This stretch of the coast of Texas is marked by a series of lakes between which and the beach the ground is marshy and in some places soft. "Beginning at the High Islands the country for twenty miles to the eastward is a continuous marsh four to five miles wide, and at only two places is it practicable to cross the marsh with horses. The adjacent Gulf shore is a fine beach, with a narrow sand and shell levee immediately back of it, having an elevation of one to three feet above the high-water line."

At a point about twenty-one miles east of the High Islands the beach terminates, and the remaining twelve miles of Gulf shore, on approaching Sabine Pass, is soft marsh. The course of the party was consequently deflected beyond the water line across Texas Point, over which by a very boggy trail Mr. Ogden reached Sabine City. His report on the results of the reconnaissance points out that beyond the broad belt of marsh the "Mound Prairie" affords firm foundation for the erection of signals. The prairie can be reached through the marsh by the two gaps or intervals before mentioned, the ground being such as to admit the passage of horses.

Assistant Ogden was accompanied in this reconnaissance by Mr. C. H. Sinclair. After noting and reporting what seemed practicable as facilities for the determination of points by which to define the shore line of the Gulf, and of the lakes and other features in its immediate vicinity, the party proceeded eastward from Sabine Pass, and reached the Mermentou River on the 21st of January. At the settlement it was ascertained that in advancing towards Vermilion Bay eight bayous would be encountered, and that they could be crossed only by swimming the saddle horses used in the reconnaissance. This was deemed inexpedient, as the weather was cold and forage too scant for animals under hardship. Assistant Ogden and his aid, in consequence, left their horses in pasture at Grand Cheniere, and hired a small light skiff that could be hauled up on the Gulf beach at the approach of bad weather. Big Freshwater Bayou was reached on the 28th of January, and from thence the party, going eastward, arrived at the western entrance of Vermilion Bay at the end of January. After making the usual notes, Mr. Ogden returned by the same route, and was again at Galveston in the middle of February.

The field report shows that for triangulation between Galveston and Vermilion Bay the main reliance must be on facilities afforded by the "Ridge Country," the characteristics of which are thus referred to by Assistant Ogden: "The ridges consist of narrow strips of firm land running parallel to the coast and to each other, or at very slight angles, but some appear as if thrown out of line in the vicinity of the water courses. They are from forty to four hundred meters wide, and have elevations of from three to five feet. Between the ridges there is always a strip of marsh, usually soft, and only passable on certain trails. Some of these marsh strips are several miles in width.

"Very few trees are found in this section except on the ridges nearest to the coast, and even on those the growth is stunted and sparse as far east as Le Bove Bayou, which is six miles west of Mermentou River. At that bayou a growth of large live oaks commences and continues eastward over Grand Cheniere and Pecan Island."

The report of Mr. Ogden was accompanied by copious notes and by a sketch showing a scheme of triangulation. The residents along the route followed manifested uniform kindness to the party. Thanks are due also for the attentions given by Mr. William Lane, superintendent of the United States engineer works at Bolivar Point, and by the assistant engineer, Mr. Heyward, at Sabine City. Mr. Frank Gonzales, Judge Henry, and L. J. Tansey, of Leesburg, Mr. William Griffith, and other residents to the eastward of Grand Cheniere showed special courtesies to the party in furtherance of the objects of the reconnaissance.

Triangulation of Laguna Madre, Tex.—At the opening of the fiscal year this work was resumed by Assistant R. E. Halter. In extending the triangulation south of Baffin's Bay, the only suitable place found for the camp was upwards of seven miles from any landing by which the party could receive supplies. The region is very sandy and access difficult. Among other hinderances are patches of salt, boggy slush, in passing which the men of the party sunk half-knee deep while carrying lumber for the erection of signals. Very little water was found in the laguna, and in some places there was none during the stay of the party. As before remarked, the depth depends almost entirely upon the wind, the shallow water in considerable stretches being forced southward by northers that while the wind prevails leaves the bottom of the laguna bare. At times the places thus left dry are fordable where the bottom is not boggy. The statistics of work done by the party in the course of the fiscal year are:

Signals erected	17
Stations occupied	13
Angles measured	169
Number of observations	12,244

In the course of the season now entered upon it is hoped that the triangulation may be completed to the Rio Grande boundary.

Magnetic observations.—In accordance with the system arranged last year by which observations for the magnetic declination, dip, and intensity will be restricted to stations that most readily combine in results with the results already obtained, a series of points was marked in May last for occupation in the course of the calendar year. The field work was intrusted to Mr. James B. Baylor. At the end of May, Dollar Point, on the coast of Texas, was occupied, and in the following month Austin, Hempstead, and Groesbeck, in the interior, and also a position at Vinita, in Indian Territory. (See sketch No. 25.)

At the close of the fiscal year, Mr. Baylor was at Fort Worth, which station with others, making in the aggregate about thirty positions between the Gulf of Mexico and Ogden, near Salt Lake, will be included in the work of the season. The statistics of the work and particulars in regard to the stations will be given in my next annual report.

SECTION X.

CALIFORNIA.—(SKETCHES NOS. 17, 18, AND 19.)

San Diego Harbor, Cal.—To meet inquiries, which were frequent in the course of the winter, concerning the present condition of the harbor at San Diego, the party of Lieut. Commander G. W. Coffin, U. S. N., Assistant Coast and Geodetic Survey, was detailed at the end of January, 1878, to make a partial survey of the bar and parts of the bay. Soundings were continued in that locality until the end of March, 1878, when the steamer returned to the vicinity of Catalina Island, where the party had been previously engaged. The work done there will be the subject of mention under a subsequent head.

The survey made by Lieutenant-Commander Coffin at San Diego shows that the curves of different depths have not materially changed within the last six years. It is also noticed in his report that freshets in San Diego River seem not to have affected the harbor or the outside bar. The soundings were made during a very severe winter, in the course of which the hydrographic party had reason to note the character of the place as a harbor of refuge. Special mention in regard to safety and smoothness is made in the report of Lieutenant-Commander Coffin. The statistics of the work are:

Miles run in sounding	352
Angles measured	2,424
Number of soundings	10,202

Lieutenant-Commander Coffin was assisted in the survey at San Diego by Lieuts. W. W. Gilpatrick, F. J. Drake, C. W. Jarboe, and W. H. Driggs, U. S. N., and by Master R. Mitchell, U. S. N. In this and other localities in which the party worked in the course of the year, ninety signals were erected for the adjustment of soundings.

Triangulation across the Santa Barbara Channel.—In order to complete the geodetic connection between the Santa Barbara Islands and stations of the primary triangulation along the adjacent coast of California, arrangements were made in the spring of the present year for occupying a point on Santa Barbara Island.

Mr. D. B. Wainwright, to whom this work was committed, erected signals at San Pedro, and Las Bolsas on the main. A third was put up on Catalina Peak, and the fourth on the summit of San Clemente Island.

In June, after closing the angular measurements at Santa Barbara Island, Mr. Wainwright moved to the station on San Clemente, and was there engaged at the close of the fiscal year to which this report corresponds in time.

Catalina Peak will be occupied, if practicable, in the course of the season. As usual, the fogs have been heavy and so continuous as to afford only occasional opportunity for the measurement of angles. The work now in progress it is hoped will serve for determining in relative position all the islands in the vicinity of the Santa Barbara Channel.

Hydrography of the Santa Barbara Channel, Cal.—With his party, in the steamer McArthur, Lieut. Frank Courtis, U. S. N., Assistant Coast and Geodetic Survey, filled five sheets with soundings between the opening of the fiscal year and the end of December, 1877. Two of these repre-

sent the inshore hydrography to the eastward of Point Concepcion, and a third shows the character of the Santa Barbara Channel between the main and San Miguel and Santa Rosa Islands. On the fourth sheet the hydrography of the vicinity of Point Arguello is developed, and the space between that survey and Point Concepcion is represented by the fifth sheet of soundings. Tides were observed at Gaviota while the hydrography was in progress. Lieutenant Courtis was assisted by Lieuts. E. H. C. Leutze and E. K. Moore, U. S. N., and by Masters J. H. Bull and R. H. Galt, U. S. N.

At the eastern end of Santa Barbara Channel the hydrography was resumed early in April, with the same party, in charge of Lieutenant Leutze, and the work was continued through the fiscal year. Soundings inside of the Santa Barbara Islands are now continuous between Point Dume and Point Concepcion. During the present season the tides have been observed at Point Hueneme. In addition to the officers already mentioned as attached to the McArthur, Lieutenant Leutze has been assisted by Lieut. E. S. Prime, U. S. N., who joined the party in March last.

Two hydrographic sheets have been filled with soundings by the work done between March and July of the present year. A third sheet of work is in progress to represent soundings to the eastward of Anacapa Island. The aggregate statistics of work done under the direction of Lieutenants Courtis and Leutze are:

Miles run in sounding	1,410
Angles measured	4,192
Number of soundings	14,214

Topography of Catalina Island, Cal.—The detailed survey of this island was resumed by Assistant Stehman Forney, on the 9th of April, and was continued during May and June. Operations have been much retarded by fog and rain, but the work will advance during the summer. At the opening of the present fiscal year it was estimated that the plane-table work could be finished by the 1st of September. This report, as usual, takes account of work done previous to the 1st of July; hence, the details prosecuted after that date will be the subject of notice in my next annual report. At the end of June, of the present year, the statistics of work done in the survey of Catalina Island were:

Signals erected	64
Miles of shore line surveyed	6
Square miles of topography	18½

Contour lines on the plane-table sheet of this season represent elevations of as much as sixteen hundred and seventy feet.

Hydrography near Catalina Island, Cal.—In September, 1877, after the completion of work, which will be mentioned under a separate head in the next section, the steamer Hassler returned to San Francisco and underwent needful repairs. Lieutenant-Commander Coffin, with his party in that vessel, left port on the 5th of December, and commenced soundings in the approaches to Catalina Island. The work there was continued until the 22d of January, when the party was assigned to service at San Diego, as already stated in a preceding item in this report.

At the end of March, 1878, the Hassler returned to Catalina, and resumed the hydrography of the approaches to that island. By the close of June the work done had included the western approach, and soundings along the north and south sides, to points about midway in the length of the island.

While working on the north side the Hassler was anchored every night either in Isthmus Cove or Dakin's Cove. Respecting these, as places of shelter, Lieutenant-Commander Coffin remarks: "Vessels will find in either a good lee with comparatively smooth water. As the island is high, and the approaches bold, vessels may with safety get close in where they will find the wind very much broken. These are excellent summer anchorages, as Long Point and Red Point project sufficiently to turn aside the strongest northwest winds, and in neither of these two coves is there ever much swell.

"On the north side of the island only one anchorage can be used (Catalina Harbor), and that, in my opinion, is not so good a southeast or northwest anchorage as Isthmus Cove, in which there is more room for veering. There are many good boat landings on the north side of Catalina Island, but only two on the south side."

At the end of the fiscal year for which this report is made up, the statistics of hydrographic work in the vicinity of the island were:

Miles run in sounding	239
Angles measured	2,066
Number of soundings	1,427

Lieuts. W. W. Gilpatrick, F. J. Drake, C. W. Jarboe, and W. H. Driggs, U. S. N., and Master R. Mitchell, U. S. N., were attached to the hydrographic party in the steamer Hassler.

Topography of Point Arguello, Cal.—At the opening of the fiscal year which began July 1, 1877 Assistant A. W. Chase was advancing the topographical survey of the coast of California in the vicinity of Point Arguello, as stated in my last annual report. The detailed work was continued northward to a station several miles above Point Padernales. In general all the features of ground within two miles of the coast line are represented on the plane-table sheets. For the use of the light-house service, tracings have been furnished to Lieutenant-Colonel Williamson, engineer of the twelfth district. The topographical survey was closed at the end of August.

Survey south of Point Sal.—For defining the coast northward of the limits reached in the work mentioned under the last head, Assistant W. E. Greenwell took the field in April last. His own triangulation of a previous year had reached to stations above Point Arguello. Others were occupied to the northward as far as Point Sal. The plane-table survey had been previously advanced as far as the Ynez River, where the topography was taken up by Mr. Greenwell and extended northward within the limits of his triangulation. The work was in progress at the close of the fiscal year, and will be the subject of further mention in my next annual report. The early part of the fiscal year was passed by Assistant Greenwell in the vicinity of Los Angeles, in the endeavor to advance the coast triangulation, but continued hazy weather made it inexpedient to continue at that site of work.

In the course of the autumn and winter a detailed survey was made of the vicinity of the town of Santa Barbara. All the artificial features which were mapped at that time have been marked on the original sheet of the neighborhood of the town.

Topography south of Point Sur, Cal.—The topography of the coast of California, in the vicinity of Point Sur, was resumed by Assistant A. F. Rodgers early in November, 1877. Along the entire region the coast is a succession of heights separated by deep and narrow gorges. These are filled with a growth of timber, and the sides of the gorges are too steep to admit of the use of instruments for delineating the adjacent features of topography.

In previous seasons the plane-table survey of the coast had been extended from Monterey Bay to a point about four miles north of Point Sur. For continuing the work, Mr. Rodgers arranged a temporary camp to the northward of the point and mapped in the ground southward of Sierra Cañon, at which the previous detailed survey had been suspended. In the latter part of January he joined with the local survey of Point Sur, and then continued southward and eastward of Cooper's Point, which is about six miles distant from the first-named headland. The plane-table sheet represents the coast features, the outlying rocks, and the contours found within about a mile and a quarter of the shore line.

Until the close of the fiscal year, Mr. D. B. Wainwright served as aid in this party. He was then assigned to the charge of work which will be the subject of notice in my next annual report. Assistant Rodgers at the same time resumed the reconnaissance for trans-continental triangulation. The statistics of the topographical work near Point Sur are:

Shore line surveyed, miles	14½
Creeks, miles	8
Roads, miles	5
Area of topography, square miles	12

Tidal observations.—Details pertaining to the tidal records of the Pacific coast have been directed for some time by Prof. George Davidson, Assistant Coast and Geodetic Survey. Among these was a plan for the transfer of the tide gauge from Fort Point, on the south side of the Golden Gate, to Saucelito, which is inside of San Francisco Bay, and not far from the north side of the

entrance. When the self-registering tide gauge was started at Saucelito a temporary observer was stationed at Fort Point for the purpose of establishing good tide levels between the two places. The object was satisfactorily accomplished in November, 1877, when the Fort Point record was closed. The tide gauge at Saucelito is now working well under the care of Mr. E. Gray.

Triangulation.—Assistant George Davidson, while occupying Mount Helena and Mount Diablo, observed on signals over lines ranging in length from one hundred and twenty to one hundred and forty-two miles. The horizontal lines were measured with a twenty-inch theodolite, in which, as the work advanced, Professor Davidson noticed certain peculiarities of the microscope micrometers, and in order to verify the results obtained with the instrument, a careful examination was instituted. For testing the graduation, upwards of fifty-two thousand readings with the micrometers were recorded. In a special report, Assistant Davidson furnished data which will not only be applied in the reduction of the angular measurements and the azimuth determinations, but afford means for guidance in the construction of new instruments, and point out the class of errors to be looked for in all theodolites.

The field records of work done in the Davidson quadrilateral have been duplicated and placed in the office. These comprise in twenty-seven volumes the observations for horizontal and vertical angles, and for time, latitude, and azimuth; also descriptions of the stations.

In previous reports mention has been made of the necessity for irrigation in some of the valleys of California, and of the part taken by Mr. Davidson in developing methods for the purpose. His remarks on the systems now in practice in Europe and India, when published, led to an official request, which was met by an oral address before the legislature of California. In this, Assistant Davidson brought before the public a comprehensive review of the subject as applicable to the great valleys of that State.

Under the direction of Mr. Davidson the compilation of material for the Coast Pilot of Oregon, Washington Territory, and California has been continued by Assistant Gershom Bradford. The surface and subcurrents of San Francisco Bay have been plotted for each quarter of the flood and ebb tides; and to aid in the study of the currents, Assistant Davidson projected a chart on which the bottom of the bay is shown by lines of contours derived from the hydrographic sheets.

Subassistant B. A. Colonna and Mr. J. F. Pratt aided in the operations conducted by Assistant Davidson in this section and also in Section XI. Mr. D. B. Wainwright, during part of the season, was employed in the sub-office at San Francisco.

On the 6th of May last, Mr. Colonna observed the transit of Mercury across the sun's disk from a station near the Central Pacific Railroad. Mr. Pratt observed the transit with a separate instrument. The station occupied has an elevation of rather more than seven thousand feet above the ocean level. Assistant Davidson had previously arranged for these observations, but was at that date on his way to attend the International Exposition at Paris, where he will inspect and report upon instruments intended for geodetic purposes and practical astronomy, hydrography, and other branches of work cognate to the operations of the survey.

The field work conducted by Assistant Davidson in the autumn of 1877 will be mentioned in the next chapter. During his absence in Europe, the office details in San Francisco were directed by Assistant Rodgers. The calls for information contained in the records and original sheets of the survey, involve as heretofore a wide range in subjects, and much care and attention additional to the work of the assistant who meets the inquiries.

Reconnaissance.—When my report of last year closed; Assistant Cleveland Rockwell was engaged in selecting points for the main triangulation which is to pass northward from the vicinity of Point Arenas. Beginning at Sanel Mountain and Walalla, both of which had been observed upon from primary stations to the southward, the country was closely examined to the northward, and two points were chosen for completing the quadrilateral. While passing on towards Mendocino City several subsidiary peaks were visited as alternative points in the scheme, but stations beyond were finally chosen. In this region it was found extremely difficult to observe on distant points because of the density of the redwood forest. North of the Noyo River the Cahto range of mountains crossed the direction of the intended chain of quadrilaterals. Of necessity a point of that range was selected, and to regain the scale desirable in regard to length of lines, a lateral station was added on the eastern side of the scheme, by means of which suitable lines were projected

to the vicinity of Shelter Cove and to a peak at the south fork of Eel River. Both of these last-named points had been under consideration when Assistant Rockwell visited the same region last year.

Above Shelter Cove a good figure was obtained by including as stations of the scheme Rainbow Mountain and Lariby Butte, but with a disadvantage in respect of distance from the coast. At Humboldt Bay a station was included, and in connection with it a summit on the east side of Mad River. North of these points careful search was extended, but without success in the addition of quadrilaterals. In general, therefore, after marking two summits to the northward, Mr. Rockwell went back and occupied an interior point at which four triangles would converge. The last figure added to the scheme carries the plan of triangulation to the vicinity of Cape Blanco. Some of the points from which observations were made are upwards of four thousand feet above the level of the sea. During the summer the heat of the rocks in the ascent was found very oppressive by the party. Much of the region traversed is extremely rough and destitute of trails.

On the 9th of October, while Mr. Rockwell was yet in the field, the rains set in and the streams were quickly filled so that crossing became dangerous. As it was adjudged expedient to defer operations, the party returned by way of Portland and Astoria to San Francisco.

From the records kept by Assistant Rockwell a full report was made showing the conditions of each of the selected stations with regard to accessibility, facilities for wood and water, and the amount of cutting needful to open the several lines of sight. Some of the points included in the scheme are bare of trees, other summits are covered by brush, and some of them by heavy timber.

Of upward of fifty stations, from which angles were measured with a small theodolite, nineteen were included in the plan for triangulation. Incidental determinations were also made, for which Mr. Rockwell plotted seventy other points in approximate positions.

Transit of Mercury.—As already mentioned, the transit of the 6th of May was successfully observed by Assistant Colonna and Mr. Pratt at Summit, a station on the Central Pacific Railroad. The position selected for the instruments is seventy-one hundred and fourteen feet above the sea level, and only about one hundred feet above the railroad track. A hill near by would have been occupied in preference on account of the greater elevation, but as the snow was five feet deep in that entire region on the 1st of May, it was found impracticable to reach the summit of the hill with the requisite instruments. Hence the transit instrument was mounted on a pier near the tank-house on the north side of the railroad. Time was observed on the night of the 1st, and on the 2d of May, telegraph wires from the Western Union office were in connection with the chronograph at the observing station. For this essential of the operations special direction had been given by James Gamble, esq., general superintendent of the Pacific Division of the Telegraph Company.

On the 5th of May all the instruments were in readiness, and time signals were exchanged between Mr. Colonna and the United States Naval Observatory at Washington.

The morning of the 6th opened cloudless; the temperature in the observing tents was only 27° Fahr., and the chronograph ink was frozen hard. All the appliances were, however, carefully and gradually warmed and adjusted for final use.

At noon Assistant Colonna observed the sun's limbs and the planet as they crossed the middle wires of the transit instrument. Mr. Pratt at the same time recorded and attended to the chronograph. The day remained cloudless, and the afternoon observations were recorded under very favorable conditions. At night, observations for time were repeated. The exchanges by telegraph with the Observatory at Washington were maintained until the 10th of May. Subsequently the longitude was determined by telegraphic exchanges between Mr. Pratt, who returned to San Francisco for that purpose, and Mr. Colonna, who remained at the transit station near Summit. Mr. Pratt also made time observations at the Coast Survey station in San Francisco, and there was efficiently assisted in the several operations by Messrs. E. F. Dickins and T. A. Harrison, who cheerfully volunteered their services.

Assistant Colonna at all favorable intervals recorded observations for the latitude of the transit station, and concluded that work on the night of the 13th of June. Mr. Robert Hughes, telegraph operator at Summit, served as recorder of the observations made by Mr. Colonna for latitude and longitude. The computations were made without delay at San Francisco and forwarded to the office.

SECTION XI.

OREGON AND WASHINGTON TERRITORY.—(SKETCHES NOS. 19 AND 20.)

Hydrography of the Columbia River approaches, Oreg.—This work was prosecuted during the summer of 1877 by Lieut. Commander G. W. Coffin, U. S. N., Assistant Coast and Geodetic Survey, with a party in the steamer Hassler. Until August of that year, heavy rains and dense fogs retarded the progress of the hydrography, but the weather was unusually good during the month preceding the 1st of September when the soundings were completed. This survey has been much in request, the unknown character of the changes being serious hindrances to the shipping interest of Astoria and Portland. For some time vessels have been very careful not to approach the bar close to, and consequently could not avail themselves of opportunities for entering, the changes of some years not being developed previous to the execution of the work now under notice. The last edition of the chart shows that the character of the bottom at the bar changes almost uniformly with the curves of equal depths, and that these curves conform nearly to the outline of the coast. Hence the bar may be approached even in thick weather if due precaution is observed in sounding. The statistics of the recent survey are:

Miles run in sounding	507
Angles measured	1,707
Number of soundings	2,791

Lieutenant-Commander Coffin was assisted in work at the Columbia River approaches by Lieuts. Richardson Clover, F. J. Drake, C. W. Jarboe, and W. H. Driggs, U. S. N., and by Master R. Mitchell, U. S. N.

Survey of the Columbia River, Oreg.—At the opening of the fiscal year to which this report corresponds in time, Assistant J. J. Gilbert was engaged in prosecuting the hydrography of the Columbia River. In July and August, 1877, the work steadily advanced, and early in September of that year the soundings were completed from Mount Coffin to a point one mile above Kalama. At the same period the topography of the shores was brought to Kalama from the vicinity of Cottonwood Island. During September and October Mr. Gilbert was engaged in erecting signals and preparing lines of sight for the triangulation above Kalama. Tides for half a lunation were recorded simultaneously at Cathlamet, Oak Point, Rainier, Saint Helen's, Vancouver, and Warrendale for determining a plane of reference for the hydrographic sheets of the Columbia.

Field work was continued until the end of October, when the schooner Kincheloe was laid up for the winter. Assistant Gilbert then returned to Olympia and completed the records and computations. The hydrography is represented on two sheets; a third shows the topographical features between Cottonwood Island and Kalama. The general statistics are:

Signals erected	4
Miles of shore line traced	19
Sloughs, outline, in miles	7
Miles of road	39
Area of topography, square miles	15
Miles run in sounding	166
Angles measured	2,226
Number of soundings	13,382

The subsequent operations of the party of Assistant Gilbert will be detailed under another head in this section.

Reconnaissance.—At the opening of the fiscal year, Assistant J. S. Lawson was yet in the field engaged in selecting stations for primary triangulation to include the great inland waters of Washington Territory. In July of the present year, Mr. Lawson returned to Point Partridge, and from thence commenced opening a line of sight towards Watmough Head. The only anchorage available for the brig Fauntleroy being affected by irregular currents and tide rips, it proved to be impracticable to work from the vessel. Mr. Lawson was in consequence constrained to pitch a

camp on the west side of Whidbey Island at a point about midway in the avenue which required opening. Meanwhile the vessel was taken to a secure anchorage in Port Townshend.

The primary station on Discovery Island was next examined, and avenues were opened for observing from that point on signals at Point Partridge, New Dungeness, and Striped Peak. Mount Shepherd being in view, a signal was put up on that summit to be observed on from the station at Discovery Island.

Proceeding westward, Assistant Lawson met and overcame many obstacles in bringing into view a signal on Sherringham Point. From the last-mentioned position the most distant land in view to the westward was the Sombrio Ridge, which being evenly wooded offered no particular site for a triangulation point. After moving the vessel into Clallam Bay, Mr. Lawson chose the highest part of the ridge, and by observation found it to be nearly midway between Pillar Point and Slip Point. There the party worked from a camp, and cleared avenues of sight to the adjacent stations.

In the Strait of Fuca fogs were prevalent and permanent during July and August, 1877. In the following months rains were frequent, and as a result nearly all the hands of the party were sick. Field operations were discontinued on the 6th of October. A few days after, the vessel was laid up for the winter at Olympia and the crew was discharged.

Assistant Lawson resumed the reconnaissance for primary stations early in April of the present year, and occupied Point Partridge with the theodolite, but because of haze or mist the signals adjacent to that station were rarely seen during that and the following month. In June a few observations were recorded in the measurement of horizontal angles.

The results gained by continuing field work during the summer will be a subject of mention in my next annual report.

Reconnaissance for base lines.—Many of the known sites in Washington Territory had been under consideration when Assistant George Davidson was continuously engaged in carrying on the triangulation over the waters of Washington Sound, but difficulties more or less serious appeared in each case when the ground was examined. Of the sites deemed practicable one on Whidbey Island was reserved for final examination, and to that end Mr. Davidson took the field in August, 1877, attended by Subassistant B. A. Colonna and Mr. J. F. Pratt. In September a trial base was opened through timber on the northwest part of Whidbey Island. The line extends from a point on Deception Pass to the wooded ridge due south, and is nearly five and a half miles in length. It opens broadly upon the Strait of Fuca, and connects well with the triangulation, but the site presents many difficulties that could be surmounted only by great labor and corresponding outlay in means.

In the Upper Willamette Valley the conditions found were far more favorable. Early in October, Assistant Davidson established the general location of a base line about fifteen miles in length, and from which the development of the main triangulation would be rapid and satisfactory. The lateness of the season precluded the definitive location of the line and of the southwest station of the first quadrilateral, but a detailed report has been made in regard to the site and its advantages.

Hydrography of Admiralty Inlet, Wash. Ter.—For the work in this quarter a party was organized at the outset of the fiscal year, and placed under the charge of Lieut. Richard M. Cutts, U. S. N., Assistant Coast and Geodetic Survey. As soon as practicable, soundings were begun in Commencement Bay, and that branch of the inlet was thoroughly developed. After furnishing a tracing of the resulting chart to the general superintendent of the Northern Pacific Railroad, the party in the schooner Yukon resumed operations at the northern end of Admiralty Inlet, where some progress had been made in June, 1877. The vessel was laid up at the end of November, after being at work afloat during every day available for soundings. In the course of the season, the hydrography of the inlet was advanced southward from Battery Point to the entrance of Colvos Passage, and south of Dolphin Point ten lines of soundings were run at about equal intervals quite across the inlet. Soundings were recorded along the east and west shores, between Brace Point and Robinson's Point, and also to the eastward of Dash Point.

In April of the present year soundings were resumed in Admiralty Inlet, and were continued during the summer.

The statistics of the hydrographic sheets which have been completed and turned in are:

Miles run in soundings	464
Angles measured	3,086
Number of soundings	12,247

At the close of the fiscal year Lieutenant Cutts transferred his party to the schooner Earnest, for continuing the hydrography. In the operations of the year he has been assisted by Lieuts. A. B. Wyckoff and U. Harris, U. S. N.

Triangulation and topography of Puget Sound, Wash. Ter.—At the outset of the fiscal year which began on the 1st of July, 1877, Subassistant Eugene Ellicott was prosecuting the topographical survey of the shores of Commencement Bay, including the tide flats at the mouth of Puyallup River, which is navigable through eight or ten miles of the lower part of its course. This stream emerges from the glaciers of Mount Rainier, and empties into the southeastern extremity of Commencement Bay. The region is fertile, and the coal mined within thirty miles of the bay is said to be the best yet found on the Pacific coast.

After completing the work last mentioned, Mr. Ellicott moved his party to Anderson Island, and starting at a line of the triangulation in that vicinity extended the triangulation of the southern extremity of Puget Sound quite into Budd's Inlet.

At the same time the shore lines of the Narrows were traced, and going southward that work was prosecuted as far as Ketron Island, below Steilacoom. The town and its environs are included in this survey.

When the working season closed in this section, Mr. Ellicott stored his camp equipage at Seattle, and was employed during the winter in completing his office work, and inking the sheets of the survey. Early in April, 1878, the party was reorganized, and took the field for the triangulation of Carr's Inlet, one of the large interior branches of Puget Sound. The work was extended to the head of the inlet, and also through the narrow passage westward of McNiel Island. The topographical survey was then resumed at the mouth of Nisqually River, and carried westward to Budd's Inlet. By this last-mentioned work the triangulation and topography of the main passage have been made continuous between Seattle and Olympia.

At the close of the fiscal year Mr. Ellicott was engaged in pushing the detailed survey so as to include the remaining branches at the southwest end of Puget Sound. The statistics of work reported at the end of June of the present year are:

Signals erected	79
Stations occupied	69
Objects observed on	85
Angles measured	590
Number of observations	14,022
Shore line traced, miles	140
Rivers and tidal creeks, miles	52
Roads, miles	21
Area of topography, square miles	79

The progress made during the last fiscal year in defining the shores of the upper part of Puget Sound gives promise of the completion of that work in the course of the present year. The shores of Possession Sound (called also the Lower Sound) are now attracting settlers, especially in the fertile region about Skagit River. As soon as practicable the work of the survey will be extended to that quarter.

Triangulation and topography of Hood's Canal, Wash. Ter.—Early in May of the present year Assistant J. J. Gilbert reorganized his field party, and with a suitable outfit proceeded to the entrance of Hood's Canal. A steam launch had been previously provided for the transportation of the party. Near the entrance of the canal, stations occupied in a previous year were identified. Points adjacent were selected, and additional stations on both sides of the canal, in going southward or westward toward Hazel Point. During May and June the triangulation was extended about fourteen miles. The topography of the shores was commenced in the latter part of May, and when the fiscal year closed six miles of the course of the canal had been surveyed.

The statistics of the field work are:

Signals erected	20
Stations occupied	16
Objects observed on	87
Angles measured	115
Number of observations	6,942
Shore line surveyed, miles	17
Area of topography, square miles	3

When the last report was received from the field, the prospect was good for progress during the summer and autumn.

Inspection.—Under special directions Assistant A. F. Rodgers proceeded from San Francisco early in September, 1877, and visited the several parties at work on the shores of Puget Sound, Wash. Ter. The work of triangulation and topography in progress between Battery Point and Nisqually River was carefully examined, as were also the records. Budd's Inlet was visited, and others in that vicinity, with a view of deciding on the practicability of extending the triangulation of the sound from its present southern limit across the land known as the "Portage," between the waters of the inlet and Hood's Canal. The Nisqually was found to be very narrow, with several outlets opening upon extensive flats covered with drift timber and snags. Leaving this quarter, Mr. Rodgers next moved to Commencement Bay, and examined the ground bordering the terminus of the Northern Pacific Railroad at New Tacoma. In the construction of the wharf there, piles one hundred feet in length were used, but, as on the coast of California, it was noticed that the *teredo* is very destructive. Piles are eaten away in limits of time varying from six months to as much as six years. The railroad company's latest structure at New Tacoma rests upon coppered piles.

From Seattle Mr. Rodgers went north, through Possession Sound, and noted the character of the streams that empty into it. The shores of Lake Washington were examined, and the course of the Skagit River, which, it is said, was once navigable by light-draught steamboats upwards of thirty miles above its mouth. From the Skagit the reconnaissance was continued northward to La Conner on the southern extremity of Fidalgo Island, and from thence Mr. Rodgers returned to Seattle and Tacoma by way of Port Susan and Possession Sound. Subsequently he met Assistant J. J. Gilbert, by appointment, at Kalama, on the Columbia River, and examined the details of the plane-table work then in progress. Proceeding next by way of Portland, Mr. Rodgers went up the river to Dallas City, and during the delay incident to the transfer of freight from the upper river walked over the Cascade Portage on his return. This is a stretch of six miles in which the transfer of an enormous yield of grain is greatly interrupted in its passage to the coast.

Early in October Assistant Rodgers returned to Kalama and examined the river shores as far up as Columbia City, at which point he took passage for Portland, and from thence returned to San Francisco. His field operations on the coast of California have been described in the preceding chapter.

SECTION XII.

ALASKA TERRITORY.

Coast of Alaska.—The compilation of material for the Coast Pilot of Alaska has been continued by Assistant W. H. Dall. Hydrographic notes, and others pertaining to navigation, have been completed for the coast and channels south of Cape Spencer. The meteorological appendix contains an exhaustive summary of all observations recorded in Alaska, and a general discussion of the weather, currents, and periods of navigation pertaining to that region, illustrated by charts of Isothermal and Isobaric lines, profiles, &c. This paper embodies much interesting matter heretofore unpublished.

Further progress has been made by Mr. Dall in collecting the titles of maps, charts, and publications descriptive of the hydrography of that coast. Some twelve hundred titles have been collected. Most of the works cited have been examined and notes taken of such of the contents as bear upon navigation. In addition to matters of general interest, Mr. Dall has in recent years

contributed for public use several papers of value bearing on the scientific results of explorations in Alaska. Among these are notes descriptive of the human remains found in the caves of the Catharina Archipelago and of the Aleutian Islands.

Tidal observations.—The self-registering tide gauge lent to the Hawaiian Government survey in the summer of 1876 for use at the Sandwich Islands, in the Pacific Ocean, reached Honolulu in good order, and as soon as practicable was set up in that harbor by the Superintendent of the Survey, W. D. Alexander, esq. It is known that the apparatus worked well during the past year, although at the end of June, 1878, the tidal rolls from the gauge had not reached the Coast Survey Office. They are doubtless on the way to Washington, and will be expected at the office before the close of the year.

Data furnished by the records of the Honolulu station will give ready means for investigating the relations of our Western coast tides to those of the open sea in the Pacific Ocean.

SECTION XIII.

KENTUCKY AND TENNESSEE.—(SKETCHES NOS. 23 AND 24.)

Geographical positions.—For the determination of latitude and longitude at several points in the vicinity of the Mississippi River, a party was organized early in July, 1877, to work under the direction of Assistant G. W. Dean. The requisite instruments were sent from the office, and Assistant William Eimbeck placed in position at Columbus, Ohio, such as were intended for use at that station. A second set of instruments in charge of the aid, Mr. H. W. Blair, was set in place at Nashville, Tenn. To facilitate in longitude determinations, President Orton, of the Western Union Telegraph Company, afforded the use of the lines and the transmission of messages free of charges as heretofore; and by the kindness of Colonel Van Horne, general superintendent of the Southern Division, a wire was promptly run from the telegraph office to the astronomical station. All the instruments were adjusted and observations commenced by the 27th of July. On the first clear night after that date, clock signals were exchanged between the observers at Columbus and Nashville. In the course of the month of August, signals were exchanged also between the same observers and an observer at the United States Naval Observatory in Washington City.

Early in September, Assistant Eimbeck, aided by Mr. F. H. Parsons, removed the instruments from Columbus and adjusted them at Paducah, Ky., from which point he exchanged time signals with the Nashville station on three successive nights preceding the middle of the month. The latitude observations at Paducah were completed in the week following; and a few days after, the instruments there in use were transferred and set up at Cairo, Ill. Until the 11th of October, the weather was unfavorable for longitude exchanges, but in clear intervals preceding the middle of that month the requisite observations were recorded. Meanwhile, Assistant Dean and Mr. Blair, at Nashville, successfully observed several series of star transits for determining their personal equation; and subsequently several sets of observations were obtained with the Personal Equation apparatus. At the same time a second series of longitude signals was satisfactorily exchanged between the Nashville station and the Naval Observatory at Washington.

After completing the records at Cairo, Assistant Eimbeck occupied a station at Hickman, Ky., and during three nights, closing with the 27th of October, successfully exchanged time signals with the observers at Nashville. Removing next to Memphis, Tenn., signals were well exchanged during five nights with the Nashville station. Mr. Eimbeck, after closing work at Memphis, repaired to Nashville to observe for personal equation, but, as in a previous attempt for the same purpose, continuous bad weather made it inexpedient to wait there for the desired opportunity.

Late in November the astronomical instruments and telegraph apparatus were stored in a room of the State House at Nashville. When the weather permitted, the observations needful there for the determination of latitude were made by Mr. Blair. The record at Nashville was completed on the 2d of December. Between that date and the previous July, longitude signals were exchanged with the Naval Observatory on thirteen nights, and between Nashville and Columbus on fourteen nights. Cairo, Paducah, and Hickman were determined by signals exchanged between each point and the observer at Nashville on three nights, and in like manner the position of the observing point at Memphis was fixed by exchanging signals on five nights.

The clock and instrumental corrections were determined from six hundred and ninety-four transits of twenty-one circumpolar and seventy-three standard stars near the zenith, each star being usually observed on fifteen or twenty-five threads. The micrometer value was tested by observing on circumpolar stars during several nights. For latitude, Mr. Blair recorded ninety-eight observations on twenty-three pairs of stars. The original records and duplicates, including also eighty chronograph sheets pertaining to the operations for determining longitude, have been deposited in the office.

At Memphis, Assistant Eimbeck established the astronomical station in the yard of the custom house. Good observations for latitude upon seventeen pairs of stars were recorded on four nights preceding the middle of November, and others were at the same time made for determining the micrometer value.

During the winter Mr. Eimbeck was employed in the office, but when means became available for further field work he was assigned to duty in the selection of points for the trans-continental triangulation, as will be mentioned under a separate head in this report.

As already mentioned under the head of Section VIII in this report, the astronomical observations needful for determining geographical positions along the Mississippi were resumed at Nashville, Tenn., in May, 1878. Assistant Dean and Subassistant Edwin Smith, who had been detailed for the work in Section VIII, made separate determinations of the local time by observing the same stars over the same meridian on the same nights. The instruments used in these trials were employed by the same observers for the subsequent longitude determinations. Four nights were occupied in observing at Nashville for personal equation. Mr. Smith then proceeded to Helena, Ark., as already stated under another head. Mr. Dean remained at Nashville, and exchanged telegraphic signals for difference of longitude during three consecutive nights. The wires employed gave a continuous line of three hundred and forty-seven miles, and no telegraphic repeaters were used in the circuit.

The month of June opened with cloudy weather at Nashville, but on three nights preceding the 21st Assistant Dean exchanged clock signals by telegraph with Subassistant Smith, who was then at Natchez, Miss. The distance being about six hundred and ninety miles, a telegraphic repeater was found necessary at Montgomery, Ala.

Early in July exchanges of signals were obtained during three nights for determining the longitude of Vicksburg. The same telegraphic repeater was used at Montgomery, the line to Vicksburg being about six hundred miles in length.

Subassistant Smith occupied a station at Greenville, Miss., in the middle of July. The line from Nashville being seven hundred and fifty miles long, and part of it imperfectly insulated, repeaters were found to be requisite at Montgomery and Vicksburg. The usual exchanges were recorded during three nights. Success in overcoming the difficulties on this line were due to the personal exertions and courtesy of Mr. James Compton, district superintendent of the Western Union Telegraph Company. The condition of the line between Vicksburg and Greenville was such that although ordinary messages could be transmitted the line could not have served for longitude determination without the improvements made by Mr. Compton. About twenty miles of the line passes over the Mississippi "bottom lands," the depth of water on which is frequently as much as thirty feet, and along that stretch many of the insulators are of necessity secured to trees.

At the end of July Mr. Smith rejoined Assistant Dean at Nashville, and repeated observations for personal equation.

The telegraph lines of the Western Union Company were as usual at the service of the observers when needed for longitude determinations, and all messages relating to the details of the work were sent and received without charge. With uniform liberality all facilities were afforded to the observers by Superintendent Trabue and his assistants at Nashville, and by operators at the stations at which longitudes were determined.

The personal equation between Assistant Dean and Subassistant Smith was ascertained from the record of eighty-eight star transits observed jointly on seven nights. Mr. F. H. Parsons served as recorder in the astronomical party at Nashville.

Assistant Dean is now completing arrangements for the determination of longitude at points intermediate between Washington and Atlanta, Ga.

Magnetic observations.—Subassistant Andrew Braid observed for the magnetic declination, dip, and intensity at eleven stations in the interior States and Territories. As only one station of the series occupied, falls in this section, the observations at Nashville, Tenn., will be noticed under Section XV, in which most of the stations were selected. At the outset of the season the points to be occupied were chosen so that the results might be most effective when collated with results previously found at other stations.

Geodetic survey.—Early in the spring of the present year, Prof. A. H. Buchanan commenced observations for latitude and azimuth at the north end of the Lebanon base line in Tennessee. The computations of the work were completed and sent to the office at the end of May. Professor Buchanan then took the field and erected nine signals suitably related to the base in position. The lines of triangulation thus provided for will average about twenty-five miles in length. From most of the stations the capital at Nashville is in view. At the end of the fiscal year arrangements were in progress for beginning the measurement of horizontal angles at stations in the vicinity of the base line.

Assistant Richard D. Cutts, as heretofore, has advised in regard to the details and progress of this work. As the scheme of triangulation has been well considered, it is expected that a good advance will be made in the course of the summer. The statistics of work done after the 1st of July will be given in my next annual report.

Geodetic survey in Kentucky.—For continuing the reconnaissance for points in the State of Kentucky, Prof. W. B. Page took the field early in May, and going northward and westward, extended the scheme of triangulation. At the end of June of the present year, stations in the vicinity of the Ohio River had been selected and the work was continued during the summer. The progress made after the 1st of July will be the subject of mention in my next annual report.

The region traversed this season by Professor Page is heavily wooded, and in general the summits are flat. These conditions have necessitated the erection of scaffolds of considerable elevation so that lines of sight may pass over the timber. In general the scaffolds have been attached to the trunks of large trees at the needful elevation, and the structures are put up at a trifling expense.

In approaching the Ohio River, in the vicinity of Louisville, careful examination will be made for the site of a base line. The bottom lands along the river readily afford lines of sufficient length that would admit of easy measurement, but the adjacent stations decided upon for the triangulation necessitate the direction of the base for its proper connection with the scheme of work. At the close of the fiscal year the prospect was good for the location of a suitable line near the Ohio.

SECTION XIV.

OHIO, INDIANA, ILLINOIS, WISCONSIN, AND MICHIGAN.—(SKETCHES NOS. 21, 22, AND 23.)

Geodetic operations in Ohio.—Prof. R. S. Devol, of the University of Ohio, having been jointly recommended for the charge of the geodetic work in that State by the governor and by the authorities of the university, was authorized in May last to make arrangements for the selection of stations for such triangulation as might at the outset be of most advantage in the construction of maps to represent the geological survey.

Assistant Richard D. Cutts conferred personally with Professor Devol, at Athens. A careful examination of the different State maps of Ohio, showed that the southern section offered advantages for the determination of points, and as Columbus had been determined in longitude by Coast Survey observers, it was deemed most expedient to examine the hilly, rolling country for a series of quadrilaterals between Athens and Columbus. At the close of the university session, Professor Devol took the field to employ the summer in the needful reconnaissance. The results of his examination will be mentioned in my next report.

Geodetic operations in Indiana.—In common with other States, application on behalf of Indiana was made several years ago by the governor for the benefit which had been elsewhere afforded by the accurate determination of geographical points, on which to base maps representing the results of the geological survey. The request was promptly entertained, but owing to the want of means, arrangements for commencing work were of necessity deferred until May of the present year.

Meanwhile, Prof. John L. Campbell, of Wabash College, had been recommended by the governor and by the State geologist, for the charge of the field work, which, as in similar cases, could be carried on only during the college vacation. Late in May, Assistant Richard D. Cutts proceeded to Indianapolis and conferred personally with the State geologist, Prof. E. T. Cox, and also with Professor Campbell, in regard to the topographical features of the State, with a view of starting triangulation in the locality most favorable for progress. The immediate wants and interests of the geological survey were at the same time kept in view. After full discussion it was decided to begin reconnaissance in the hilly country south of Indianapolis, and, if practicable, to select stations for a chain of quadrilaterals extending to the Ohio River.

A short time before the close of the fiscal year, Professor Campbell was relieved from college duties and at once took the field. His plan of work will include the selection of a site for a base line between Indianapolis and New Albany. Of the progress in the summer of the present year further mention will be made in my next annual report.

Reconnaissance.—The reconnaissance for stations in Southern Illinois was resumed by Assistant G. A. Fairfield on the 10th of May. In the preceding season the even surface of the ground eastward of Saint Louis and the want of means for identifying positions some miles distant, made the selection of stations impracticable. To overcome in some measure the difficulty thus presented, Mr. Fairfield procured a few rockets and used them as night signals. By this device, the direction was obtained towards a point at which it was desirable to place a signal, and at which, by a structure of moderate height, the adjacent signals might be brought into view. A signal was erected at Sugar Loaf station, and arrangements were made for a structure at Parkinson. Night signals were exchanged between those points, and the practicability of the line was established. At Sherman Park a signal was put up, and was seen from Parkinson, the distance being eight miles. The lines for this triangle were traced out in June, after due tests with the spirit-level. Assistant Fairfield, at the date of his last report, was engaged in connecting other stations with those which he had already selected. The work will be continued during the summer, and mention of the results will be made in my next annual report.

Geodetic operations in Wisconsin.—The progress made in triangulation, after completing the reconnaissance in Wisconsin, was a subject of notice in my annual report for 1876. During the year following, means not being available, the work was suspended, but was resumed in May of the present year by Prof. J. E. Davies, of the University of Wisconsin, who had the triangulation in charge in the first-mentioned year. In the autumn of that year connection was made by the measurement of horizontal angles between the base line, near Spring Green, and the astronomical station at Madison, but the operations of that year closed without means for the measurement of the base, which, of necessity, was deferred until the present year. Late in May last Professor Davies made the needful preparations, and was joined on the 2d of June by Assistant Richard D. Cutts. The measurement was made with the sliding-contact apparatus, the management of which at the outset was directed by General Cutts. When the work was fully under way the measurement was left to Professor Davies, who completed the work in June. The ends of the line were secured for use, at any time hereafter, by structures of solid masonry below the ground surface. Upon these, monuments of Joliet limestone were placed, with the proper inscriptions. The site of the line is unusually level and favorable for rapid work. The record of the measurement shows that the operation was performed with care, and that a close degree of accuracy may be accorded to the results.

The course of the triangulation now in progress in Wisconsin will extend to the Mississippi River, between Prairie du Chien and Dubuque.

Magnetic observations.—In this section the magnetic elements were determined by Subassistant Andrew Braid, at Madison and at La Crosse, in Wisconsin. As the larger number of stations occupied fall into the next section, a summary notice will be made under that head of all the determinations made by the party in the course of the season.

SECTION XV.

MISSOURI, KANSAS, IOWA, NEBRASKA, MINNESOTA, AND DAKOTA.—(SKETCH No. 25.)

Magnetic observations.—It is generally known that the variation of the compass, at positions off the coast, can be marked on charts with reasonable accuracy, but not as the result of observations recorded at sea. To obtain data of such importance, the magnetic declination must be known not only at field points adjacent to our coasts, but also for many places in the interior of the continent of North America. Hence, for whatever purpose observations have been made on land, within the last two centuries, the determinations all become of great account, as items of the large aggregate, needful for accuracy in marking compasses on the coast charts of the United States. Of late years, calls for information in regard to the variation of the compass have become frequent, suggesting that determinations, instead of being incidental among other field operations, should be committed to an observer qualified to return results from a number of stations in each season. Subassistant Andrew Braid was assigned to this service, and was carefully trained in the use of instruments by Prof. J. E. Hilgard, the assistant in charge of the office. The distribution of magnetism over the territory of the United States, having been a subject of study for years in the office, Mr. Hilgard was able to indicate readily the points which should be occupied first, in order to gain the utmost with a limited outlay in time and means. After due practice, under the inspection of Assistant Hilgard, Mr. Braid took the field and determined the magnetic elements at Nashville, Tenn.; Cairo, Ill.; La Crosse, Wis.; Minneapolis, Minn.; Sibley, Des Moines, Davenport, and Keokuk, Iowa; Omaha, Nebr.; Lawrence, Kans.; and at Vinita, in the Indian Territory.

The work done at these places consisted, as at Madison, of observations for magnetic declination, dip, and horizontal intensity. Not less than three days were spent at each station in recording successive observations, and at some of them operations were kept up during four and five days.

For determining the horizontal force, both deflections and oscillations of the magnet were observed, except at La Crosse, Sibley, Keokuk, and Vinita, where the record was made only of oscillations.

This field work was prosecuted by Mr. Braid between July and the middle of December. During the season the weather was for the most part favorable for magnetic observations, yet on a few days the instruments were affected by irregular disturbances, or "magnetic storms." Rains were very frequent, but caused interference in the work only by delay in obtaining the astronomical azimuth.

At all the stations, care was taken to set up the instruments at some considerable distance from structures of iron, and all the stations were carefully marked.

Arrangements have been made for securing similar determinations at another series of stations in the course of the ensuing season, the points as before being selected so as to add directly to the means for tracing lines along which the variation of the magnetic needle is equal. Sketch No. 24, given in my report of last year, shows the results obtained thus far from repeated discussion of all the observations for magnetic declination that have been found consistent with each other.

COAST AND GEODETIC SURVEY OFFICE.

Assistant J. E. Hilgard continued in charge of the Coast and Geodetic Survey Office until near the close of the year for which this report is submitted. He was relieved on June 17, having been commissioned to execute important duties which required the presence of a representative of the Coast Survey in Europe.

These were, first, to co-operate as a member with the International Committee on Weights and Measures in perfecting the organization of its establishment near Paris, and to assist in initiating the practical operations agreed upon; second, to make at London a recomparison of the Coast Survey standard yard (bronze No. 11) with the British Imperial standard, in order to establish with certainty a difference which had been indicated by comparisons with other original standards; and, third, to attend the annual session of the International Geodetic Association.

During Mr. Hilgard's absence, Assistant Charles A. Schott was directed to take charge of the office.

Assistant Edward Goodfellow continued on duty in the office throughout the year, aiding the assistant in charge in the details of administration and in the correspondence of the office, which is constantly increasing with the growing demand for the information accumulated by the Coast and Geodetic Survey.

The temporary assignment to duty, under office direction, of several field officers gave Mr. Hilgard opportunities of prosecuting, with their aid, some scientific investigations designed to effect improvements in the instruments and in the methods and processes of the survey.

Assistant O. H. Tittmann reported to the office in August, and, after making comparisons of the secondary six-meter base bars with the standard bar No. 1, he designed and had constructed a convenient apparatus for comparing base bars, and executed the comparisons, an account of which will be prepared for future publication.

He also supervised the construction of the new geodesic level, determined its constants, and made tests of the graduation of theodolites No. 88 and No. 117, of several micrometer screws, and of ocean salinometers and deep-sea thermometers. Early in December Assistant Tittmann was temporarily detached from the office to make an exact levelling between the tidal bench-marks at Albany and Stuyvesant, on the Hudson; on his return, in January, he was occupied with the computations of this work; he then took up the completion and revision of the new Coast Survey Catalogue of Latitude Stars, which is printed as Appendix No. 7 of the report for 1876. He had also in charge the designs for an enlarged construction of the level of precision.

Early in May Mr. Tittmann was detached from the office and charged with the geodetic triangulation westward from the eastern base of the Rocky Mountains.

Mr. H. W. Blair, after completing at the office the computations of his recent field work, was assigned to duty under Mr. Hilgard's immediate direction in February, and assisted in the revision of the star catalogue for observations of latitude. He also made experiments with an optical densimeter devised by Mr. Hilgard, and with a new form of drop-cylinder water-specimen cup described in Appendix No. 14, Report for 1877, and, beside other measurements of precision, made elaborate comparisons between standard yards bronze No. 11 and iron No. 57 to verify the relative change heretofore indicated in these two important standards, which were in 1856 presented to the United States by Great Britain as original and verified copies of the Imperial yard. Early in April Mr. Blair was ordered to duty in the field.

Assistant A. T. Mosman, who reported for office duty in the beginning of March, was occupied in making a descriptive catalogue of the instruments belonging to the survey. Towards the close of that month he was detached in order to continue the primary triangulation from the Blue Ridge westward.

Assistants J. A. Sullivan and F. D. Granger were assigned to the computing division during the few weeks they were on duty in the office. Late in March they were ordered to field service.

Mr. E. Hergesheimer, upon his recovery from severe illness, reported for duty under Mr. Hilgard's immediate direction in January, and executed drawings of chart of Saint John's River and of Rockaway Inlet for publication by photolithography. He was subsequently occupied in composing guide maps for topographical drawing adapted to secure uniformity in the mode of representing topographical details, and in making micrometric comparisons of certain graduated scales.

Hydrographic Division.—In this division, which remained under the direction of Commander E. P. Lull, U. S. N., Hydrographic Inspector, aided by Lieuts. W. I. Moore and W. H. Parker, U. S. N., on duty during part of the year, the draughtsmen's work is summarily stated as follows:

Mr. E. Willenbucher has protracted, plotted, and drawn fifteen hydrographic sheets, made the verification of thirty-six hydrographic sheets for registry, verified reductions of hydrography for eleven charts, examined seven charts for additions and corrections of aids to navigation, made eight projections for hydrographic parties, and executed miscellaneous diagrams and reductions for the office and tracings to meet calls for information.

Mr. W. C. Willenbucher has protracted, plotted, and drawn twelve hydrographic sheets; verified, inked, and finished ten hydrographic sheets; brought up the progress sketches to date, and has made such miscellaneous tracings and reductions as from time to time were required.

Mr. Julius Sprandel was unable by reason of failing health to give much time to office work. He had protracted, plotted, and drawn three hydrographic sheets previous to his death, which ook place August 29, 1877.

The services of the hydrographic inspector are elsewhere referred to in this report.

Computing Division.—The direction of this division of the office was continued with Assistant C. A. Schott. The distribution of work among the computers was kept up as in former years as far as the small number of computers would allow, there being but three during December and January. Mr. Main's health, reported unsatisfactory at the close of the last report, did not improve during the summer, and his services were necessarily discontinued from September until May, when they were resumed with reduced daily time of attendance and corresponding pay. Mr. C. H. Van Orden was on duty in the computing division during July; Mr. M. W. Wines, during July, August, and September; Mr. B. Bradbury, jr., during part of September; Mr. C. A. Ives, during part of October and November. Assistant F. D. Granger reported for duty February 1, 1878, and was detached March 28; Assistant J. A. Sullivan reported for duty March 4, and was detached April 15; Mr. P. Lobanoff was engaged on special duty from May 1 to the close of the fiscal year. The limited force of regular computers was just sufficient to attend to the more important current work.

Much of the time of the chief of the division is necessarily taken up with preparing replies to official correspondence; with directing, distributing, supervising, and reporting on the various computations; with furnishing geographical positions of light-houses, magnetic variation, and corrected longitudes for charts.

The discussion of the secular change of the magnetic declination was extended by Mr. Schott to stations in Brazil, Mexico, Alaska, and the Sandwich Islands, and the data for many stations in the United States were brought forward. This valuable discussion appears in a paper published separately from this report.

In September, Mr. Schott visited the magnetic observatory located at the University of Wisconsin, and provided for its more effective working. A full report on this observatory, including its history, description, and adjustment of instruments, determination of constants, and discussion of results has been prepared by him. The results obtained during the first two years of its existence are nearly ready, and will be printed in a subsequent report.

The usual annual magnetic observations at Washington were made by Mr. Schott in June, 1878. He has also prepared a new register of magnetic data, in which the stations are arranged by States, and for each in chronological order. He also observed and prepared the report on his own and the observations of other officers of the Coast Survey on the transit of Mercury across the disk of the sun on May 6, Appendix No. 7.

On June 24, 1878, Assistant Schott was appointed assistant in charge of the office during the absence of Assistant Hilgard, retaining at the same time the special charge of the computing division.

The work in detail performed by each computer during the fiscal year 1877–'78 is as follows:

Mr. James Main computed the observations for latitude and azimuth at Young Mountain, N. C., 1876, and those for time and azimuth at Middle Base, Helena, Ark., 1878, and at King, N. C., 1877. He also furnished star places for field parties.

Dr. Gottlieb Rumpf computed the following secondary and tertiary triangulations: State triangulation of Wisconsin, 1875–'76; near Santa Monica, Cal., 1875; coast between Monterey and Point Sur, Cal., 1875; local triangulation at Madison, Wis., 1877; vicinity of Mare Island, Cal., 1877; south side of Long Island, near Rockaway, N. Y., 1875; vicinity of Buzzard's Bay, Mass., 1870–'71; Mississippi River above New Orleans, La., 1876–'77; James River, Va., 1875–'76–'77; Santa Barbara Island, Cal., 1871; Boston Harbor, Mass., 1877; coast near Fort Ross, Cal., 1876–'77; and nearly completed the triangulation between Nisqually and Budd's Inlet, Wash. Ter., 1877. He also prepared abstracts of horizontal-angles survey of New Jersey, 1876–'77; performed miscellaneous geodetic computations and supplied geodetic data for field parties, or in reply to inquiries made by office correspondents.

Mr. Edward H. Courtenay was engaged in the preparation of the geographical register containing the results of the triangulations; completed the fitting on of the old secondary and tertiary triangulation, sea coast of North Carolina, between Upper Currituck Sound and Core Sound; revised abstracts of horizontal directions and of vertical angles at primary stations in the Blue Ridge, Virginia, North Carolina, and South Carolina, also in Georgia; computed the magnetic ob-

servations, absolute values taken at Madison, Wis., in 1876 and in 1877; collected and tabulated magnetic constants for three magnetometers; assisted in the solution of normal equations resulting from primary triangulation; computed magnetic observations made lately by Assistants Boutelle, Mosman, Smith, and Braid; adjusted primary triangulation about Mount Washington, N. H., and nearly completed the computation of the new triangulation of the Delaware River, near Philadelphia, 1877–'78. He also assisted in preparing the annual statistics of geodetic work, and had general charge of copying and clerical work.

Mr. M. H. Doolittle made abstracts of resulting directions of primary stations, Johns, Ga., 1875; Moore, N. C., 1876–'77; King, N. C., 1876–'77; adjusted that part of the primary triangulation between Pine Log, Ga., and Brandon, Ala.; computed the triangulation of the Connecticut River between the Sound, and north of Hartford, 1861–'62–'75; assisted in the preparation of the annual statistics, made miscellaneous magnetic computations; read off the photographic traces of tabulated results of differential magnetic observations made at Madison, Wis., between March, 1877, and March, 1878; computed position of primary stations about the Santa Barbara Islands, Cal., 1876–'77, and fitted on part of the secondary and tertiary work of the same locality, and nearly completed the computation for length of the Lebanon base line, Tenn., 1877.

The computing division also had temporary assistance during the year as follows:

Mr. C. H. Van Orden, attached to the division during July, 1877, attended to miscellaneous copying, preparation of sketches, and miscellaneous geodetic computations.

Mr. M. W. Wines, attached to the division between July 1 and September 26, 1877, attended to copying of data for field parties, and to special clerical work under the direction of the assistant in charge of the office.

Mr. B. Bradbury, jr., was engaged in reading off magnetic traces between September 8 and 27.

Mr. C. A. Ives reported for duty October 19, computed positions of a number of tertiary points, attended to clerical work, and prepared information for various field parties. He was detached from the office November 30.

Assistant F. D. Granger was temporarily assigned to duty in the computing division. Between February 1 and March 28, 1878, he assisted in preparing abstract of directions of primary stations, King, N. C., 1876–'77; Anacapa, Cal., 1876; Laguna, Cal., 1877; Santa Barbara, Cal., 1876. He also adjusted three quadrilaterals of the Santa Barbara channel survey, and prepared geodetic information for a field party.

Assistant J. A. Sullivan was temporarily assigned to duty in the computing division between March 4 and April 15, when he computed the triangulation of light-houses on Delaware Bay, executed by himself in 1877. He also assisted in proof-reading of annual report.

Mr. P. Lobanoff was employed since May 1 on reading off and tabulating photographic traces of magnetic observatory at Madison; he also assisted occasionally in tracing descriptive sketches of stations for field parties.

Tidal Division.—Mr. R. S. Avery remained in charge. He inspected all records of tidal observations when received from the hydrographic parties or from the permanent tidal stations; furnished tidal data for use on the charts and to meet calls for information; arranged the observations for reduction; prepared for publication the predictions of tides for the Atlantic and Pacific Coast Tide Tables for 1879; and studied improvements in the construction of self-registering tide gauges.

Referring to the establishment of a self-registering gauge, loaned by the Coast Survey, at a station in the Sandwich Islands, in connection with the government survey of those islands under W. D. Alexander, esq., it is suggested by Mr. Avery that it would be desirable to establish a tidal station at a point similarly located in relation to the tides of our Atlantic coast. The Bermuda Islands would seem to be favorably placed for this purpose, being separated from our coast by the deep channels of the Gulf Stream.

In the following table is given a statement of the names of stations at which self-registering gauges have been established, with the number of days upon which observations were recorded since the last report:

Section.	Name of station.	Name of observer.	Kind of gauge.	Permanent or temporary.	Time of occupation.		Total days.
					From—	To—	
I	North Haven, Me	J. G. Spaulding	Self-registering	Permanent	Apr. 26, 1877	Apr. 28, 1878	367
I	Providence, R. I	R. A. Wood	...do	Temporary	Dec. 30, 1875	June 1, 1878	731
II	Governor's Island, N. Y	R. T. Bassett	...do	Permanent	May 31, 1877	May 30, 1878	364
II	Sandy Hook, N. J	J. W. Banford	...do	...do	June 1, 1877	June 1, 1878	365
III	Fort Monroe, Va	W. J. Bodell	...do	...do	June 1, 1877	June 1, 1878	365
VI	Fernandina, Fla	H. W. Bache	...do	...do	June 1, 1877	Sept. 3, 1877	95
VI	......do	...do	...do	...do	Dec. 30, 1877	Apr. 1, 1878	92
VIII	New Orleans, La	G. Faust	Staff	Temporary	June 30, 1877	Dec. 31, 1877	184
X	Saucelito, Cal	E. Gray	Self-registering	Permanent	June 1, 1877	June 1, 1878	365
XI	Port Townshend, Wash	L. Wilson	...do	Temporary	Mar. 1, 1877	Apr. 1, 1877	31

Each observer forwards to the office, with the gauge record, his tabulation of the times and heights of high and low waters, the comparative readings of staff and automatic record, and the hourly ordinates deduced from the tidal curve. These tabulations, when verified, serve as a basis for the office reductions.

In the work of the division, Mr. Avery had the aid of the following-named persons:

Mr. John Downes was employed chiefly in the tabulation and reduction of the tidal observations made upon the Atlantic coast and upon the tidal predictions of this coast for 1879.

Mr. L. P. Shidy made reductions of short series of observations for many stations on the Atlantic and Pacific coasts, predicted tides for ports at which the diurnal inequality was large, and aided in miscellaneous computations.

Miss M. Thomas continued to work mostly on hourly readings and copying, and aided in the predictions. She had also charge of the office library until January 28.

Drawing Division.—This division has continued under charge of Mr. W. T. Bright. The clerical and other miscellaneous duties have been performed during the year by Mr. G. A. Morrison.

The force of principal draughtsmen consisted of Messrs. A. Lindenkohl, H. Lindenkohl, C. Junken, L. Karcher, and P. V. Erichsen, who have drawn the more important topographic and hydrographic charts for publication by copper-plate engraving or photolithography. The work performed by each is stated in Appendix No. 4.

Messrs. C. A. Meuth, H. Eichholtz, and E. Molkow have chiefly been engaged in bringing up by hand the aids to navigation upon the published charts. This information is received in the division, and the necessary corrections and additions made previous to its insertion upon the printing-plate. By this means the sale agents are supplied with the latest data. They have also furnished the information asked for by bureaus of the government and citizens, in the form of tracings and diagrams.

Mr. E. J. Sommer has been employed in the division since April, and has principally been engaged upon drawings for charts to be photolithographed.

Mr. B. Bradbury, jr., aid, who was temporarily attached to the division at the date of last year's report, was assigned to field duty in August.

The list of charts and sketches which have been completed or in progress during the year, given in Appendix No. 4, will at a glance show the large amount of work accomplished.

From a very recent experimental beginning in the production of charts by photolithography, this process has grown to be a most important auxiliary in getting rapidly before the public the results of the field work. In addition to the compilation and publication of twenty-one charts by this means, progress has been made upon seventy-eight charts of various scales, of which number forty-two have been completed, including the photolithographic class.

Appendix No. 3 gives a statement of information furnished by the division upon the application of officers of the United States Engineer Corps, officers of the Army, light-house engineers and inspectors, and other persons, for tracings and copies of special maps and charts. Some eighty-four calls of this description were met during the year.

Forty-seven diagrams of various descriptions were constructed, together with drawings of numerous instruments of precision and engravings upon stone, to illustrate the papers accompa-

nying the annual reports. The matter of correcting the original copper-plate engravings for the latest determination of longitude has continued to receive attention. Projections for new charts have been constructed upon six copper plates. As the work of the survey advances, it becomes in many cases necessary to revise former projects for charts that were constructed upon data derived from reconnaissances and imperfect maps.

In the course of the year about thirty-five projects have been reconstructed and adjusted to suit better the wants of the navigator.

The numerous calls by the field parties for projections on drawing paper have been promptly met, forty-three having been constructed upon different scales. The large number of sketches showing the progress of the field work have been duplicated for the files of the Superintendent and computing division, and the annual progress sketches have received the usual additions.

Notwithstanding the great amount of this miscellaneous work, the division has furnished and has on hand an abundant supply of material for engraving, and has made progress commensurate with the field work in many sections as the data became available.

Engraving Division.—The direction of the work in this division remained with Assistant L. A. Sengteller during the fiscal year. Towards its close, having received instructions to resume field duty on the Pacific coast, he made arrangements for the transfer of the charge of the division to his successor, Assistant J. S. Bradford.

In a careful and elaborate report of the work executed in detail by the several engravers, and of the condition of the engraved plates, Mr. Sengteller refers to the efforts made to bring up to date the plates of those charts most in demand by navigators. These efforts have been so far successful that all of the principal coast charts are now brought up to the current year, and a "note of issue" engraved upon each plate, bearing the date to which it has been corrected; this has been done also with the plates of the principal harbor charts, sailing charts, and general coast charts.

Messrs. J. Enthoffer, A. Sengteller, H. C. Evans, J. J. Young, W. A. Thompson, and R. F. Bartle have continued as topographical engravers.

Messrs. E. A. Maedel, A. Petersen, H. M. Knight, J. G. Thompson, and F. Courtenay have continued as letter engravers.

Messrs. E. H. Sipe and W. H. Davis have been employed as miscellaneous engravers.

Mr. J. J. Young, during the greater part of the year, was engaged in etching the views for the Coast Pilot.

The duties of copyist in the division were successively discharged by Messrs. E. Molkow, G. A. Morrison, and (since his reappointment February 1) by Mr. L. C. Kerr. The compilation of an historical record of every plate was begun by Mr. W. B. French, who was detailed specially for that duty, and carried it on at intervals since November, 1877; upon the detachment of Mr. French, for field service early in March, the work was for a time suspended, but towards the end of May resumed by Mr. John H. Smoot.

In Appendix No. 5 will be found a detailed statement of plates completed, continued, or begun during the year, together with work in detail performed by each engraver.

Electrotype and Photographing Division.—Under the direction of Dr. A. Zumbrock, who remained in charge, fifty-seven altos and twenty-eight bassos or printing plates were made from the original engraved plates; one hundred and fifty-four prints, fifty-three negatives, and fifteen positives were furnished for the use of the draughtsmen and engravers. Dr. Zumbrock was aided by Mr. Frank Over.

Miscellaneous Division.—The organization of this division, up to May of the present year, involved the direction of details in the care of the office buildings, the printing and distribution of the charts and reports, the charge of the stationery and instruments, and the care of the office disbursements. These duties were faithfully and acceptably performed by Mr. John T. Hoover until failing health obliged him to relinquish them. His death, deeply lamented by all of his associates in the office, has been referred to in another part of this report.

A rearrangement of the work of the division having been found to a certain extent desirable, Mr. M. W. Wines was assigned to the charge of the details relating to the office building, the care of the stationery, and the printing and distribution of the charts and reports; Mr. W. A. Herbert to the keeping of the office accounts; Mr. G. N. Saegmüller to the care of the instrument records.

The duty of registering and filing for reference the original sheets, topographic and hydrographic, and the records of observations as received from the field parties continued to be performed by Mr. G. A. Stewart.

From the chart room, under the immediate care of Mr. Thomas McDonnell, there were distributed, during the year, upwards of twenty-one thousand copies of charts; over five thousand of this number were placed with agents for sale at the principal sea-ports; upwards of four thousand were furnished to meet the demands of the several departments of the government; thirty-three hundred were supplied to members of Congress.

The distribution of the Coast Survey Reports amounted to one thousand and three copies.

There were printed during the year twelve thousand and sixty-seven impressions from engraved plates; the number of lithographed copies of charts published was thirty-eight hundred and nineteen.

Mr. Frank Moore, assisted by Mr. D. N. Hoover, served as copper-plate printer; Mr. H. Nissen attended to the preparation of the backed drawing paper for field parties, and to the various requirements of the folding-room.

The work of the instrument-shop was performed by Mr. G. N. Saegmüller, chief mechanician, and by Messrs. John Clark, W. Jacobi, and E. Eshleman.

Among the subjects which occupied Mr. Saegmüller's time when not engaged in routine work, may be mentioned the improvements in the construction of the dividing engine to render it capable of executing more accurate circular graduations, and the regraduation, with very satisfactory results, of one of the fourteen-inch theodolites. The design and construction of a prismatic double-image eye-piece for simultaneous use by two observers in determining personal equation is also due to him. This is described in the Transactions of the International Geodetic Association for 1878.

Mr. John Clark has been engaged on improvements in the primary-base apparatus, assisting in its comparisons with the standards mentioned elsewhere; in constructing the level of precision described in Appendix No. —, and in other work requiring great skill and experience.

Mr. Werner Suess was employed on piece work, under the immediate direction of the assistant in charge, in constructing various self-registering magnetic and thermometric instruments, portable chronographic apparatus, and the optical densimeter for ascertaining the density of sea water, mentioned elsewhere in this report and described in Appendix No. 10 of the report for 1877. He also aided Assistant Schott in perfecting the building and apparatus of the Magnetic Observatory at Madison, Wis.

Mr. A. Yeatman, carpenter, assisted during part of the year by F. E. Lackey and C. Webster, made the wood-work for instruments, packed them for transportation, and did all of the carpentry required by the office.

The clerical duties in the office of the assistant in charge were performed by Messrs. M. W. Wines, W. B. French, and W. A. Herbert, with the occasional aid of Messrs. F. H. Parsons and George A. Morrison.

As heretofore, the expenditures of the year for every purpose connected with field work and hydrography, have been strictly controlled by preceding estimates, of which each item as to its character and amount has been separately examined. Party estimates are revised in all cases when reductions are deemed expedient, and are approved by the Superintendent in advance of any outlay whatever. The control thus provided for is made complete by the attention of the disbursing agent, J. W. Porter, by whom the accounts of the chiefs of parties are kept in conformity with the approved estimates.

My correspondence with the surveying parties, needful references pertaining to the progress of work under my official instructions, and to administrative details generally, have been facilitated as usual by the care of Assistant W. W. Cooper, and by his summary of the operations of the year in the field and afloat.

Respectfully submitted.

CARLILE P. PATTERSON,
Superintendent Coast and Geodetic Survey.

Hon. JOHN SHERMAN,
Secretary of the Treasury.

APPENDICES.

APPENDIX No. 1.

Distribution of surveying parties upon the Atlantic, Gulf of Mexico, and Pacific coasts of the United States during the fiscal year 1877–'78.

Sections.	Parties.	Operations.	Persons conducting operations.	Localities of work.
SECTION I.				
Maine, New Hampshire, Vermont, Massachusetts, and Rhode Island, including coast and sea-ports, bays and rivers.	No. 1	Hydrography....	Lieut. J. F. Moser, U. S. N., assistant: Masters J. B. Murdock and F. E. Greene, U. S. N.	Soundings extended in the seaward approaches of Mount Desert Island, Me. (See also Section V.)
	2	Topography	Charles Hosmer, assistant	Survey of the shores of Skilling River at the head of Frenchman's Bay, Me. (See also Section VIII.)
	3	Topography	A. W. Longfellow, assistant......	Topography of the north shore of Blue Hill Bay, Me.
	4	Hydrography.....	Lieut. J. M. Hawley, U. S. N., assistant; Masters G. C. Hanus and A. H. Cobb, U. S. N.; Ensign Albert Mertz, U. S. N.	Hydrography of Jericho Bay, including the vicinity of Deer Isle and Isle au Haut, coast of Maine. (See also Section VI.)
	5	Tidal observations	J. G. Spaulding..................	Series of tidal and meteorological observations continued at North Haven, in Penobscot entrance.
	6	Geodetic	Prof. E. T. Quimby	Triangulation at Gunstock Mountain and Gilford, for determining points in New Hampshire.
	7	Triangulation	F. Blake, jr., assistant............	Determination of numerous points for the Harbor Commissioners' survey of Boston Upper Harbor.
	8	Triangulation	G. A. Fairfield, assistant	Positions determined of light-houses near Provincetown, Plymouth, Cohasset, and Marblehead, coast of Massachusetts. (See also Section XIV.)
	9	Hydrography.....	Master Robert Platt, U. S. N., assistant; Ensign J. P. Underwood, U. S. N.	Observations on the sea currents of the Gulf of Maine.
	10	Tides	....	Tidal observations continued with self-registering gauge at Providence, R. I.
SECTION II.				
Connecticut, New York, New Jersey, Pennsylvania, and Delaware, including coast, bays, and rivers.	No. 1	Topography and hydrography.	Joseph Hergesheimer, subassistant.	Shore-line survey and soundings in Duck Island Harbor, Conn. (See also Sections VI and VII.)
	2	Topography	R. M. Bache, assistant............	Detailed topographical survey of the north and west approaches to New Haven, Conn. (See also Section VI.)
	3	Topography	J. W. Donn, assistant	Topography of the western shores of Jamaica Bay, including Rockaway Inlet; plane-table survey of Coney Island, including the shores of Sheepshead Bay and Gravesend Bay, Long Island, N. Y.
	4	Hydrography.....	Lieut. Washburn Maynard, U. S. N., assistant; Masters W. F. Low and S. H. May, U. S. N.	Hydrography of the western part of Jamaica Bay, including Rockaway Inlet and its approaches, Long Island, N. Y.
	5	Special	C. S. Peirce, assistant	Pendulum experiments at New York City for determining the force of gravity.
	6	Tides	R. T. Bassett; J. W. Banford	Tidal observations continued with self-registering gauges at Governor's Island, in New York Harbor, and at Sandy Hook, New York Bay.
	7	Topography	H. L. Whiting, assistant; W. C. Hodgkins, aid.	Topographical survey of the east side of Hudson River between Croton and Peekskill, N. Y.

APPENDIX No. 1—Continued.

Sections.	Parties.	Operations.	Persons conducting operations.	Localities of work.
SECTION II—Continued.	No. 8	Special	O. H. Tittmann, assistant; Andrew Braid, subassistant; J. B. Baylor, aid.	Lines of level run between tidal bench-marks at Stuyvesant and Albany. (See also Sections III, IX, XIII, XIV, and XV.)
	9	Triangulation	Richard D. Cutts, assistant; C. H. Sinclair, aid.	Primary triangulation extended in this section by observations at Mount Tom, in Massachusetts, and at Mount Equinox, in Vermont. (See also Sections VIII, IX, XIII, and XIV.)
	10	Astronomical observations.	Edwin Smith, subassistant; J. B. Baylor, aid.	Determinations of latitude and longitude at points near the boundary line between Pennsylvania and New York. (See also Sections III, VIII, and IX.)
	11	Geodetic	Prof. Edward A. Bowser	Points determined by triangulation in the northern part of New Jersey.
	12	Geodetic	Prof. Lewis M. Haupt	Triangulation continued in the eastern part of Pennsylvania.
	13	Astronomical observations.	Edwin Smith, subassistant; J. B. Baylor, aid.	Latitude, longitude, and the magnetic elements determined at Harrisburg, Pa. (See also Sections III, VIII, and IX.)
	14	Triangulation	S. C. McCorkle, assistant	Determination of points for the special survey of Delaware River at Philadelphia.
	15	Physical survey	H. Mitchell, assistant; H. L. Marindin, assistant; J. B. Weir, aid.	Special observations in regard to the tides and currents of the Delaware River at and near Philadelphia.
	16	Triangulation	J. A. Sullivan, assistant	Positions determined of light-houses in Delaware Bay.
SECTION III.				
Maryland, Virginia, and West Virginia, including bays, sea-ports, and rivers.	No. 1	Topography	C. M. Bache, assistant	Plane-table survey continued eastward of Norfolk, Va., between Elizabeth River and Cape Henry.
	2	Tides	W. J. Bodell	Tidal observations with self-registering gauge continued at Fort Monroe, Chesapeake Bay, Va.
	3	Special hydrography.	Lieut. Fred. Collins, U. S. N., assistant; Master Francis Winslow, U. S. N.	Observations on salinity and density of the waters of Chesapeake Bay. (See also Sections IV and VI.)
	4	Special	Charles Junken	Vertical heights of the great flood in the Potomac River determined at several points and referred to bench-marks.
	5	Magnetic observations.	Charles A. Schott, assistant	Determination of the magnetic declination, dip, and intensity at the permanent station on Capitol Hill, Washington, D. C.
	6	Levels	Edwin Smith, subassistant	Lines run with the spirit-level from Hagerstown westward, in Maryland. (See also Sections II and VIII.)
	7	Levels	Andrew Braid, subassistant	Continuation of lines of level in Western Maryland towards Cumberland. (See also Sections II, IX, XIII, XIV, and XV.)
	8	Triangulation	A. T. Mosman, assistant	Stations occupied on the Blue Ridge, Va., for triangulation connecting the Kent Island base line with the base line near Atlanta, Ga.
SECTION IV.				
North Carolina, including coast, sounds, sea-ports, and rivers.	No. 1	Hydrography	Lieut. Fred. Collins, U. S. N., assistant; Masters Francis Winslow and H. H. Barroll, U. S. N.; J. R. Barker, draughtsman.	Special examination of the coast between Cape Henry and Cape Fear, and compilation of marine notes for the Coast Pilot. (See also Sections III and VI.)
	2	Triangulation	F. H. Gerdes, assistant; L. C. Kerr.	Life-saving stations determined in position between Cape Henry and Cape Hatteras.
	3	Topography	C. T. Iardella, assistant	Plane-table survey of the shores of Cape Fear River below Wilmington, N. C.
	4	Triangulation	C. O. Boutelle, assistant; C. A. Ives, aid; J. B. Boutelle.	Stations occupied in North Carolina for closing the triangulation between the base in Kent Island, Md., and the base near Atlanta, Ga. (See also Section VIII.)

APPENDIX No. 1—Continued.

Sections.	Parties.	Operations.	Persons conducting operations.	Localities of work.
SECTION V.				
South Carolina and Georgia, including coast, sea-water channels, sounds, harbors, and rivers.	No. 1	Hydrography	Lieut. J. F. Moser, U. S. N., assistant; Masters A. C. Dillingham, J. B. Murdock, and F. E. Greene, U. S. N.	Hydrography of the coast of South Carolina above Murrell's Inlet, including Little River and its approaches. (See also Section I.)
SECTION VI.				
East Florida, from Saint Mary's River to Anclote Keys, on the west coast, including the coast approaches, reefs, keys, seaports, and rivers.	No. 1	Hydrography	Lieut. Fred. Collins, U. S. N., assistant; Masters Francis Winslow and H. H. Barroll, U. S. N.; J. R. Barker, draughtsman.	Coast examinations between Fernandina, Fla., and the Dry Tortugas, and marine notes for completion of the Coast Pilot. (See also Sections III and V.)
	2	Tides	H. W. Bache	Tidal observations with self-registering gauge at Fernandina, Fla.
	3	Hydrography	Lieut. Commander C. M. Chester, U. S. N., assistant; Lieuts. Uriel Sebree and A. V. Wadhams, U. S. N.; Master T. G. C. Salter, U. S. N.; Ensign C. H. Amsden, U. S. N.	Hydrography of the eastern coast of Florida from Mosquito Inlet southward and eastward to Cape Cañaveral.
	4	Triangulation, topography, and hydrography.	F. W. Perkins, assistant (part of season); W. I. Vinal, subassistant (part of season); Ensign J. P. Underwood, U. S. N.; C. A. Ives, aid (part of season).	Survey of the shores of Saint John's River, Fla., south of San Patricio Point, and soundings extended from that point southward to stations beyond Tocoi, Fla. (See also Section VIII.)
	5	Topography and hydrography.	R. M. Bache, assistant; W. I. Vinal, subassistant (part of season); Lieut. Thomas N. Lee, U. S. N.; C. A. Ives, aid (part of season).	Survey of Indian River below Cape Cañaveral, and soundings completed in Banana River, Fla. (See also Section II.)
	6	Hydrography	Lieut. J. M. Hawley, U. S. N.; Masters G. C. Hanus, A. H. Cobb, and Albert Mertz, U. S. N.	Hydrography of Charlotte Harbor, Fla. (See also Section VII.)
	7	Triangulation ...	Joseph Hergesheimer, subassistant.	Triangulation of Sarasota Bay, Fla. (See also Sections II and VII.)
SECTION VII.				
West Florida, from Anclote Keys to Perdido Bay, including coast, ports, and rivers.	No. 1	Topography and hydrography.	Joseph Hergesheimer, subassistant.	Shore-line survey and soundings in Crooked River, adjacent to Saint George's Sound, Fla. (See also Sections II and VI.)
	2	Hydrography	Lieut. J. M. Hawley, U. S. N., assistant; Masters G. C. Hanus, A. H. Cobb, and Albert Mertz, U. S. N.	Supplementary soundings in Duer's Channel and off Light-House Point, Saint George's Sound, Fla. (See also Section VI.)
SECTION VIII.				
Alabama, Mississippi, Louisiana, and Arkansas, including Gulf coast, ports, and rivers.	No. 1	Hydrography	Lieut. Commander C. D. Sigsbee, U. S. N., assistant; Lieuts. S. M. Ackley and W. O. Sharrer, U. S. N.; Masters H. M. Jacoby and Henry McCrea, U. S. N.; Ensign G. H. Peters, U. S. N.	Deep-sea soundings, with observations for temperature and density, in the waters of the Gulf of Mexico.
	2	Triangulation, topography, and hydrography.	W. H. Dennis, assistant; F. C. Donn, aid.	Triangulation and topography of Barataria Bay, La., completed and connected with the survey of Mississippi River. Supplementary soundings in the upper branches of the bay. Triangulation of the Mississippi River in the vicinity of Natchez, Miss.
	3	Hydrography	Lieut. W. I. Moore, U. S. N., assistant; Lieuts. W. F. Low and S. H. May, U. S. N.	Soundings in the approaches and entrance, and inside of Barataria Bay, La.
	4	Special	G. Faust	Continuous observations with a tide-gauge, and record of the height of the water of the Mississippi at New Orleans, La.
	5	Azimuth and bases	H. G. Ogden, assistant; C. H. Sinclair, aid; W. B. French.	Measurement of base lines and azimuth at Donaldsonville, La., and at Natchez and Vicksburg, Miss. (See also Section IX.)

APPENDIX No. 1—Continued.

Sections.	Parties.	Operations.	Persons conducting operations.	Localities of work.
SECTION VIII—Continued.	No. 6	Triangulation	F. W. Perkins, assistant	Triangulation of the Mississippi River above Donaldsonville, La. (See also Section VI.)
	7	Triangulation	Charles Hosmer, assistant	Triangulation for the survey of the Mississippi River in the vicinity of Vicksburg and Greenville, Miss. (See also Section I.)
	8	Astronomical observations.	Edwin Smith, subassistant	Latitude observations at Natchez, Vicksburg, and Greenville, Miss., and at Helena, Ark. Longitude of the same places determined by telegraphic exchanges with Nashville, Tenn., in Section XIII. Observations for azimuth at Vicksburg and Helena. (See also Sections II and III.)
	9	Triangulation	C. H. Boyd, assistant; C. H. Van Orden, aid.	Measurement of a base line near Helena, Ark., and triangulation of the Mississippi River upwards to Bennett's Landing.
	10	Triangulation	F. D. Granger, assistant; Prof. A. H. Buchanan, part of season.	Triangulation continued in Northern Alabama, and reconnaissance in Northern Mississippi.
	11	Triangulation	C. O. Boutelle, assistant	Reconnaissance, and triangulation extended westward from stations in the vicinity of Huntsville, Ala. (See also Section IV.)
SECTION IX.				
Texas and Indian Territory, including Gulf coast, bays, and rivers.	No. 1	Reconnaissance	H. G. Ogden, assistant; C. H. Sinclair, aid.	Examination of the coast of Texas for triangulation to extend from Galveston Bay, Tex., towards Vermilion Bay, La. (See also Section VIII.)
	2	Triangulation	R. E. Halter, assistant	Triangulation of Laguna Madre, Tex., continued southward of Baffin's Bay.
	3	Magnetic observations.	J. B. Baylor, aid	Magnetic declination, dip, and intensity determined at Dollar Point, San Antonio, Hempstead, and Groesbeck, Tex. (See also Section II.)
	4	Magnetic observations.	....	Magnetic elements determined at Vinita, Ind. T. (See also Sections II, III, XIII, XIV, and XV.)
SECTION X.				
California, including the coast, bays, harbors, and rivers.	No. 1	Hydrography	Lieut. Commander G. W. Coffin, U. S. N., assistant; Lieuts. W. W. Gilpatrick, F. J. Drake, C. W. Jarboe, and W. H. Driggs, U. S. N.; Master R. Mitchell, U. S. N.	Hydrographic survey of the bar and of parts of the harbor of San Diego, Cal. (See also Section XI.)
	2	Triangulation	D. B. Wainwright, aid	Triangulation for determining the position of San Clemente, and of Santa Barbara Island, off the coast of California.
	3	Hydrography	Lieut. E. H. C. Leutze, U. S. N., assistant; Lieuts. E. K. Moore and E. S. Prime, U. S. N.; Masters J. H. Bull and R. H. Galt, U. S. N.	Hydrography of the Santa Barbara Channel, from Point Dume westward to Santa Cruz Island, Cal.
	4	Topography	Stehman Forney, assistant	Plane-table survey continued on Catalina Island, Cal.
	5	Hydrography	Lieut. Commander G. W. Coffin, U. S. N., assistant; Lieuts. W. W. Gilpatrick, F. J. Drake, C. W. Jarboe, and W. H. Driggs, U. S. N.; Master R. Mitchell, U. S. N.	Hydrography of approaches to the western part of Catalina Island. (See also Section XI.)
	6	Topography	A. W. Chase, assistant	Topographical survey of the coast in the vicinity of Point Arguello, Cal.
	7	Hydrography	Lieut. Frank Courtis, U. S. N., assistant; Lieuts. E. H. C. Leutze and E. K. Moore, U. S. N.; Masters J. H. Bull and R. H. Galt, U. S. N.	Hydrography of the Santa Barbara Channel between Santa Rosa Island and Point Concepcion, Cal.

APPENDIX No. 1—Continued.

Sections.	Parties.	Operations.	Persons conducting operations.	Localities of work.
SECTION X—Continued.	No. 8	Triangulation and topography.	W. E. Greenwell, assistant	Detailed survey of the coast of California from Ynez River northward towards Point Sal.
	9	Topography	A. F. Rodgers, assistant; D. B. Wainwright, aid (part of season).	Topography of the coast of California north and south of Point Sur.
	10	Tides	E. Gray	Tidal observations, with self-registering gauge, continued at Saucelito, inside of San Francisco Bay, Cal.
	11	Triangulation	George Davidson, assistant; B. A. Colonna, subassistant; J. F. Pratt, aid.	Supplementary observations for completing the measurement of horizontal angles at Mount Helena and Mount Diablo, Cal. (See also Section XI.)
	12	Reconnaissance	Cleveland Rockwell, assistant	Examination for points of primary triangulation along the coast between Point Arenas, Cal., and Cape Blanco, Oreg.
	13	Astronomical observations.	B. A. Colonna, subassistant; J. F. Pratt, aid.	Latitude and longitude determined at Summit, Nev., and observations recorded of the transit of Mercury.
SECTION XI.				
Oregon and Washington Territory, including coast, interior bays, ports, and rivers.	No. 1	Hydrography	Lieut. Commander G. W. Coffin, U. S. N., assistant; Lieuts. R. Clover, F. J. Drake, C. W. Jarboe, and W. H. Driggs, U. S. N.; Master R. Mitchell, U. S. N.	Hydrography of the approaches to Columbia River, Oreg. (See also Section X.)
	2	Topography and hydrography.	J. J. Gilbert, assistant	Detailed survey of the shores and soundings in the Columbia River, Oreg., extended upwards from the vicinity of Mount Coffin to Kalama.
	3	Reconnaissance	James S. Lawson, assistant	Selection of points for primary triangulation of the Strait of Fuca and Washington Sound, Wash. Ter.
	4	Reconnaissance	George Davidson, assistant; B. A. Colonna, subassistant; J. F. Pratt, aid.	Examination of sites for base lines in upper Willamette Valley, Oreg., and on Whidbey Island, Wash. Ter. (See also Section X.)
	5	Hydrography	Lieut. Richard M. Cutts, U. S. N., assistant; Lieuts. A. B. Wyckoff and U. R. Harris, U. S. N.	Hydrography of Admiralty Inlet, Wash. Ter.
	6	Triangulation and topography.	Eugene Ellicott, subassistant	Triangulation and topography of the southern end of Puget Sound from Commencement Bay to Budd's Inlet, Wash. Ter.
	7	Triangulation and topography.	J. J. Gilbert, assistant	Survey of the northern part of Hood's Canal, Wash. Ter.
	8	Inspection	A. F. Rodgers, assistant	Reconnaissance and inspection of plane-table operations on the shores of Columbia River and in Washington Territory. (See also Section X.)
SECTION XII.				
Alaska Territory		Office work	W. H. Dall; Marcus Baker	Compilation of the titles of charts, &c., illustrative of the coast features, and hydrography of Alaska, and material descriptive of the resources of the Territory.
SECTION XIII.				
Kentucky and Tennessee	No. 1	Astronomical observations.	George W. Dean, assistant; William Eimbeck, assistant; H. W. Blair, subassistant; F. H. Parsons; Prof. A. H. Buchanan, part of season.	Latitude and longitude determinations at Memphis and Nashville, Tenn.; at Cairo, Ill.; at Hickman and Paducah, Ky.; and telegraphic exchange of signals from Nashville for the longitude of points in Section VIII.
	2	Magnetic observations.	Andrew Braid, subassistant	Magnetic declination, dip, and intensity observed at Nashville, Tenn. (See also Sections II, III, IX, XIV, and XV.)
	3	Geodetic	Prof. A. H. Buchanan	Latitude and azimuth determinations near Lebanon, Tenn., and signals erected for the triangulation in that vicinity. (See also Section VIII.)
	4	Geodetic	Prof. W. B. Page	Reconnaissance, and selection of stations for triangulation in Kentucky, between Cumberland Gap and the Ohio River.

APPENDIX No. 1—Continued.

Sections.	Parties.	Operations.	Persons conducting operations.	Localities of work.
SECTION XIV.				
Ohio, Indiana, Illinois, Wisconsin, and Michigan.	No. 1	Geodetic	Prof. R. S. Devol	Reconnaissance in Ohio for triangulation to determine geographical positions between Athens and Columbus.
	2	Geodetic	Prof. J. L. Campbell	Reconnaissance in Indiana for triangulation north of Jeffersonville.
	3	Reconnaissance	G. A. Fairfield, assistant	Selection of stations for triangulation in Southern Illinois. (See also Section I.)
	4	Geodetic	Prof. J. E. Davies	Determination of points by triangulation near Madison, Wis.
	5	Magnetic observations.	Andrew Braid, subassistant	Magnetic declination, dip, and intensity observed at Madison and La Crosse, Wis. (See also Sections II, III, IX, XIII, and XV.)
SECTION XV.				
Missouri, Kansas, Iowa, Nebraska, Minnesota, and Dakota.	No. 1	Magnetic observations.	Andrew Braid, subassistant	Magnetic elements determined at Minneapolis, Minn.; Sibley, Des Moines, Davenport, and Keokuk, Iowa; Omaha, Nebr.; and at Lawrence, Kans. (See also Sections II, III, IX, XIII, and XIV.)

APPENDIX No. 2.

Statistics of field and office work of the United States Coast Survey to the close of the year 1877.

Description.	Total to December 31, 1876.	1877.	Total to December 31, 1877.
RECONNAISSANCE.			
Area in square statute miles	202,166	11,159	213,325
Parties, number of, in year	....	4	....
BASE LINES.			
Primary, number of	13	0	13
Subsidiary, number of	105	2	107
Primary, length of, in statute miles	79	0	79
Subsidiary and line measures, length of, in statute miles	238½	5	243½
TRIANGULATION.			
Area in square statute miles	98,133	8,392	106,525
Stations occupied for horizontal angles, number of	8,559	301	8,860
Geographical positions determined, number of	15,797	739	16,536
Stations occupied for vertical angles, number of	485	22	507
Elevations determined, number of	1,183	110	1,293
Lines of spirit-levelling, length of	633½	43½	677
Parties (triangulation and levelling), number of, in year	....	29	....
ASTRONOMICAL WORK.			
Azimuth stations, number of	135	2	137
Latitude stations, number of	226	11	237
Longitude stations (telegraphic), number of	83	9	92
Longitude stations (chronometric and lunar), number of	110	0	110
Astronomical parties, number of, in year	....	6	....
MAGNETIC WORK.			
Stations occupied, number of	385	9	394
Permanent magnetic stations, number of, in year	....	2	....
Magnetic parties, number of, in year	....	4	....
TOPOGRAPHY.			
Area surveyed in square miles	26,101	218	26,319
Length of general coast, in miles	5,969	27	5,996
Length of shore line, in miles (including rivers, creeks, and ponds)	72,562	1,032	73,594
Length of roads in miles	38,173	258	38,431
Topographical parties, number of, in year	....	18	....
HYDROGRAPHY.			
Parties, number of, in year	....	18	....
Number of miles run while sounding	300,373	12,094	312,467
Area sounded, in square miles	72,756	1,810	74,566
Miles run, additional, of outside or deep-sea soundings	51,963	6,467	58,430
Number of soundings	13,995,310	578,873	14,574,183
Soundings in Gulf Stream for temperature	4,072	0	4,072
Tidal stations, permanent	213	12	225
Tidal stations occupied temporarily	1,613	43	1,656
Tidal parties, number of, in year	....	31	....
Current stations occupied	416	52	468
Current parties, number of, in year	....	2	....
Number of deep-sea soundings in year	....	1,038	....
Specimens of bottom, number of	10,500	189	10,689
RECORDS.			
Triangulation, originals, number of volumes	2,186	135	2,321
Astronomical observations, originals, number of volumes	1,221	36	1,257
Magnetic observations, originals, number of volumes	363	13	376
Duplicates of the above, number of volumes	2,324	111	2,435
Computations, number of volumes	2,489	103	2,592
Hydrographical soundings and angles, originals, number of volumes	7,228	283	7,511
Hydrographical soundings and angles, duplicates, number of volumes	839	126	965
Tidal and current observations, originals, number of volumes	3,011	80	3,091
Tidal and current observations, duplicates, number of volumes	1,965	66	2,031

APPENDIX No. 2—Continued.

Description.	Total to December 31, 1876.	1877.	Total to December 31, 1877.
RECORDS—Continued.			
Sheets from self-registering tide-gauges, number of	2, 460	92	2, 552
Tidal reductions, number of volumes	1, 643	40	1, 683
Total number of volumes of records	23, 269	993	24, 262
MAPS AND CHARTS.			
Topographical maps, originals	1, 530	13	1, 543
Hydrographic charts, originals	1, 427	45	1, 472
Reductions from original sheets	791	23	814
Total number of manuscript maps and charts, to and including 1876	2, 552	23	2, 575
Number of sketches made in field and office	2, 948	49	2, 997
ENGRAVING AND PRINTING.			
Engraved plates of finished charts, number of	213	6	219
Engraved plates of preliminary charts, sketches, and diagrams for the Coast Survey Reports, number of	568	4	572
Electrotype plates made	1, 216	73	1, 289
Finished charts published	198	25	223
Preliminary charts and hydrographical sketches published	494	6	500
Printed sheets of maps and charts distributed	364, 698	20, 578	385, 276
Printed sheets of maps and charts deposited with sale agents	132, 606	5, 694	138, 300
LIBRARY.			
Number of volumes	6, 219	186	6, 405
INSTRUMENTS.			
Cost of	$117, 497 16	$4, 197 19	$121, 694 35

APPENDIX No. 3.

Information furnished from the Coast and Geodetic Survey Office, by tracings from original sheets, &c., in reply to special calls, for the fiscal year ending with June, 1878.

Date.		Name.	Data furnished.
1877.			
July	25	United States Light-House Board	Chart of upper part of Tampa Bay, Fla., scale 1-40,000, prepared from photographic reductions of the original hydrographic sheets.
Aug.	2	John L. Edwards, esq., Jacksonville, Fla	Approximate distances on the Saint John's River from Railroad Wharf, Jacksonville, to Enterprise, on Lake Monroe.
	6	Charles Crook, civil engineer	Shore line survey of 1835, reduced to 1-40,000 scale, of the south side of Long Island, from Rockaway Beach to Coney Island.
	8	George R. Howell, esq	Geographical positions of triangulation points in South and East Hampton, Long Island.
	20	Elisha R. Potter, esq., Rhode Island	Topographical survey, vicinity of Point Judith, from Tower Hill to Green Hill Pond, Rhode Island.
	24	George W. Atherton, esq., New Brunswick, N. J	Topographical survey of Squan Inlet, N. J., survey of 1868.
Sept.	4	T. T. Price, esq., Tuckerton, N. J	Length of sea-coast of New Jersey from Cape May light-house to Sandy Hook Spit.
	7	O. H. Kelley, secretary National Grange of the Patrons of Husbandry.	Hydrographic survey of Saint George's Sound and approaches, from East Pass to Dog Island Bar.
	18	S. T. Abert, United States civil engineer	Hydrographic survey of Perquimons River, Albemarle Sound, N. C.
	27	Elisha R. Potter, esq., Kingston, R. I	Topographical survey of the interior of Rhode Island, from Boston Neck to Shunnock Hill, survey of 1839.
Oct.	15	M. Robertson, esq., of Maryland Steamboat Company	Hydrographic survey of Secretary Creek, Choptank River, Md.
	18	F. Richardson, esq	Topographical survey of the environs of Norfolk eastward to Lawson's Lake and Kempsville, Princess Anne County, Va.
	18	Woolman & Rose, Camden, N. J	Positions of life-saving stations, coast of New Jersey, from Sandy Hook to Cape May; also proofs of unfinished charts Nos. 21 and 22, 1-80,000, with curves of depths of inside waters added by hand.
	20	D. M. Carter, esq., of North Carolina	Hydrographic reconnaissance of upper part of Alligator River, N. C., and proof of General Coast Chart V, with curves of depths of inside soundings added.
	22	Nichols & Richards, law firm, New York	Topogr phical survey of Coney Island, made in 1835, and copy of same published in 1778, by Des Barres.
	25	Hon. C. E. Hooker, of Mississippi	Hydrographic survey, approaches and entrance to Pearl River, La. and Miss.
	25	Lieut. Charles Bird, acting assistant quartermaster, U. S. A., Fort Columbus, New York Harbor.	Hydrographic survey, part of Buttermilk Channel, New York Harbor.
Nov.	1	Maj. Gen. John Newton, Corps of Engineers	Results of six current stations in Kill van Kull and lower part of Newark Bay, made in August and September, 1856, accompanied by graphic illustrations.
	2	Harbor Commission of Massachusetts	Hydrographic and topographic survey of Wild Harbor, Buzzard Bay.
	9	John J. Evans, esq., Washington City	Copy of hydrographic surveys north coast of Cuba, El Moro to Playa de Marianao, and across the Yucatan Channel, 1867 and 1872.
	12	W. J. Lewis, esq	Topographic and hydrographic survey of Duck Island Harbor, Long Island Sound.
	21	G. R. Talcott, esq., James River improvement force	Shore line and triangulation points of the James River from Deep Water light to Jamestown Island. Shore line and triangulation points of the James River from Westover to beyond Harrison's Bar. Shore line and triangulation points of the James River from City Point to beyond Bermuda Hundred. Shore line and triangulation points of the James River from Aikin to Cox's Landing, including Dutch Gap Canal.
	28	Hon. E. Pilsbury, mayor of New Orleans	Shore line and hydrography of Mississippi River, city front of New Orleans.
Dec.	13	W. W. Dewhurst, esq., Jacksonville, Fla	Hydrographic survey of part of the Saint John's River, from Jacksonville to Cross's Landing.

APPENDIX No. 3—Continued.

Date.		Name.	Data furnished.
1877.			
Dec.	17	W. W. Dewhurst, esq., Jacksonville, Fla	Additional hydrography of the Saint John's River, from Cross's Ledge to Sadler's Point.
	28	Harbor Board of Baltimore, Md	Hydrographic survey of Baltimore Harbor and part of Patapsco River, including Ridgeley's Cove, from survey made in 1845.
1878.			
Jan.	7	Maine Historical Society	Topographic survey of approaches to Saco River, Me.
	16	Dr. M. F. Bonzano, director of mint, New Orleans	Unfinished proof of coast chart No. 95, Mississippi River, La., from Grand Prairie to New Orleans, with title and scales added by hand.
	21	Horatio Seymour, jr., State surveyor and engineer, New York.	Hydrographic survey of northern shore of Long Island Sound between Delancey Point and Rye Neck, 1837.
	29	Donald G. Mitchell, esq	Topographic map of "Edgewood," New Haven, Conn., from the survey by the United States Coast Survey, 1876.
Feb.	4	William Curlett, esq., secretary Ocean Tow-Boat Company, New Orleans.	Hydrographic survey of Cubits Crevasse, Mississippi River, La.
	8	J. S. Purviance, esq., Live Oak, Fla	Hydrographic survey of approaches to the Suwannee River, Fla.
	8	H. G. Scofield, civil engineer	Tracing from topographical survey of Bridgeport, Conn., showing location of triangulation points.
	14	W. C. Cranmer, civil engineer	Location of triangulation stations along the Schuylkill River, with list of their geographical positions.
	16	Maj. Gen. A. A. Humphreys, Chief of Engineers	Hydrographic survey of Rockaway Inlet and part of Jamaica Bay, N. Y.
	25	Lieut. J. H. Willard, Corps of Engineers	Sketch showing location of tide-gauges and limits wherein each was used for the reduction of the soundings between Hudson City and Troy, N. Y.
	27	Lieut. G. M. Wheeler, Corps of Engineers	Sketches and results of the determination of Verdi station, Nev.
Mar.	1	John Penn Curry, New York	Computation of acre area of Staten Island, N. Y.
	6	United States Light-House Board	Hydrographic survey of the Thames River, Conn., from Norwich to Winthrop's Point, compiled to the scale of 1-10,000.
	9	John Winthrop Harris, esq., Dorchester, Mass	Topographic survey of the western side of Campobello Island, New Brunswick.
	9	Hon. James B. Eustis, Senator from Louisiana	Unfinished proofs of Coast Chart No. 91, Lakes Borgne and Pontchartrain, with title, scales, and notes added.
	12	C. M. Soria, esq	Unfinished proof of Coast Chart No. 91, Lakes Borgne and Pontchartrain, with title, scales, and notes added.
	18	Maj. Gen. John Newton, Corps of Engineers	Compiled sketch showing changes in the shore line of Rockaway Inlet, Long Island, from the surveys made in 1835, 1841, 1856, and 1877.
	20	W. C. Cranmer, civil engineer, Philadelphia, Pa	Hydrographic and topographic survey of the Schuylkill River from South street bridge to the Delaware.
	23	S. T. Abert, United States civil engineer	Hydrographic surveys of North Landing River to Coanjock Bay, N. C., made in 1877; North River, N. C., as far as surveyed, made in 1850; and Scuppernong River to town of Columbia, N. C., made in 1849.
	27	Maj. J. W. Barlow, Corps of Engineers	Hydrographic survey of the Thames River, Conn., from Norwich to Winthrop's Point, compiled to the scale of 1-10,000.
	28	Henry Nutt, esq., Wilmington, N. C	Unfinished proofs of General Coast Chart No. VI, scale 1-400,000, from Cape Hatteras to Cape Romain, and Coast Chart No. 37, Cape Henry to Currituck Beach light-house.
Apr.	3	C. A. Locke, engineer State geological survey of Georgia.	Colored progress sketch, sea-coast of Georgia from Savannah River to Saint Mary's River, scale 1-200, 000.
	17	J. B. Eads, chief engineer of South Pass jetty improvement.	Hydrographic survey of 1875 of the South Pass, from the Throat of the Pass to East Point.
May	3	C. A. Thompson, esq., Santa Barbara, Cal	Unfinished proof of Santa Barbara Channel, with title, notes, &c., added by hand,
	9	Capt. C. A. Abbey, United States Revenue Service	Unfinished proofs of Pamplico Sound, N. C., upon scales of 1-400,000 and 1-80,000, brought up to date by hand.
	18	Prof. G. H. Cook, State geologist of New Jersey	Topographic survey of the shores of the Hudson River from opposite Hoboken to Hastings, on the scale of 1-10, 000.
	23	Hon. J. H. Pugh, M. C., from New Jersey	Charts of the coast of New Jersey showing positions of life-saving stations.
June	8	Richard M. Pancoast, C. E., Camden, N. J	Comparative map of the coast of New Jersey, vicinity of Absecon and Dry Inlets, from the surveys of 1841 and 1869.

APPENDIX No. 4.

DRAWING DIVISION.

Charts completed or in progress during the fiscal year ending with June, 1878.

1. Hydrography. 2. Topography. 3. Drawing for photographic reduction. 4. Details upon photographed and pantographed outlines. 5. Inking and lettering sheets. 6. Verification.

Titles of charts.	Scale.	Draughtsmen.	Remarks.
Sailing chart A, Cape Sable to Cape Hatteras, N. C	1-1,200,000	2. A. Lindenkohl	Additions.
General coast chart No. III, Gay Head to Cape Henlopen, Del.	1-400,000	1. A. Lindenkohl. 1. H. Lindenkohl	Do.
General coast chart No. V, Cape Henry to Cape Lookout, N. C.	1-400,000	1. A. Lindenkohl. 1 and 2. H. Lindenkohl.	Do.
General coast chart No. VI, Cape Hatteras to Cape Romain, S. C.	1-400,000	2. A. Lindenkohl	Do.
Coast chart No. 4, Penobscot Bay, Me	1-80,000	L. Karcher (general lettering). 1. C. Junken	Continued.
Coast chart No. 10, Cape Cod Bay, Mass	1-80,000	1. C. Junken	Additions.
Coast chart No. 11, Monomoy and Nantucket Shoals to Muskeget Channel, Mass.	1-80,000	6. C. Junken	Do.
Coast chart No. 14, Point Judith and Block Island to Plum Island, R. I.	1-80,000	3. C. Junken	Do.
Coast chart No. 15, Plum Island to Welch's Point, Conn. and N. Y.	1-80,000	6. A. Lindenkohl. 6. H. Lindenkohl. 1, 2, and 3. C. Junken.	Do.
Coast chart No. 16, Welch's Point to New York, Conn. and N. Y.	1-80,000	6. A. Lindenkohl. 6. H. Lindenkohl. 1 and 3. C. Junken.	Do.
Coast chart No. 20, New York Bay and Harbor, N. Y	1-80,000	1. A. Lindenkohl	Do.
Coast chart No. 22, Barnegat Bay to Absecon light, N. J	1-80,000	1. C. Junken	Continued.
Coast chart No. 23, Absecon to Cape May, N. J	1-80,000	1 and 2. C. Junken	Do.
Coast chart No. 38, Currituck Beach light-house to Bodie's Island light-house, N. C.	1-80,000	1. A. Lindenkohl	Do.
Coast chart No. 40, Albemarle Sound, Atlantic Ocean, to the Pasquotank River, N. C.	1-80,000	1. A. Lindenkohl	Do.
Coast chart No. 42, Pamplico Sound, Roanoke Island to Hatteras Inlet, N. C.	1-80,000	1. H. Lindenkohl	Do.
Coast chart No. 43, Pamplico Sound, Ocracoke Inlet to mouth Pamplico River, N. C.	1-80,000	1. H. Lindenkohl. 1. A. Lindenkohl	Additions.
Coast chart No. 44, Pamplico and Neuse Rivers, N. C	1-80,000	1. A. Lindenkohl	Do.
Coast chart No. 54, Long Island to Hunting Island, S. C	1-80,000	1. A. Lindenkohl. 1. H. Lindenhohl. 1. C. Junken.	Do.
Coast chart No. 58, Cumberland Sound, Saint John's River, Fla.	1-80,000	2. H. Lindenkohl	Do.
Coast chart No. 59, Atlantic coast of Florida, Saint Augustine to Halifax River, Fla.	1-80,000	1 and 2. A. Lindenkohl. 1 and 2. H. Lindenkohl.	Commenced.
Coast chart No. 70, Key West, Marquesas Keys, &c., Fla.	1-80,000	1. C. Junken	Additions.
Coast chart No. 77, Tampa Bay and coast north and south, Fla.	1-80,000	1 and 2. A. Lindenkohl. 2. H. Lindenkohl.	Completed.
Coast chart No. 81, Apalachee Bay, Saint Mark's, &c., Fla.	1-80,000	2. P. Erichsen. 1. C. Junken	Do.
Coast chart No. 82, Saint Mark's to Saint George's Island, Fla.	1-80,000	1. C. Junken	Do.
Coast chart No. 83, Apalachicola Bay to Cape San Blas, Fla.	1-80,000	2. H. Lindenkohl	Do.
Coast chart No. 86, Choctawhatchee Inlet to Pensacola entrance, Fla.	1-80,000	1. A. Lindenkohl	Do.
Isle au Haut and Eggemoggin Reach, Me	1-40,000	1. A. Lindenkohl. 1. H. Lindenkohl. 1. L. Karcher.	Continued.
Belfast Bay and Penobscot River, Me	1-40,000	6. P. Erichsen	Additions.
Sheepscot and Kennebec Rivers, Me	1-40,000	3. C. Junken	Do.
Mystic River, Fisher's Island Sound, N. Y	1-40,000	1. L. Karcher	Do.

APPENDIX No. 4—Continued.

Titles of charts.	Scale.	Draughtsmen.	Remarks.
Delaware River, Fisher's Point to Cooper's Point	1-10, 000	1 and 2. L. Karcher	Photolithograph; additions.
Delaware River, Cooper's Point to Kaighn's Point	1-10, 000	1 and 2. L. Karcher	Do.
Patuxent River, lower sheet, Md	1-60, 000	2. H. Lindenkohl	Additions.
Potomac River No. 2, Piney Point to Lower Cedar Point	1-40, 000	3. C. Junken	Do.
Potomac River No. 3, Lower Cedar Point to Indian Head	1-40, 000	3. C. Junken	Do.
James River No. 1, Thimble light to Deep Water light	1-40, 000	1. C. Junken. 2. A. Lindenkohl	Completed.
James River No. 2, Point of Shoals light to Sloop Point	1-50, 000	1 and 2. H. Lindenkohl. 1. C. Junken	Photolithograph; completed.
James River No. 3, Sloop Point to City Point	1-40, 000	1 and 2. A. Lindenkohl. 1. C. Junken	Completed.
Do	1-50, 000	1 and 2. H. Lindenkohl	Photolithograph; completed.
Core Sound, N. C	1-40, 000	1. H. Lindenkohl	Additions.
Beaufort Harbor, N. C	1-40, 000	1. H. Lindenkohl	Do.
Savannah River, Ga	1-40, 000	1 and 2. C. Junken	Do.
Saint John's River, Fla	1-80, 000	1. L. Karcher. 1 and 2. E. Hergesheimer	Photolithograph; completed.
Tortugas Harbor, Fla	1-40, 000	1 and 2. A. Lindenkohl. 1 and 2. H. Lindenkohl.	Completed.
Subsketch for Tortugas Harbor and approaches	1-10, 000	1. A. Lindenkohl	Do.
Tampa Bay, Fla	1-40, 000	4. L. Karcher	Additions.
MISSISSIPPI RIVER. (Photolithographs in thirteen sheets.)			
Sheet No. 1, Fort Jackson to Point Pleasant, including Quarantine and Buras Settlement.	1-20, 000	1 and 2. H. Lindenkohl. 2. E. J. Sommer	Completed.
Sheet No. 2, Point Pleasant to Little Texas, Grand Prairie, and Guyot Settlement.	1-20, 000	1 and 2. H. Lindenkohl. 2. E. J. Sommer.	Do.
Sheet No. 3, Huling tract to Point Celeste, including Pointe à la Hache, Barthelmy, and Union Settlement.	1-20, 000	1 and 2. H. Lindenkohl. 2. E. J. Sommer.	Do.
Sheet No. 4, Point Celeste to Beau Sejour, including Poverty Point.	1-20, 000	1 and 2. H. Lindenkohl. 2. E. J. Sommer.	Do.
Sheet No. 5, Beau Sejour to Stella Plantation, including Jesuits' Bend.	1-20, 000	1 and 2. H. Lindenkohl. 2. E. J. Sommer.	Do.
Sheet No. 6, Stella Plantation to Powder House, including English Turn.	1-20, 000	1 and 2. H. Lindenkohl. 2. E. J. Sommer.	Do.
Sheet No. 7, Powder House to New Orleans	1-20, 000	1 and 2. H. Lindenkohl. 2. E. J. Sommer	Do.
Sheet No. 8, New Orleans to Soniat Plantation, including Carrollton, Jefferson, and Kennerville.	1-20, 000	1 and 2. H. Lindenkohl. 2. E. J. Sommer.	Do.
Sheet No. 9, Soniat Plantation to Myrtle Land Plantation, Saint Charles Railroad Station, and Hahnville.	1-20, 000	1 and 2. H. Lindenkohl. 2. E. J. Sommer.	Do.
Sheet No. 10, Myrtle Land Plantation to Reserve Plantation, including Bonnet Carré Point, and Bonnet Carré.	1-20, 000	1 and 2. H. Lindenkohl. 2. E. J. Sommer.	Do.
Sheet No. 11, Bonnet Carré to Grandview Reach, including Bell Point and Willow Bend.	1-20, 000	1 and 2. H. Lindenkohl. 2. E. J. Sommer.	Do.
Sheet No. 12, Chapman Plantation to Brilliant Point, including College Point.	1-20, 000	1 and 2. H. Lindenkohl. 2. E. J. Sommer	Do.
Sheet No. 13, Saint James estate to Point Houmas, including Brilliant Point and White Hall.	1-20, 000	2. H. Lindenkohl. 2. E. J. Sommer	Do.
Monterey Bay, Cal	1-40, 000	2. A. Lindenkohl. 2. L. Karcher	Additions.
Santa Barbara, Cal	1-10, 000 1-20, 000	1 and 2. H. Lindenkohl	Photolithograph; completed.
Anchorage southeast end San Clemente Island	1-20, 000	1 and 2. H. Lindenkohl	Do.
San Luis Obispo Bay, Cal	1-40, 000	2. E. Hergesheimer	Commenced.
City of San Francisco	1-10, 000	2. C. A. Meuth (adding proposed water front by United States commission).	Completed.
San Francisco entrance, Cal	1-50, 000	2. A. Lindenkohl	Additions.
Santa Barbara Channel No. 2, San Pedro Harbor to Point Concepcion.	1-200, 000	2. A. Lindenkohl	Continued.
Point Pinos to Bodega Head	1-200, 000	2. H. Lindenkohl	Do.
Columbia River entrance	1-40, 000	L. Karcher (comparative charts)	Commenced.
Columbia River No. 3, Tenasillihee Island to Grim's Island, Oreg.	1-40, 000	2. A. Lindenkohl. 1 and 2. H. Lindenkohl.	Completed. Commenced.
Columbia River No. 4, Grim's Island to Talama, Oreg	1-40, 000	2. A. Lindenkohl. 2. H. Lindenkohl	Do.

APPENDIX No. 4—Continued.

Titles of charts.	Scale.	Draughtsmen.	Remarks.
Puget Sound, Wash. Ter	1-200, 000	1 and 2. A. Lindenkohl	Additions.
Commencement Bay, Puget Sound	1-20, 000	A. Lindenkohl. H. Lindenkohl	Photolithograph; commenced.
Northwest coast, Cape Flattery to Dixon Entrance	1-1, 200, 000	1 and 2. A. Lindenkohl	Additions.
Northwest coast, Dixon Entrance to Cape Saint Elias	1-1, 200, 000	1 and 2. A. Lindenkohl	Do.
Map of Alaska and adjoining territory		2. A. Lindenkohl. 2. H. Lindenkohl	Photolithograph; completed.
Current chart Gulf of Maine	1-1, 200, 000	1. H. Lindenkohl. 1. L. Karcher	Completed.
Topographic sheet, Rockaway Inlet, &c	1-5, 000	5. C. A. Meuth	
Topographic sheet, coast of New Hampshire	1-10, 000	5. C. A. Meuth	
Topographic sheet, Port Jefferson	1-10, 000	5. C. A. Meuth	
Topographic sheet, Union River Bay, Me	1-10, 000	5. C. A. Meuth	
Topographic sheet, James River	1-10, 000	5. H. Lindenkohl	
Topographic sheet, Saint John's River, &c	1-20, 000	5. C. A. Meuth	
Topographic sheet, east coast of Florida	1-20, 000	5. C. A. Meuth	
Topographic sheet, Cubits Crevasse, Mississippi River	1-10, 000	5. C. A. Meuth	
Topographic sheet, Monterey Bay to Carmel River	1-10, 000	5. H. Lindenkohl	
Topographic sheet, Fort Ross to Salt Point, Cal	1-10, 000	5. H. Lindenkohl	
Topographic sheets, Puget Sound, Wash. Ter	1-20, 000	5. C. A. Meuth	
Chart Gulf of Mexico	1-10, 000, 000	1. A. Lindenkohl	Photolithograph; completed.
Drawing of plans of instruments of precision for the Annual Report.		P. Erichsen	
Drawing of plan of Magnetic Observatory, Madison, Wis		P. Erichsen	
Specimen sheets for topographic drawing		H. Lindenkohl	
Quincuncial projection		H. Lindenkohl	Photolithograph; completed.
North Atlantic Ocean, track chart		H. Lindenkohl. L. Karcher	Do.
General progress sketch	1-5, 000, 000	A. Lindenkohl	Completed.
Magnetic map		A. Lindenkohl	Do.

APPENDIX No. 5.

ENGRAVING DIVISION.

Plates completed, continued, or commenced during the fiscal year ending with June, 1878.

1. Outlines. 2. Topography. 3. Sanding. 4. Lettering. 5. Miscellaneous.

Title of plate.	Scale.	Engravers and work.
COMPLETED.		
Sailing chart A (2 plates)	1–80,000 1–400,000 1–1,200,000	1, 2, and 3. W. A. Thompson. 4. J. G. Thompson. 5. J. G. Thompson and W. A. Thompson.
Coast charts.		
No. 6, Kennebec entrance to Saco River	1–80,000	4. E. A. Maedel and J. G. Thompson, 5. W. A. Thompson and E. A. Maedel.
No. 7, Seguin Island to Kennebunkport	1–80,000	4. J. G. Thompson. 5. J. G. Thompson and H. M. Knight.
No. 37, Cape Henry to Currituck Beach	1–80,000	3. W. A. Thompson. 4. E. A. Maedel and J. G. Thompson. 5. W. A. Thompson.
Harbor charts.		
Entrance to Tampa Bay	1–40,000	4. H. M. Knight.
Columbia River, No. 3 (preliminary edition)	1–40,000	1, 3, and 4. H. M. Knight.
16 Atlantic Coast Pilot charts	1–80,000	2 and 3. W. A. Thompson. 4 and 5. A. Petersen, J. G. Thompson, E. H. Sipe, and W. H. Davis.
13 Atlantic Coast Pilot views (6 plates)	....	J. J. Young (etching). W. H. Davis (lettering).
2 views Suez Canal (1 plate)	....	J. J. Young (etching). W. H. Davis (lettering).
CONTINUED.		
General coast charts.		
No. 1, West part Isle au Haut to Cape Cod	1–400,000	4 and 5. J. G. Thompson.
General coast charts, west coast.		
Point Vincent to Point Concepcion	1–200,000	1 and 2. H. Lindenkohl. 4. A. Petersen.
Coast charts.		
No. 4, Penobscot Bay	1–80,000	4. E. A. Maedel.
No. 21, Sandy Hook to Barnegat Inlet (nearly completed)	1–80,000	1 and 2. W. A. Thompson. 2 and 3. H. C. Evans. 4. E. A. Maedel and F. Courtenay.
No. 22, Barnegat Inlet to Absecon Inlet (nearly completed).	1–80,000	2 and 3. H. C. Evans. 4. F. Courtenay.
No. 58, Cumberland Sound, Saint John's River, &c	1–80,000	1 and 2. A. Sengteller. 4. F. Courtenay.
No. 70, Key West, Marquesas Keys, &c	1–80,000	1, 2, 3. and 4. H. M. Knight. 4. F. Courtenay.
No. 83, Apalachicola Bay	1–80,000	2. A. Sengteller.
Harbor charts.		
Frenchman's Bay and Somes' Sound	1–40,000	2. R. F. Bartle.
Penobscot River and Belfast Bay	1–40,000	1. W. A. Thompson.
Lake Champlain, No. 1 (nearly completed)	1–40,000	3. W. A. Thompson. 4. A. Petersen
Lake Champlain, No. 2 (nearly completed)	1–40,000	4. E. A. Maedel.
Lake Champlain, No. 3	1–40,000	4. F. Courtenay.
James River, No. 1	1–40,000	1 and 2. J. Enthoffer.
Entrance to San Francisco Bay (in contours)	1–40,000	1. J. J. Young.
COMMENCED.		
General coast charts.		
No. VI, Cape Hatteras to Cape Romain	1–400,000	1 and 2. R. F. Bartle. 4. J. G. Thompson.
Coast charts.		
No. 23, Absecon Inlet to Cape May	1–80,000	1. R. F. Bartle.
No. 38, Currituck Beach to Oregon Inlet	1–80,000	1. J. J. Young. 4. A. Petersen.
No. 59, Saint Augustine Inlet to Halifax River	1–80,000	1 and 2. A. Sengteller. 4. E. A. Maedel.
No. 77, Tampa Bay	1–80,000	1. R. F. Bartle. 4. J. G. Thompson.
Harbor charts.		
James River, No. 2	1–40,000	1 and 2. J. Enthoffer.
37 Atlantic Coast Pilot views (13 plates are etched, but not lettered).	....	J. J. Young (etching).

APPENDIX No. 6.

OBSERVATIONS OF THE TRANSIT OF MERCURY, MAY 6, 1878, MADE AT SUMMIT STATION, CENTRAL PACIFIC RAILROAD.—REPORT BY B. A. COLONNA, ASSISTANT.

COAST AND GEODETIC SURVEY, SUB-OFFICE,
San Francisco, Cal., June 23, 1878.

SIR: I have the honor to submit to you my report of the observations of the transit of Mercury, May 6, 1878, made at Summit Station, Central Pacific Railroad.

Owing to a delay in receiving your telegraphic instructions of April 13, we had only about ten days to collect instruments, attend to the many minor details of such an expedition, get to Summit, clear away the snow for a place for the observatories, erect them, put up piers, run a telegraphic wire from the telegraph office at Summit to the station to be occupied, and so on.

Mr. Pratt, with his usual ingenuity, devised a transit machine run by clock work, representing the sun and Mercury with their apparent motions, giving us a very good artificial transit for practice. He did the mechanical work himself, and had it ready in time for us to have two days' practice with it before we left the city. We again practiced with it at Summit.

With Mr. Pratt's aid, and with that of Carl Rahsskopff, instrument maker, we found ourselves sufficiently advanced in preparation to leave San Francisco on the 30th of April. Through the kindness of Assistant Davidson's friend, Col. George Gray, chief engineer of the Southern Pacific Railroad, I procured an order from Mr. John Corning, the assistant general superintendent of the Central Pacific Railroad, for the transportation of our instruments by passenger train. We arrived at Summit at about 11 o'clock p. m. on the night of the 30th of April, and in the dark unloaded our instruments ourselves. Our reception was certainly a very cool one, made all the more so by contrast, for we had only a few hours ago passed through the Sacramento Valley, where the flowers were in bloom and the freshness of spring was to be seen on all sides; now, the snow sheds cut off even the starlight and left us in utter darkness.

I desired very much, in accordance with your instructions and Assistant Davidson's counsel, to get the instruments mounted on a hill about 800 feet higher than the railroad track at Summit, but it was impracticable, for the snow, about five feet deep on a level and in many places drifted to greater depths, presented difficulties that could not, with the means at hand, be overcome without much time. So I was reluctantly compelled to mount them as shown in the accompanying sketches of the station and vicinity.

On the 1st day of May the transit pier was set and the transit mounted. That night I put it in the meridian and observed for time.

On the 2d, Mr. Pratt ran the telegraph wires from the Western Union Telegraph office to the observatory and put up the chronograph, and we were ready to get the Washington Naval Observatory time signals on the 3d of May—these preparations and the subsequent exchanges of time signals being greatly facilitated by the courtesy of Mr. James Gamble, general superintendent of the Pacific Division of the Western Union Telegraph.

The evening of the 4th found us with our instruments all mounted. Besides the observation of the contacts, preparations had been made to observe for time and the sun's limbs and Mercury, with an instrumental outfit consisting of—

1st. For time observations, meridian telescope C. S. No. 1, made by Würdemann.

2d. Three chronometers, viz: Sidereal break-circuit No. 3479; mean-time chronometer No. 231; mean-time chronometer No. 1702.

3d. A chronograph.

4th. The "Hassler telescope," made by Utzschneider and Fraunhofer and equatorially mounted. This telescope is 3.75 feet focal length, three inches clear aperture, and Mr. Pratt used it for observing the contacts with an eye-piece whose magnifying power was 200 diameters.

5th. A $5\frac{3}{8}$-inch telescope, the private property of Assistant Eimbeck, on the new equatorial mounting belonging to the Coast Survey, and here used for the first time. It rested on a wooden stand which rested on a rock foundation, and was filled with stone to steady it. This telescope has a clear aperture, as stated above, and a focal length of 7 feet. I used it for observing the contacts, with an eye-piece the magnifying power of which was 120 diameters.

6th. A barometer, kindly loaned by Assistant Aug. F. Rodgers, there being none in this office that would read low enough for the altitude of Summit.

All of these instruments were mounted in tents or surrounded by tent-walls, and generally well protected from the wind; about 5 feet of packed snow had to be shoveled to get space enough for the tents.

On the 5th we received time signals from Washington, attended to various minor details, and spent much time in practice on the automatic transit machine. We did not compare notes, converse, or in any manner endeavor to get markings that would coincide, for I thought that two independent judgments of the times of contact would be more desirable. Nor did we, after the observations, converse upon the subject until each had prepared his own notes and placed them upon record.

The weather was bright and pleasant, so bright, in fact, that we were obliged to wear colored glasses to prevent becoming snow-blind. The snow had melted down to about 3 feet around the instruments. On the 5th it became partially overcast with cirrus and cirro-stratus clouds. The wind blowing fresh from the southwest, the barometer had fallen 0.15 inch. It cleared off, however, in the afternoon, so that the usual time observations were made.

The sun rose on the morning of the 6th in a cloudless sky. We were at the instruments at 5.30 a. m. The night had been quite cold, and the thermometer now stood at 27°. There was much ice. The chronograph was cold and the ink was frozen hard. We began by thawing out things generally, and placing the telescopes so that they would gradually warm, in order not to shiver the colored glasses when we began pointing.

The chronograph was so arranged that we both marked upon it, Mr. Pratt with one pen and I with the other.

Sidereal break-circuit chronometer No. 3479 was in my circuit, and placed near me, where I also had Mr. Crittenden to mark the time by eye and ear whenever I recorded on the chronograph; the click of my key was the signal for him to mark the time.

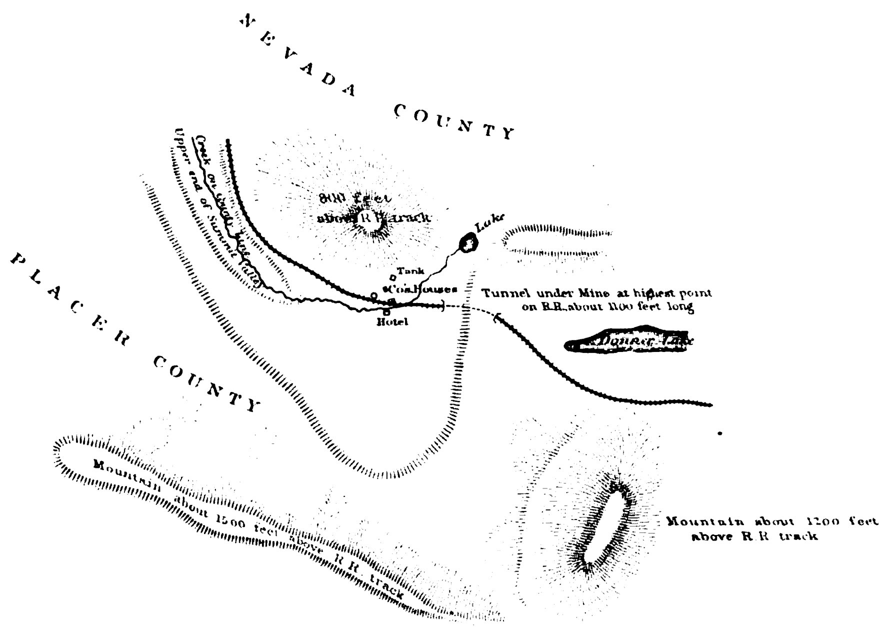
Mr. Berndt held a lamp beneath the ink-wells to prevent the ink from freezing and make it flow freely. From time to time during the last half hour before ingress we watched a little outside the sun's limb to ascertain if we could see anything of the planet. A minute or so before contact we finally assumed our positions at the instruments, and the observations at ingress were made as recorded.

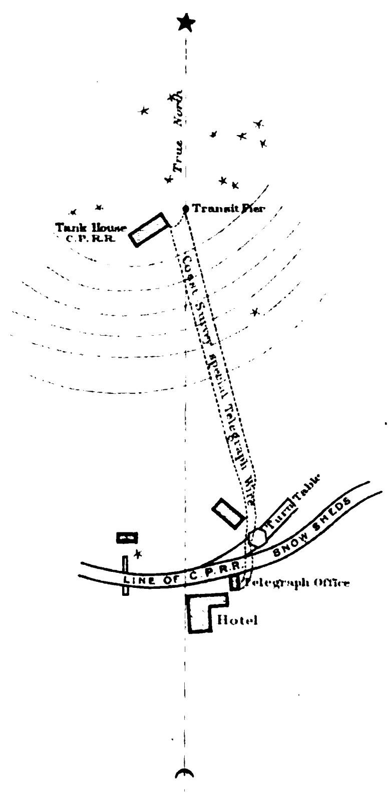
At noon I observed the sun's first and second limbs and the planet, as they crossed the three wires nearest the center of the transit, Mr. Pratt recording and attending to the chronograph.

The day remained cloudless to its close, and the afternoon observations were made in the same manner and under the same favorable conditions as those of the morning.

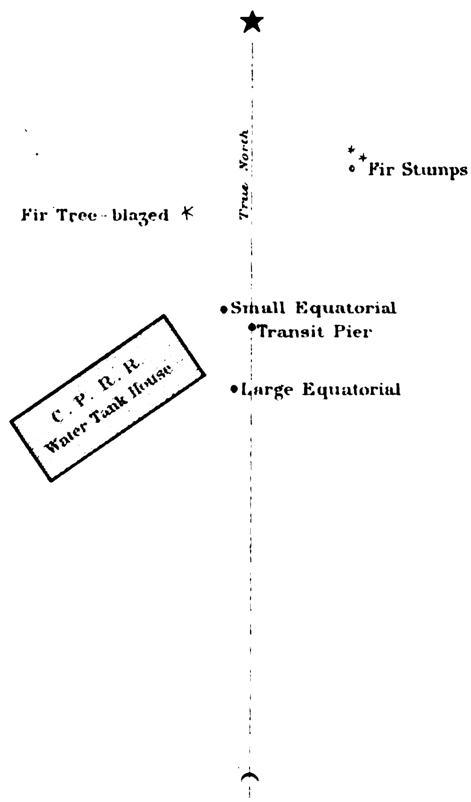
The phenomena of Mercury's appearance and disappearance reminded me of what I have seen in North Carolina and elsewhere, when, in a boat, on a bright day, looking over the water and across the marsh, say at a tree distant $2\frac{1}{2}$ miles, and where from one-fourth to one-half mile of the distance is over reedy marsh, then you have the same unsteady boiling and jumping as on the sun's limb. As you approach, the tree comes into view, the top first, a kind of black phantom, so to speak, that flits before your eyes and repeats itself. Now, if you rise in the boat gradually, assuming an erect posture, the tree takes form and becomes permanently visible just as Mercury did at ingress. Receding from the object, and being erect, you have the appearance of egress when you take your seat. The waves of heated air on the marsh that roll, as it were, over the tree top, are rather longer than the waves that are apparently on the sun's limb.

Mercury was rather egg-shaped, or elongated, when projected against the sun near its limb just after ingress and just before egress, the smaller end being next the nearest point in the sun's circumference. At night I continued my observations for time. The sidereal chronometer (3479) loses rapidly with the thermometer at 27°, while at about 52°, or a little over, it has scarcely any rate.





Scale 400 Feet to One Inch



Scale 80 Feet to One Inch

I have never had the pleasure of spending a day under a clearer or brighter sky. By 9^h 30^m a. m. the wind, which was not very strong even at sunrise, had fallen almost to a calm, all things combining to make it a most suitable day for the observations.

Upon the conclusion of the work I notified you, and you authorized a latitude and longitude station to be made. Time signals were received from Washington up to the 10th.

On the night of the 9th, Mr. Pratt started for San Francisco to make the necessary arrangements for exchanging time for telegraphic longitude, and to make the time observations at the Coast Survey Observatory at Washington Square.

Six good nights of time, and exchange of like dates, having been obtained with San Francisco by the 2d of June, I considered it ample for the work in hand.

When Mr. Pratt left Summit our mutual understanding was that the observations should be made by eye and ear. When he came to attempt it, he found that, the clock being in a case, the tick could not be heard without the use of an armature, which is very objectionable.

As there was no break-circuit chronometer for him, and his time exchange had to be made through the clock, he very properly decided that it would be better to abandon the eye and ear method and use the chronograph; but I was not aware of the change of method until I returned to San Francisco. It was deemed desirable, therefore, to determine our absolute personal equations, which was done by means of the Coast Survey personal-equation apparatus.

At the observatory in San Francisco Mr. Pratt made some changes in the switches, &c., and altered the chronograph from a cylindrical to a fillet. He also introduced a third pen for his absolute equation, thus making one chronograph do the work of two.

As for my own absolute equation, the apparatus recorded the absolute time of transit over the middle thread, and I marked the same by eye and ear, Mr. Pratt attending to the chronograph, and Mr. T. A. Harrison, late of the Survey, very kindly recording for me without remuneration.

Mr. E. F. Dickins, Subassistant Coast Survey, on leave, volunteered his services. He was, as usual, precise and methodical; his services were timely and very acceptable.

The longitude work having been completed, I converted the meridian telescope from a transit into a zenith telescope and began observing for latitude.

On the night of the 11th of June I considered that I had sufficient observations for the purpose, and on the 13th the instrument was packed, the observatory was taken down, and everything removed to the station in time for the morning train.

I desire to mention the services of Mr. Robert Hughes, who, in addition to his duties as telegraph operator, recorded both the longitude and latitude work.

I dwell with great pleasure on the trust which you have been pleased to repose in me. At all times your generous support of my endeavors is a sufficient guarantee for my best and entire services.

Very respectfully, yours,

B. A. COLONNA,
Assistant Coast Survey.

CARLILE P. PATTERSON,
Superintendent United States Coast and Geodetic Survey,
Washington, D. C.

[Extract from record book.]

TRANSIT OF MERCURY.

First external and internal contacts.

Observation of first external and first internal contacts of Mercury with the sun's limb, May 5 (astronomical reckoning), 1878. The time of sidereal chronometer 3479, Frodsham, break-circuit was marked by Mr. Thomas T. Crittenden by eye and ear when the key clicked, and by the chronograph also.

By an oversight, the chronograph was stopped and the fillet broken before the hours and minutes were written, and it had to be done from Crittenden's marking.

	h.	m.	s.		
(*a*)	22	10	40	Crittenden's markings by eye and ear at sound of key.	At 21^h 30^m the barometer read 22.97.
(*b*)	22	11	23		Ther:
(*c*)	22	13	09		D. B. 27½°.
(*d*)	22	13	29.5		W. B. 25½°.

B. A. COLONNA, *Observer*.

(*a*) First appearance, waves of fire rolling over it.
(*b*) So well defined that it cannot be mistaken.
(*c*) First light ripples cross.
(*d*) Well-defined line of light.

The above readings, taken from the fillet, are as follows:

	h.	m.	s.	
(*a*)	22	10	40.07	Fillet readings by Mr. J. F. Pratt.
(*b*)	22	11	23.28	
(*c*)	22	13	08.89	
(*d*)	22	13	29.52	

Was prepared to receive Washington signal of mean noon, but it did not come for May 5–6.

Noon, May 5–6, 1878.—The transit instrument being in the meridian, the following observations were made on the limbs of the sun and on Mercury. The telescope, 2⅝ inches clear aperture, had no eye-piece of sufficient magnifying power to enable me to take the limbs of Mercury, and it was observed as though it were a star. The three threads in the center of the field were used.

	West end.	East end.	
Levels.	35.0	34.3	Taken just before observing the following.
	33.0	35.8	

☉				*Mercury.*			☉		
h.	m.	s.		h.	m.	s.	h.	m.	s.
2	54	22.81	☿	2	55	07.36	2	56	37.6
2	54	34.49		2	55	18.51	2	56	48.87
2	54	4.69*		2	55	29.11	2	56	59.93

Clamp west—Barometer, 23.0 inches.

		°
Thermometer.	D. B.	46.0
	W. B.	37.5

	West end.	East end.	
Levels.	35.8	30.5	Taken just after completing above observations.
	25.0	41.0	

The above times were marked on chronograph, and not by eye and ear. Mr. Pratt attended to the chronograph. I herewith send fillet.

COLONNA.

* Probably 45.69.

TRANSIT OF MERCURY.

Second internal and external contacts.

B. A. COLONNA, *Observer.* T. T. CRITTENDEN, *Recorder.*

On hand, and watching Mercury, off and on, for 30 minutes before the second internal and external contacts. The edge of the sun was boiling much worse than it was this morning, and the planet itself at times appeared distorted. As it approached the edge of the sun it appeared to move much slower.

This marking is by Mr. Crittenden. The times are also recorded by chronograph.

h.	m.	s.		
5	40	45½	*a'*	Ther.: D. Bulb, 46½°. W. Bulb, 40.
5	41	03	*b'*	
5	41	25	*c'*	
5	43	30½	*d'*	Bar.: 23.0 inches.
5	43	47	*e'*	

(*a'*) First appearance of rippling black and white balls following one another between Mercury and sun's limb.

(*b'*) Contact actually made.

(*c'*) Contact very evident.

(*d'*) Waves of light flow over Mercury's limb at fourth contact.

(*e'*) Mercury has disappeared beneath waves of fire, as it were. Then looked, but see nothing more of the planet. Had hoped to see it after it had left the sun.

SUMMIT STATION, CENTRAL PACIFIC RAILROAD.

Summit astronomical reckoning, May 6, 1878.

J. F. PRATT, *Observer.* B. A. C., *Recorder.*

Receipt of Washington mean-time signal. May 6.—Using sidereal chronometer No. 3479, by Frodsham, and fillet chronograph by Hipp, over Western Union overland wire No. 6. (See fillet.)

Washington Naval Observatory.

	h.	m.	s.
Mean-time signal	6	00	00.0
Chronometer 3479	6	06	39.9

Comparison of chronometers.

		h.	m.	s.
Sid. chr. 3479	1	6	25	21.5
		3	28	17.0
M. T. chr. 231	2	6	27	45.0
		3	30	40.0
Sid. chr. 3479	1	6	32	17.5
		3	34	55.0
M. T. chr. 1705	2	6	35	21.0
		3	37	58.0

Remarks on the transit of Mercury.

MAY 5–6, 1878.

Ingress.—First contact, external. The sky was beautifully clear and the thermometer stood at 27½°. I was using Assistant Eimbeck's telescope made by Steinheil, No. 5248, of 5⅜ inches clear aperture and seven feet focal length, and a magnifying power of 120 diameters. The colored glasses used gave a yellowish-white light. The equatorial mounting rested on a wooden tripod,

which was filled with earth and stones to keep it steady. The whole was protected from the wind and weather by Professor Davidson's large observing tent.

For half an hour before first contact I occasionally scanned the sun's limb in the vicinity of the point of contact to see if any signs of the planet could be seen. Having rested my eyes for a few minutes preparatory to the final observation, I had not been looking a minute before I saw Mercury's limb and recorded the time marked (*a*).

It appeared in the eye-piece of my telescope as in this sketch, except that the shaded part was invisible. Fiery waves appeared at this instant to roll along behind and across it, and the next instant to disappear. This immediately ceased, and it was at this instant that I marked (*a*), which I consider very near the true time of first contact. Between the fiery waves were black balls of irregular shape and continually changing.

At the time (*b*), which was marked when the planet was so far on that the most casual observer could not fail to see it, it presented the appearance as in sketch marked (*b*). I do not attach any particular value to this marking.

I had intended marking the time when the planet was apparently bisected by the sun's limb, but it was boiling so that I found that no such estimate could be well made.

Ingress, second contact, internal.

Marked the time (*c*). At this contact the planet had partially assumed its round appearance, the cusps did not meet sharp and clear, but were preceded by fiery waves. At the instant that these seemed about to begin, I marked (*c*), as seen in sketch. I was about on time, for the next instant the first fiery waves ran across and contact was probably over.

When these waves ceased, and there was a well-defined bright line of light between the limbs of the sun and of Mercury, which could not be mistaken, I marked (*d*). To this marking I attach no particular value.

Egress, third contact, internal.

The telescope was in all respects the same as at ingress, except that a very slight amount of moisture had formed between the two glasses. I was afraid to risk taking them apart to wipe them, although I took out the object glass and wiped the inner surface where very heavy drops of dew had formed. The moisture between the glasses could only be seen in certain lights, and I could not see that it materially affected vision. The sky continued beautifully bright, and not a cloud had been seen during the day. For 30 minutes before third contact, I watched occasionally to see if there was anything worthy of note. The sun's circumference seemed to boil worse than at ingress, and as Mercury approached the sun's limb it seemed to be slightly deformed (this sketch exaggerates it about three times), and to move much slower. The latter of course was not the case, but I record it as it appeared to me at the time.

At the time marked (*a'*) the planet appeared as in the sketch, with rippling fiery and black balls chasing each other between it and the sun's limb.

The time (*b'*) was marked when I considered third contact actually made, or at the instant when the waves of fire or balls ceased to roll between the two limbs.

The time (*c'*) was marked after third contact was so plainly visible as to be noted by a casual observer. I do not attach any particular value to it.

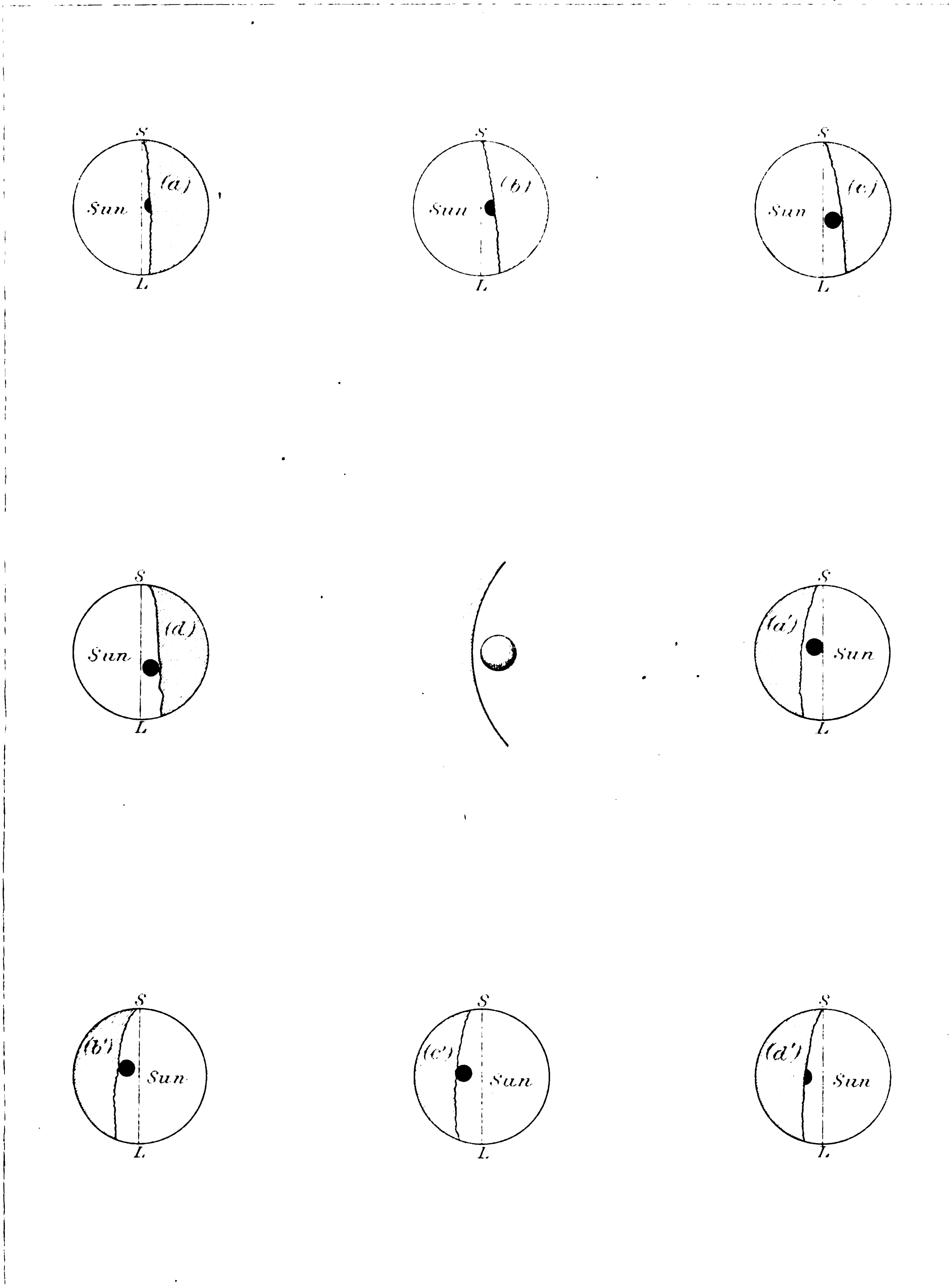
Egress, fourth contact, external.

Waves of light began to roll between Mercury and the sun, and to indicate the immediate approach of fourth contact when I marked (*d'*), as in the sketch. The limb of Mercury next the sun was now slightly distorted.

At (*e'*) the planet actually disappeared in the dark space beyond the sun's circumference, and I marked it as the time of what I considered the actual fourth contact.

I then continued to search for several minutes, hoping to see the planet beyond the sun's circumference, but it had totally disappeared, and the transit was over.

B. A. COLONNA.


S
(a)
Sun
L
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(b)
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(c)
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SUMMIT STATION, CENTRAL PACIFIC RAILROAD.

J. F. PRATT, *Observer and Recorder.*

MAY 5 (astronomical), 1878.

The telescope used is an equatorially mounted Coast Survey reconnoitering glass made by Utzschneider and Fraunhofer. It has a focal length of 3.75 feet, a clear aperture of 3 inches, and was used during the whole of the transit, with a magnifying power of 200 diameters, and the instrument was protected by a canvass wall.

Time-piece, sidereal break-circuit chronometer No. 3479, by Charles Frodsham, breaking every two seconds on fillet chronograph, by Hipp, observer, recording on fillet with break-circuit key.

Ingress.

Observer closely watched the sun for ten minutes previous to the time of first contact. Its edge seemed to be boiling or undulating.

(1.) (See fillet.) First external contact, $22^h\ 10^m\ 46^s.15$. The edge of the sun was quite irregular, and Mercury had notched the sun about as much as is indicated in the engraving of the United States Naval Observatory Instructions (external contact, Fig. 2).

(2.) (See fillet.) $22^h\ 13^m\ 01^s.40$. The planet had advanced so that it looked the same as is represented in I (Naval Observatory Instructions). Planet seemed to be very slightly distorted (egg-shaped), with small end towards limb of sun.

(3.) (See fillet.) First internal contact, $22^h\ 13^m\ 14^s.95$. Could *very* faintly distinguish a *grayish* thread of light between Mercury and sun's limb.

Egress.

MAY 6 (astronomical), 1878.

Watched the sun closely for ten minutes previous to time of second internal contact. Limb of sun appeared to be a little steadier than at time of ingress, though boiling.

(1'.) (See fillet.) Second internal contact, $5^h\ 40^m\ 48^s.9$. Planet seemed *very* slightly elongated (egg-shaped), with small end towards limb of sun.

(2'.) (See fillet.) $5^h\ 40^m\ 52^s.24$. Planet seemed perfectly round.

(3'.) (See fillet.) $5^h\ 40^m\ 58^s.23$. Side of planet distinctly flattened.

(4'.) (See fillet.) $5^h\ 42^m\ 39^s.87$. Center of planet appeared to be on limb of sun.

(5'.) (See fillet.) $5^h\ 44^m\ 01^s.38$. Second external contact. During the whole day the atmosphere was clear, and not a single cloud to be seen in the sky.

APPENDIX No. 7.

OBSERVATIONS MADE AT WASHINGTON, D, C., OF THE TRANSIT OF MERCURY, MAY 6, 1878.—REPORT OF CHARLES A. SCHOTT, ASSISTANT.

COAST AND GEODETIC SURVEY OFFICE,
Washington, D. C., May 7, 1878.

DEAR SIR: With your approbation, the undersigned, associated with Assistant R. D. Cutts, made the necessary arrangements for observing the transit of Mercury of May 6, 1878, the results of which, together with those by Assistant W. Eimbeck, are herewith respectfully submitted.

In thus multiplying observations in this city, I had in view the desirability of obtaining data for assigning the probable error to the observed times of the several phases, and consequently also of increasing the value of the local results.

Assistant Cutts and myself observed at Fauth & Co.'s observatory and yard, southwest of the Capitol, 8″.7 south of the dome, or in latitude 38° 53′ 11″.7, and 9″.4 west of it, or in longitude 77° 00′ 45″.1. The United States Naval Observatory being in longitude $5^h\ 08^m\ 12^s.1$, our place of observation was $9^s.1$ east of the Naval Observatory. Assistant Eimbeck observed in the rear of the Coast Survey buildings, in latitude 38° 53′ 11″ and in longitude 77° 00′ 34″.7, hence $9^s.8$ east of the Naval Observatory.

Local time was obtained by means of the noon-ball at the Observatory, viz:

	h. m. s.
May 2, ball down, by chronometer "Dent 2256"	11 59 07.3
3	06.0
4 (a failure in dropping the ball	19.5)
6	02.5
7	00.0

Average daily rate of mean-time chronometer Dent 2256, losing $1^s.5$. Comparisons (by coincidence of beats) were made with sidereal chronometer "Kessels 1287," used by Assistant Cutts, which are given in the proper place, together with comparisons of the sidereal chronometer used by Assistant Eimbeck.

I observed the ingress with an equatorial made by Fauth & Co., of this city, of 8 feet focal length, aperture 6½ inches, object-glass thinly silvered, and opening reduced to 3 inches diameter, magnifying power about 112; used a light-colored (neutral tint) shade glass. Time was counted and noted by Mr. M. H. Doolittle.

First external contact at $10^h\ 05^m\ 06^s$ (by Dent 2256). When first seen limb of planet was considerably advanced, nearly double the amount of indentation as shown on Professor Newcomb's diagram, but the projected part apparently belonging to a smaller radius than the true one. Tried to catch the same size of phase at time of egress, but did not succeed on account of undulations; possibly the recorded minute should be one less. As it is I place no reliance on this outer contact.

First internal contact at $10^h\ 06^m\ 40^s$, light darted across between sun and Mercury, but the limbs appeared loosely connected by a patch of haziness oscillating between them; full sunlight on and contact certainly over at $10^h\ 06^m\ 53^s$. The observation of internal contact is considered reliable.

Chronometer slow of Naval Observatory mean time 57^s, hence—

h. m. s.
First contact at 10 .. 03, mean time United States Naval Observatory (doubtful).
Inner contact at 10 07 37, mean time United States Naval Observatory (satisfactory).

Egress was observed in the yard, outside the observatory, with the Coast Survey transit made by Pistor & Martins of Berlin, 8 feet focal length, aperture 6½ inches, object glass partly covered with a thin film of silver except central part of 2 inches diameter which was left clear; used a neutral tint shade-glass; magnifying power 110. Sun's limb very unsteady, and seen through a thin cloud at time of third contact. At $5^h\ 32^m\ 27^s$ (chronometer time, Dent 2256) undulations on the sun's limb occasionally reached Mercury, otherwise sunlight clearly seen between the bodies. At $5^h\ 32^m\ 50^s$ interior contact established, though exact second difficult to seize on account of boiling motion of sun's image. At $5^h\ 34^m\ 54^s$ judged to be about same phase as at ingress when Mercury was first seen in the morning; time extremely doubtful on account of undulations. At $5^h\ 35^m\ 21^s$ outer and last contact fairly made out, the large waves along the sun showing no further interruption. Chronometer slow of Naval Observatory mean time 58^s, hence—

	h. m. s.	
Second interior contact at	5 33 48,	mean time United States Naval Observatory (satisfactory).
Last contact at	5 36 19,	mean time United States Naval Observatory (tolerably satisfactory.)

Respectfully submitted by—

CHAS. A. SCHOTT,
Assistant Coast and Geodetic Survey.

CARLILE P. PATTERSON, Esq.,
Superintendent United States Coast and Geodetic Survey.

TRANSIT OF MERCURY.

UNITED STATES COAST AND GEODETIC SURVEY OFFICE,
Washington, May 7, 1878.

Place of observation, Fauth & Co.'s observatory, Washington.
Latitude, 38° 53′ 11.7″ N.
Longitude, $09^s.1$ east of Naval Observatory.
Observer, Richard D. Cutts.
Recorder, John W. Parsons.

The large transit by Pistor & Martins, 6½ inches aperture, was mounted in the yard close to the observatory, but in consequence of the tremulous motion given to the telescope by the wind prevailing at the time of the ingress of the planet, one contact only was observed, and that unsatisfactorily.

Second (internal) contact.

Mr. Saegmuller, using a telescope with an aperture of 3 inches and a magnifying power of 70, observed the second (internal) contact as follows:

	h.	m.	s.
Time of contact by chronometer	6	49	01
Chronometer slow of Naval Observatory mean time	3	18	24
Mean time of contact	10	07	25
			P. M.

The third and fourth contacts were fully observed with a telescope by Fauth of Washington, having a clear aperture of 2½ inches and a magnifying power of 78.

As the telescope was small, the observer determined to note the time when the first doubt in regard to the contact arose, and again when that doubt ended, under the supposition that the mean of the two would give a better result than could otherwise be obtained with so small an objective and power.

Third (internal) contact.

	h.	m.	s.
Doubt commenced	2	16	32
Doubt ended	2	16	44
Time of contact by chronometer	2	16	38
Chronometer slow of Naval Observatory mean time	3	17	10
Mean time of contact	5	33	48

Fourth (external) contact.

	h.	m.	s.
Doubt commenced	2	18	59
Doubt ended	2	19	24
Time of contact by chronometer	2	19	11
Chronometer slow of Naval Observatory mean time	3	17	10
Mean time of contact	5	36	21

The time was determined by Kessels' sidereal box-chronometer, and was referred to Observatory time.

The definition of Mercury was sharp and steady. As the planet approached the undulating limb of the sun, the clear outline commenced to tremble or break; and at this moment the time was called, and again called, when the outline ceased to be visible. A thin cloud crossed the sun during this contact, which, however, did not interfere with the observation.

The same course was adopted for the last (external) contact, the time being noted when the inner limb of Mercury first became agitated, and again at its total extinguishment.

RICH'D D. CUTTS.

UNITED STATES COAST AND GEODETIC SURVEY OFFICE,
Washington, D. C., May 7, 1878.

DEAR SIR: I take pleasure in submitting to you a report of the observation of the transit of Mercury May 6, 1878, as made by me at a station near southeast corner Coast Survey Office, in latitude 38° 53′ 11″.2 (north); longitude 77° 00′ 34″.7 west of Greenwich.

The instrument employed was zenith telescope C. S. No. 3, by Troughton & Simms, of $3\frac{1}{8}$ inches clear aperture, and about 42 inches focal length. The magnifying power used was 48 diameters for the morning observations, *i. e.*, first and second contacts, and a power of 120 diameters in the afternoon observations, or third and fourth contacts.

Time was noted by sidereal chronometer Hutton No. 220, adjusted so as to indicate approximately local mean time.

The four contacts were observed and noted by chronometer as follows, viz:

First contact (ingress)	= $10^h\ 05^m\ 00^s.0$.	Observed too late; notch much smaller than expected.
Second contact (ingress)	= $10^h\ 07^m\ 40^s.0$.	Sun's limb just complete.
Third contact (egress)	= $5^h\ 35^m\ 06^s.0$.	Instant of formation of ligament.
Fourth contact (egress)	= $5^h\ 38^m\ 04^s.0$.	Observed too late.

During the morning observations the condition of the atmosphere proved very favorable, the well-defined solar limb appeared quite steady. In the afternoon the contacts were observed through haze and a strongly boiling atmosphere. Especially at the occurrence of the last contact the sun's limb trembled and undulated violently, owing to which the observer is under the impression that the contact was announced too late by about 5 or 10 seconds. It was difficult to judge the moment of disappearance of the notch in the solar limb.

The definition of the telescope was very fair. Screens were used at the eye end of the telescope.

About an hour before observation the Hutton chronometer was compared with Dent No. 2256, the chronometer used by Mr. Schott. The comparison was as follows:

Hutton, $9^h\ 01^m\ 47^s.5$ = Dent, $9^h\ 01^m\ 00^s.0$.

The error of Dent, at comparison, was given by Mr. Schott to be = 57^s slow.

The dropping of the time ball at Naval Observatory was noted by Hutton 220 (noon):

	h.	m.	s.
May 6, 1878 =	12	00	19.7.
May 7, 1878 =	12	04	21.7.

The hourly gaining rate on mean time of Hutton, therefore, was = $10^s.08$.

Assuming the time ball to have been dropped at precisely noon May 6 and 7, the observations of the contacts corrected for rate, *i. e.*, referred to Naval Observatory mean time, stand as follows, viz:

	h.	m.	s.
First contact (ingress)	10	05	00.0
Second contact (ingress)	10	07	39.0
Third contact (egress)	5	33	50.0
Fourth contact (egress)	5	36	47.0

which times of first and fourth contacts are subject to the qualification above stated.

The occurrence of the several contacts was announced aloud by the observer, and the time noted and recorded by Louis A. Sengteller, Assistant in the Coast and Geodetic Survey.

Yours, very respectfully,

WILLIAM EIMBECK.

P. S.—I inclose herewith, at his request, Assistant O. H. Tittmann's observations of the first two contacts. The correction for rate of chronometer is insignificant for that time.—W. E.

UNITED STATES COAST AND GEODETIC SURVEY OFFICE,
Washington, D. C., May 6, 1878.

Observation of transit of Mercury.

Telescope, 3-inch aperture; 91 magnifying power; red glass over ocular.

Stop-watch set at 15^s. When line of light broke I started stop-watch at 15^s.

	(22.) h.	m.	s.
When Assistant Eimbeck's chronometer showed	10	08	00
Stop-watch showed			42.5
Time between noting the breaking of the line of light			(15.0)
and comparison of stop-watch with chronometer			27.5 seconds of

mean time = $27^s.6$ of sidereal time.

Time of first interior contact (breaking of line of light), by Eimbeck's sidereal chronometer, $22^h\ 07^m\ 32^s.4$, uncorrected for chronometer error. The telescope was mounted within 10 feet of Assistant Eimbeck's chronometer and telescope.

O. H. TITTMANN.

APPENDIX No. 8.

UNITED STATES COAST AND GEODETIC SURVEY OFFICE,
Washington, March 17, 1879.

DEAR SIR: I transmit herewith the report of Assistant Schott on the adjustment of the primary triangulation between the Kent Island and Atlanta base lines.

I have read this paper with great satisfaction, both as to the quality of the triangulation and the methods of reduction, and recommend that it be printed in the Coast Survey Report for 1878.

Yours, respectfully,

J. E. HILGARD,
Assistant in Charge.

C. P. PATTERSON, Esq., *Superintendent.*

COMPUTING DIVISION, COAST AND GEODETIC SURVEY,
February 20, 1879.

DEAR SIR: Under date of November 15, 1878, I submitted to the Superintendent a report* on the completion and results of the primary triangulation along the Blue Ridge between Washington, D. C., and Atlanta, Ga. In this report I stated that the investigation of the probable error of this triangulation was not quite complete, and when completed would be communicated. This is now done.

I also submit in paper No. 1 a copy of the triangle-side computation to show the corrections to the measured angles needed to satisfy the geometrical conditions of the triangulation. It has already been stated, in the report referred to above, that no special equation was needed to bring the Kent Island and Peach Tree Ridge base lines into accord, since the values for length of junction line, as derived from these bases, already agreed closely.

From the employment of direction instruments, it might be expected that there is no tendency either to overmeasure or to undermeasure the angles, and, in fact, if we collect the errors in the closing of the triangles, we find for the 62 triangles of the northern half of the triangulation, or between Kent Island base and line Buffalo—Moore 31 with + errors, amounting in the aggregate to 41″.5; 31 with − errors, amounting in the aggregate to 38″.6; and for the 73 triangles of the southern half or between Buffalo—Moore and the Atlanta base, 30 with + errors, amounting in the aggregate to 41″.8; 43 with − errors, amounting in the aggregate to 65″.5. The want of balance I regard as accidental.

If we arrange the errors in closing of triangles according to the magnitude of the triangle, that is, according to the value of the spherical excess, we have the sum of the squares of errors as follows:

	Northern half of triangulation.	Spherical excess.	$\Sigma\Delta^2$		Southern half of triangulation.	Spherical excess.	$\Sigma\Delta^2$
		″ ″				″ ″	
1	10 triangles	0.244 to 0.725	20.64	1	12 triangles	0.092 to 0.930	22.04
2	11 ...do	0.764 to 1.608	18.95	2	12 ...do	0.949 to 1.887	37.81
3	10 ...do	1.848 to 4.021	25.12	3	13 ...do	2.004 to 2.806	31.70
4	10 ...do	4.040 to 5.365	35.69	4	12 ...do	2.832 to 4.158	93.46
5	11 ...do	5.491 to 7.386	30.81	5	12 ...do	4.796 to 7.763	39.01
6	10 ...do	8.124 to 12.119	23.45	6	12 ...do	7.937 to 15.026	51.43

* Copy appended.

From the first part of the table we can hardly infer anything respecting a relation between the error in the closing of triangles and their magnitude, but from the second part of the table it would seem that the error of closing slightly increases with the size of the triangle. A law of this kind is very probable.

The average probable error of a resulting direction, as found from the station or local adjustment, is $\pm 0''.19$ for the northern and $\pm 0''.18$ for the southern part of the triangulation. Now these numbers are very much smaller than the final corrections to the directions as resulting from the figure adjustment; in fact on this difference depends the application of weights in the adjustment, and it forms the distinctive feature of our method of treatment as explained, Coast Survey Report for 1864, p. 129, article 9. It is here also clearly seen that any increase in the number of measures or observations, or the substitution of instruments superior to those actually used, would but very slightly increase the accuracy of the triangulation. In order to do the latter effectively we would have to look to triangles of better shape, and to more triangles entering in combination as well as to a shortening of the distance between any two base lines.

In paper No. 2 I have given a complete account of the considerations for the computation of the probable error to which a triangulation may be liable in consequence of the uncertainty in the base measure and in consequence of the uncertainties in the angular measures.

The method now presented is the result of further attention given to the subject since the last publication in the report of 1865. Referring to this appendix, we have the probable error of any measured angle for the triangulation between Kent Island and the junction line Buffalo—Moore = $\pm 0''.63$; and for the triangulation between the Atlanta base and the junction $\pm 0''.86$; also, the probable error of the length of the junction line from the first or northern half of the Blue Ridge triangulation = $\pm 0^{m}.665$, and from the second or southern half = $\pm 0^{m}.829$; and combining with these values the effect of the probable errors of the base lines, or $\pm 0^{m}.208$ and $\pm 0^{m}.085$, the respective probable errors become $\pm 0^{m}.697$ and $\pm 0^{m}.833$, hence the probable error of the junction line by combination of these two values = $\pm 0^{m}.535$, or $\frac{1}{88700}$ of its length.

The probable error of the measure of the Kent Island base is $\pm \frac{1}{228000}$ and of the Atlanta base $\pm \frac{1}{561880}$; hence the average probable error of the northern half of the Blue Ridge triangulation = $\pm \frac{1}{116900}$, and of the southern half = $\pm \frac{1}{123900}$ of the length; and for the whole of the 602 statute miles of triangulation $\pm \frac{1}{120300}$ of its length, on the average.

Paper No. 3 is submitted by special direction of the Superintendent, who desired that a complete account of the method of solution of large numbers of normal equations be presented, and I have accordingly requested Mr. M. H. Doolittle, of the Computing Division, to draw up this paper.

This method of solution of linear normal equations is rigorous and direct, and with systematic arrangement merely involves mechanical labor. It may be applied to a far greater number of equations than we have had hitherto occasion to solve (52). Its essential peculiarity consists in first obtaining an approximate solution effected by operating only with a small number of significant figures of the coefficients involved so as to bring into use Crelle's tables. Logarithms are altogether dispensed with.

An approximate value for the first unknown quantity having been obtained from the first equation, and expressed in terms of the other unknown quantities contained in that equation, it is substituted in the second equation, and from this an approximate value of the second unknown quantity is found in terms of the remaining unknown quantities. Next follows a substitution in the third equation, and so on to the end. Approximate values of all unknown quantities having thus been obtained, a rigorous substitution in the original equations is made, and the new and necessarily small residuals are treated a second time in a manner similar to the first-described process, and the small corrections now found are added to the first approximations. It is not likely that a third treatment will ever be needed. To the paper Mr. Doolittle has added some remarks pertinent to the proper solution and arrangement of conditional equations, especially for the case involving small angles. Practically, two computers are employed in the solution of any large number of equations, and they compare their results at the close of each day's work, making the commission of any mistakes, without speedy detection, almost impossible.

Yours, respectfully,

CHAS. A. SCHOTT,
Assistant in charge Computing Division.

COAST AND GEODETIC SURVEY OFFICE,
November 15, 1878.

DEAR SIR: It gives me pleasure to inform you by this preliminary report of the successful completion, a few days ago, of the computation of the primary triangulation along the Blue Ridge between Washington, D. C., and Atlanta, Ga. The primary triangulation, as measured along its axis, stretches over 602 statute miles between the Kent Island base and the Atlanta base.

The junction line, which represents about the average width of the triangulation, is nearly midway between the bases (300 miles from Kent Island and 302 from the Atlanta line), and its length between Buffalo △ and Moore △ is 29.5 statute miles, or 47.5 kilometers nearly. Computed from the Kent Island base it is 47 462^{m}.826, from the Atlanta base 47 462^{m}.842 in length, the difference being only one unit in the seventh place of decimals, or in linear measure about 15 millimeters, or a little over half an inch. This close accord must be regarded, in a measure, as accidental, though such accidents usually happen only with good work. The true estimate of the value of our work is furnished by the probable error of the length of the junction line, which is a little less than $\pm$ 0^{m}.6 or $\pm$ 2 feet, or $\pm \frac{1}{79000}$ nearly of the length, but this quantity is not yet finally settled. The close agreement at the junction rendered the extra labor of carrying a distance equation, for perfect accord, unnecessary. The least square treatment of the northern half of the triangulation involved the solution of 52, and that of the southern of 41 normal equations. Both sets were successfully solved in a very short time, and the method of solution adopted by Mr. Doolittle will be presented with my full report. The accord in the azimuths of the line is no less remarkable. From the northern mean astronomical azimuth it is 338° 26′ 34″.07, and from the southern 338° 26′ 32″.63, difference 1″.44, which includes effect of error in assumed spheroid. With respect to latitude, the northern geodetic latitudes overlap the southern ones by nearly 4″; thus latitude of Moore from North 36° 23′ 49″.905, from South 53″.851; difference 3″.946, about 121^{m}.7 or 399 feet. This I expect to convert into a small quantity by *lifting* the triangulation from the surface of the now obsolete Bessel spheroid to that of Clarke's. The longitudes agree within 1″.2 or 0^{s}.08, indicating that our approximate telegraphic longitude of Atlanta is probably nearly correct. In accordance with your verbal approval, I shall resume my discussion of 1870, and bring out the final results for the whole oblique arc.

I remain, yours, very respectfully,

C. A. SCHOTT,
Assistant in charge of Office and Computing Division.

CARLILE P. PATTERSON, Esq.,
Superintendent United States Coast and Geodetic Survey.

[PAPER No. 1.]

Adjusted primary triangles between Kent Island, Md., Washington, D. C., and Atlanta, Ga.

Name of stations.	Observed angles.	Corrections.	Spherical angles.	Spherical excess.	Plane angles and distances.	Logarithms.
	° ′ ″	″	″	″	*Meters.*	
Kent Island—North Base to South Base					8 687.5446	3.9388970 5
Taylor	38 36 52.373	−.509	51.864	.081	51.783 ″	0.2047626 7
North Base	88 35 36.913	−.233	36.680	.081	36.599	9.9998691 3
South Base	52 47 32.011	−.311	31.700	.082	31.618	9.9011567 3
Taylor to South Base					13 916.462	4.1435288 5
Taylor to North Base					11 087.061	4.0448164 5
Kent Island—North Base to South Base						3.9388970 5
Marriott	18 13 37.318	+.270	37.588	.146	37.442	0.5047559 4
North Base	50 05 05.356	−.702	04.654	.145	04.509	9.8847911 5
South Base	111 41 18.247	−.052	18.195	.146	18.049	9.9681128 2
Marriott to South Base					21 303.165	4.3284441 4
Marriott to North Base					25 808.681	4.4117658 1

Adjusted primary triangles between Kent Island, Md., Washington, D. C., and Atlanta, Ga.—Continued.

Name of stations.	Observed angles.	Corrections.	Spherical angles.	Spherical excess.	Plane angles and distances.	Logarithms.
	° ′ ″	″	″	″	*Meters.*	
Taylor to South Base						4.1435288 5
Marriott	40 10 21.280	+.243	21.523	.215	″ 21.308	0.1903782 4
Taylor	80 55 51.946	+.680	52.626	.215	52.411	9.9945370 4
South Base	58 53 46.236	+.260	46.496	.215	46.281	9.9325918 2
Marriott to South Base					21 303.165	4.3284441 3
Marriott to Taylor					18 471.361	4.2664989 1
Taylor to North Base						4.0448164 5
Marriott	21 56 43.962	−.027	43.935	.150	43.785	0.4274485 3
Taylor	119 32 44.319	+.171	44.490	.151	44.339	9.9395008 3
North Base	38 30 31.557	+.470	32.027	.151	31.876	9.7942339 3
Marriott to North Base					25 808.681	4.4117658 0
Marriott to Taylor					18 471.361	4.2664989 1
North Base to Marriott						4.4117658 1
Linstid	68 43 33.677	−.068	33.609	.370	33.239	0.0306515 0
North Base	70 56 58.975	+.763	59.738	.370	59.368	9.9755389 2
Marriott	40 19 28.501	−.738	27.763	.370	27.393	9.8109800 3
Linstid to Marriott					26 179.192	4.4179562 3
Linstid to North Base					17 922.448	4.2533973 4
North Base to Linstid						4.2533973 4
Swan Point	56 08 57.917	+1.016	58.933	.254	58.679	0.0806629 1
North Base	60 07 41.140	−.009	41.131	.255	40.876	9.9380894 4
Linstid	63 43 20.633	+.067	20.700	.255	20.445	9.9526273 9
Swan Point to Linstid					18 713.270	4.2721496 9
Swan Point to North Base					19 350.297	4.2866876 4
Swan Point to Linstid						4.2721496 9
Pool's Island	36 22 15.134	+.034	15.168	.234	14.934	0.2269388 5
Swan Point	113 07 27.589	−.434	27.155	.233	26.922	9.9636254 5
Linstid	30 30 19.239	−.862	18.377	.233	18.144	9.7055337 2
Pool's Island to Linstid					29 021.108	4.4627139 9
Pool's Island to Swan Point					16 018.515	4.2046222 6
Pool's Island to Linstid						4.4627139 9
Finlay	53 32 21.714	+1.651	23.365	.635	22.730	0.0945990 6
Pool's Island	79 44 39.791	+.509	40.300	.635	39.665	9.9930053 2
Linstid	46 42 57.730	+.510	58.240	.635	57.605	9.8621100 6
Finlay to Linstid					35 507.358	4.5503183 7
Finlay to Pool's Island					26 267.764	4.4194231 1
Linstid to Webb						4.2142059 4
Finlay	25 43		37.235	.489	36.746	0.3624285 5
Linstid	84 01 06.411	+.096	06.507	.489	06.018	9.9976289 4
Webb	70 15 16.992	+.734	17.726	.490	17.236	9.9736838 8
Finlay to Webb					37 520.051	4.5742634 3
Finlay to Linstid					35 507.358	4.5503183 6
North Base to Taylor						4.0448164 5
Linstid	34 46 24.832	+1.126	25.958	.090	25.868	0.2438671 1
North Base	32 26 27.418	+.295	27.713	.091	27.622	9.7295138 2
Taylor	112 47 05.714	+.886	06.600	.090	06.510	9.9647137 8
Linstid to Taylor					10 427.912	4.0181973 8
Linstid to North Base					17 922.448	4.2533973 4
Taylor to Marriott						4.2664989 1
Linstid	33 57 08.845	−1.195	07.650	.129	07.521	0.2529772 4
Taylor	127 40 09.967	−1.057	08.910	.129	08.781	9.8984800 9
Marriott	18 22 44.539	−.712	43.827	.129	43.698	9.4987212 2
Linstid to Marriott					26 179.192	4.4179562 4
Linstid to Taylor					10 427.912	4.0181973 7

Adjusted primary triangles between Kent Island, Md., Washington, D. C., and Atlanta, Ga.—Continued.

Name of stations.	Observed angles.			Corrections.	Spherical angles.	Spherical excess.	Plane angles and distances.	Logarithms.
	°	′	″	″	″	″	*Meters.*	
Linstid to Marriott								4. 4179562 3
Webb	76	16	06. 190	− . 103	06. 087	. 333	05. 754	0. 0125939 5
Linstid	66	18	42. 310	+ . 256	42. 566	. 332	42. 234	9. 9617745 0
Marriott	37	25	11. 128	+1. 216	12. 344	. 332	12. 012	9. 7836557 5
Webb to Marriott							24 678. 836	4. 3923246 8
Webb to Linstid							16 375. 929	4. 2142059 3
Webb to Marriott								4. 3923246 8
Hill	56	40	32. 002	+ . 408	32. 410	. 465	31. 945	0. 0780156 5
Webb	53	10	52. 339	− . 342	51. 997	. 465	51. 532	9. 9033789 6
Marriott	70	08	37. 171	− . 184	36. 987	. 464	36. 523	9. 9733801 1
Hill to Marriott							23 643. 910	4. 3737192 9
Hill to Webb							27 779. 245	4. 4437204 4
Webb to Marriott								4. 3923246 8
Soper	39	41	37. 078	− . 245	36. 833	. 486	36. 347	0. 1947169 7
Webb	102	15	58. 886	+ . 009	58. 895	. 486	58. 409	9. 9899705 6
Marriott	38	02	26. 815	−1. 085	25. 730	. 486	25. 244	9. 7897331 2
Soper to Marriott							37 758. 280	4. 5770122 1
Soper to Webb							23 810. 843	4. 3767747 7
Webb to Hill								4. 4437204 4
Soper	75	01	10. 921	− . 034	10. 887	. 423	10. 464	0. 0150165 0
Webb	49	05	06. 547	+ . 351	06. 898	. 423	06. 475	9. 8783399 6
Hill	55	53	43. 600	− . 116	43. 484	. 423	43. 061	9. 9180378 3
Soper to Hill							21 730. 859	4. 3370769 0
Soper to Webb							23 810. 843	4. 3767747 7
Marriott to Hill								4. 3737192 9
Soper	35	19	33. 843	+ . 211	34. 054	. 401	33. 653	0. 2379007 9
Marriott	32	06	10. 356	+ . 901	11. 257	. 402	10. 855	9. 7254568 2
Hill	112	34	15. 602	+ . 292	15. 894	. 402	15. 492	9. 9653921 3
Soper to Hill							21 730. 859	4. 3370769 0
Soper to Marriott							37 758. 280	4. 5770122 1
Soper to Hill								4. 3370769
Causten	69	31	00. 444	+1. 019	01. 463	. 280	01. 183	0. 0283643
Soper	47	08	46. 410	+ . 655	47. 065	. 280	46. 785	9. 8651591
Hill	63	20	11. 713	+ . 598	12. 311	. 279	12. 032	9. 9511718
Causten to Hill							17 005. 926	4. 2306003
Causten to Soper							20 730. 652	4. 3166130
Soper to Hill								4. 3370769
Stabler	17	46	18. 832	− . 001	18. 831	. 033	18. 798	0. 5153755
Soper	158	15			58. 660	034	58. 626	9. 5685460
Hill	3	57	43. 173	− . 560	42. 610	. 034	42. 576	8. 8394268
Stabler to Hill							26 363. 218	4. 4209984
Stabler to Soper							4 919. 027	3. 6918792
Webb to Hill								4. 4437204
Stabler	62	40	22. 319	− . 319	22. 000	. 536	21. 464	0. 0513924
Webb	57	28	13. 386	+ . 128	13. 514	. 536	12. 978	9. 9258856
Hill	59	51	26. 654	− . 560	26. 094	. 536	25. 558	9. 9369035
Stabler to Hill							26 363. 218	4. 4209984
Stabler to Webb							27 040. 600	4. 4320163
Webb to Soper								4. 3767748
Stabler	44	54	03. 487	− . 318	03. 169	. 080	03. 089	0. 1512678
Webb	8	23	06. 488	+ . 128	06. 616	. 079	06. 537	9. 1638367
Soper	126	42			50. 453	. 079	50. 374	9. 9039738
Stabler to Soper							4 919. 028	3. 6918793
Stabler to Webb							27 040. 606	4. 4320164

Adjusted primary triangles between Kent Island, Md., Washington, D. C., and Atlanta, Ga.—Continued.

Names of stations.	Observed angles.			Corrections.	Spherical angles.	Spherical excess.	Plane angles and distances.	Logarithms.
	°	′	″	″	″	″	*Meters.*	
Stabler to Hill								4.4209984
Peach Grove	51	03	01.001	−.142	00.859	.616	00.243	0.1091904
Stabler	63	40	03.058	−.163	02.895	.616	00.279	9.9524212
Hill	65	16	57.501	+.593	58.094	.616	57.478	9.9582683
Peach Grove to Hill							30 381.552	4.4826100
Peach Grove to Stabler							30 793.362	4.4884571
Causten to Hill								4.2306003
Peach Grove	7	24	22.078	+.320	22.398	.045	22.353	0.8897368
Causten	166	41			09.343	.045	09.298	9.3622730
Hill	5	54	28.958	−.564	28.394	.045	28.349	9.0125388
Peach Grove to Hill							30 381.559	4.4826101
Peach Grove to Causten							13 579.253	4.1328759
Stabler to Peach Grove								4.4884571
Maryland Heights	26	57	21.211	+1.049	22.260	1.634	20.626	0.3436124
Stabler	67	47	51.529	+.336	51.865	1.634	50.231	9.9665419
Peach Grove	85	14	49.788	+.988	50.776	1.633	49.143	9.9985039
Maryland Heights to Peach Grove							62 894.319	4.7986114
Maryland Heights to Stabler							67 697.625	4.8305734
Maryland Heights to Stabler								4.8305734
Bull Run	67	51	56.802	−.610	56.192	2.708	53.484	0.0332494
Maryland Heights	67	51	33.942	+.208	34.150	2.708	31.442	9.9667317
Stabler	44	16	38.016	−.234	37.782	2.708	35.074	9.8439305
Bull Run to Stabler							67 694.672	4.8305545
Bull Run to Maryland Heights							51 021.506	4.7077533
Maryland Heights to Peach Grove								4.7986114
Bull Run	85	09	50.705	−.755	49.950	1.779	48.171	0.0015492
Maryland Heights	40	54	12.731	−.841	11.890	1.778	10.112	9.8160940
Peach Grove	53	56	04.405	−.910	03.495	1.778	01.717	9.9075927
Bull Run to Peach Grove							41 328.971	4.6162546
Bull Run to Maryland Heights							51 021.506	4.7077533
Stabler to Peach Grove								4.4884571
Bull Run	17	17	53.903	−.145	53.758	.704	53.054	0.5267427
Stabler	23	31	13.513	+.570	14.083	.704	13.379	9.6010548
Peach Grove	139	10	54.193	+.078	54.271	.704	53.567	9.8153548
Bull Run to Peach Grove							41 328.971	4.6162546
Bull Run to Stabler							67 694.687	4.8305546
Stabler to Hill								4.4209984
Sugar Loaf	18	22			03.879	.618	03.261	0.5015851
Stabler	134	09	42.341	+.252	42.593	.617	41.976	9.8557474
Hill	27	28	15.028	+.353	15.381	.618	14.763	9.6639797
Sugar Loaf to Hill							60 017.917	4.7782809
Sugar Loaf to Stabler							38 593.416	4.5865132
Stabler to Peach Grove								4.4884571
Sugar Loaf	45	42			51.637	.948	50.689	0.1451693
Stabler	70	29	39.283	+.414	39.697	.948	38.749	9.9743307
Peach Grove	63	47	31.094	+.416	31.510	.948	30.562	9.9528870
Sugar Loaf to Peach Grove							40 546.850	4.6079571
Sugar Loaf to Stabler							38 593.434	4.5865134
Stabler to Bull Run								4.8305545
Sugar Loaf	98	43			42.024	1.616	40.408	0.0050584
Stabler	46	58	25.770	−.156	25.614	1.617	23.997	9.8639389
Bull Run	34	17	56.964	+.248	57.212	1.617	55.595	9.7509004
Sugar Loaf to Bull Run							50 067.023	4.6995518
Sugar Loaf to Stabler							38 593.425	4.5865133

Adjusted primary triangles between Kent Island, Md., Washington, D. C., and Atlanta, Ga.—Continued.

Names of stations.	Observed angles.	Corrections.	Spherical angles.	Spherical excess.	Plane angles and distances.	Logarithms.
	° ′ ″	″	″	″	*Meters.*	
Stabler to Maryland Heights						4. 8305734
Sugar Loaf	173 44		19. 706	. 104	″ 19. 602	0. 9623282
Stabler	2 41 47. 754	+ . 078	47. 832	. 104	47. 728	8. 6725321
Maryland Heights	3 33 53. 321	— . 547	52. 774	. 104	52. 670	8. 7936116
Sugar Loaf to Maryland Heights					29 203. 419	4. 4654337
Sugar Loaf to Stabler					38 598. 416	4. 5865132
Hill to Peach Grove						4. 4826100
Sugar Loaf	27 20		47. 757	. 946	46. 811	0. 3378392
Hill	37 48 42. 473	+ . 240	42. 713	. 946	41. 767	9. 7875081
Peach Grove	114 50 32. 095	+ . 274	32. 369	. 947	31. 422	9. 9578319
Sugar Loaf to Peach Grove					40 546. 860	4. 6079573
Sugar Loaf to Hill					60 017. 944	4. 7782811
Peach Grove to Bull Run						4. 6162546
Sugar Loaf	53 00		50. 388	1. 373	49. 015	0. 0975737
Peach Grove	75 23 23. 099	— . 339	22. 760	1. 372	21. 388	9. 9857237
Bull Run	51 35 50. 867	+ . 103	50. 970	1. 373	49. 597	9. 8941288
Sugar Loaf to Bull Run					50 067. 047	4. 6995520
Sugar Loaf to Peach Grove					40 546. 850	4. 6079571
Peach Grove to Maryland Heights						4. 7986114
Sugar Loaf	128 01		28. 070	. 790	27. 280	0. 1036115
Peach Grove	21 27 18. 694	+ . 571	19. 265	. 789	18. 476	9. 5632111
Maryland Heights	30 31 14. 532	+ . 502	15. 034	. 790	14. 244	9. 7057341
Sugar Loaf to Maryland Heights					29 203. 439	4. 4654340
Sugar Loaf to Peach Grove					40 546. 841	4. 6079570
Bull Run to Maryland Heights						4. 7077533
Sugar Loaf	75 00		37. 681	1. 195	36. 486	0. 0150357
Bull Run	33 33 59. 838	— . 858	58. 980	1. 195	57. 785	9. 7426450
Maryland Heights	71 25 27. 263	— . 338	26. 925	1. 196	25. 729	9. 9767630
Sugar Loaf to Maryland Heights					29 203. 439	4. 4654340
Sugar Loaf to Bull Run					50 067. 047	4. 6995520
Maryland Heights to Peach Grove						4. 7986114
Mount Marshall	45 15 40. 425	+ . 138	40. 563	3. 905	36. 658	0. 1485518
Maryland Heights	76 11 58. 138	— . 584	57. 554	3. 904	53. 650	9. 9872760
Peach Grove	58 32 34. 064	— . 467	33. 597	3. 905	29. 692	9. 9309588
Mount Marshall to Peach Grove					85 988. 275	4. 9344392
Mount Marshall to Maryland Heights					75 530. 439	4. 8781220
Maryland Heights to Bull Run						4. 7077533
Mount Marshall	41 01 17. 218	+ . 589	17. 807	1. 885	15. 922	0. 1828733
Maryland Heights	35 17 45. 407	+ . 257	45. 664	1. 885	43. 779	9. 7617726
Bull Run	103 41 01. 614	+ . 569	02. 183	1. 884	00. 299	9. 9874952
Mount Marshall to Bull Run					44 915. 804	4. 6523992
Mount Marshall to Maryland Heights					75 530. 404	4. 8781218
Bull Run to Peach Grove						4. 6162546
Mount Marshall	4 14 23. 207	— . 451	22. 756	. 242	22. 514	1. 1311954
Bull Run	171 09 07. 681	+ . 186	07. 867	. 241	07. 626	9. 1869892
Peach Grove	4 36 29. 659	+ . 443	30. 102	. 242	29. 860	8. 9049492
Mount Marshall to Peach Grove					85 988. 275	4. 9344392
Mount Marshall to Bull Run					44 915. 804	4. 6523992
Maryland Heights to Sugar Loaf						4. 4654339
Mount Marshall	18 25 47. 940	+ . 866	48. 806	1. 788	47. 018	0. 5001187
Maryland Heights	106 43 12. 670	— . 082	12. 588	1. 788	10. 800	9. 9812403
Sugar Loaf	54 51		03. 970	1. 788	02. 182	9. 9125604
Mount Marshall to Sugar Loaf					88 469. 367	4. 9467929
Mount Marshall to Maryland Heights					75 530. 439	4. 8781220

Adjusted primary triangles between Kent Island, Md., Washington, D. C., and Atlanta, Ga.—Continued.

Names of stations.	Observed angles.	Corrections.	Spherical angles.	Spherical excess.	Plane angles and distances.	Logarithms.
	° ′ ″	″	″	″	*Meters.*	
Sugar Loaf to Peach Grove						4.6079571
Mount Marshall	26 49 52.485	−.728	51.757	2.906	″ 48.851	0.3454880
Sugar Loaf	73 10		24.099	2.906	21.193	9.9809941
Peach Grove	79 59 52.758	+.104	52.862	2.906	49.956	9.9933477
Mount Marshall to Peach Grove					85 988.275	4.9344392
Mount Marshall to Sugar Loaf					88 469.347	4.9467928
Sugar Loaf to Bull Run						4.6995519
Mount Marshall	22 35 29.278	−.277	29.001	1.292	27.709	0.4154983
Sugar Loaf	20 09		33.711	1.292	32.419	9.5373489
Bull Run	137 14 61.452	−.288	61.164	1.292	59.872	9.8317426
Mount Marshall to Bull Run					44 915.794	4.6523991
Mount Marshall to Sugar Loaf					88 469.347	4.9467928
Mount Marshall to Bull Run						4.6523992
Fork	24 41 24.035	−1.249	22.786	1.022	21.764	0.3791369
Mount Marshall	134 42 53.217	−.632	52.585	1.022	51.563	9.8516396
Bull Run	20 35 48.021	−.326	47.695	1.022	46.673	9.5462726
Fork to Bull Run					76 414.491	4.8831757
Fork to Mount Marshall					37 837.591	4.5778087
Mount Marshall to Bull Run						4.6523992
Clark	40 54 42.447	+.550	42.997	2.066	40.931	0.1838311
Mount Marshall	86 33 27.193	−.319	26.874	2.065	24.809	9.9992153
Bull Run	52 31 56.183	+.143	56.326	2.066	54.260	9.8996512
Clark to Bull Run					68 461.375	4.8354456
Clark to Mount Marshall					54 435.412	4.7358815
Fork to Bull Run						4.8831757
Clark	84 53 37.398	−.357	37.041	2.341	34.700	0.0017275
Fork	63 10 21.046	+.307	21.353	2.342	19.011	9.9505425
Bull Run	31 56 08.162	+.469	08.631	2.342	06.289	9.7234213
Clark to Bull Run					68 461.390	4.8354457
Clark to Fork					40 581.168	4.6083245
Fork to Mount Marshall						4.5778087
Clark	43 58 54.951	−.907	54.044	1.298	52.746	0.1583754
Fork	87 51 45.081	−.942	44.139	1.298	42.841	9.9996975
Mount Marshall	48 09 26.024	−.313	25.711	1.298	24.413	9.8721405
Clark to Mount Marshall					54 435.425	4.7358816
Clark to Fork					40 581.178	4.6083246
Mount Marshall to Clark						4.7358815
Peters	44 26		43.717	1.387	42.330	0.1547621
Mount Marshall	24 30 02.461	+.221	02.682	1.387	01.295	9.6177330
Clark	111 03 18.128	−.367	17.761	1.386	16.375	9.9699928
Peters to Clark					32 238.680	4.5083766
Peters to Mount Marshall					72 549.833	4.8606364
Fork to Mount Marshall						4.5778087
Peters	21 49		29.656	.932	28.724	0.4297289
Fork	134 31 10.900	−.788	10.112	.933	09.179	9.8530989
Mount Marshall	23 39 23.563	−.534	23.029	.932	22.097	9.6034115
Peters to Mount Marshall					72 549.850	4.8606365
Peters to Fork					40 827.151	4.6109491
Fork to Clark						4.6083245
Peters	66 16		13.371	1.020	12.351	0.0383641
Fork	46 39 25.819	+.154	25.973	1.020	24.953	9.8616880
Clark	67 04 23.177	+.539	23.716	1.020	22.696	9.9642604
Peters to Clark					32 238.680	4.5083766
Peters to Fork					40 827.142	4.6109490

Adjusted primary triangles between Kent Island, Md., Washington, D. C., and Atlanta, Ga.—Continued.

Names of stations.	Observed angles.	Corrections.	Spherical angles.	Spherical excess.	Plane angles and distances.	Logarithms.
	° ′ ″	″	″	″	*Meters.*	
Fork to Clark						4.6083245
Spear	21 58 17.797	+ .318	18.115	3.591	14.524	0.4269747
Fork	79 35 40.852	+1.219	42.071	3.591	38.480	9.9927976
Clark	78 26 10.172	+ .415	10.587	3.591	06.996	9.9910926
Spear to Clark					106 683.39	5.0280938
Spear to Fork					106 265.38	5.0263918
Fork to Clark						4.6083245
Humpback	27 01 48.987	+ .566	49.553	2.462	47.091	0.3425110
Fork	98 41 42.476	+ .558	43.034	2.462	40.572	9.9949802
Clark	54 16 32.805	+1.994	34.799	2.462	32.337	9.9094680
Humpback to Clark					88 270.531	4.9458157
Humpback to Fork					72 494.233	4.8603035
Humpback to Fork						4.8603035
Spear	32 08 11.606	+ .037	11.643	2.134	09.509	0.2741452
Humpback	128 45 53.652	+ .144	53.796	2.134	51.662	9.8919430
Fork	19 05 61.624	− .661	60.963	2.134	58.829	9.5148300
Spear to Fork					106 265.35	5.0263917
Spear to Humpback					44 594.235	4.6492787
Humpback to Fork						4.8603035
Peters	93 46		18.065	1.975	16.090	0.0009414
Humpback	34 11 29.823	+ .978	30.801	1.976	28.825	9.7497042
Fork	52 02 16.657	+ .404	17.061	1.976	15.085	9.8967542
Peters to Fork					40 827.151	4.6109491
Peters to Humpback					57 279.487	4.7579991
Humpback to Clark						4.9458157
Peters	160 02		31.435	0.533	30.902	0.4668222
Humpback	7 09 40.836	+ .413	41.249	0.534	40.715	9.0957385
Clark	12 47 50.372	−1.455	48.917	0.534	48.383	9.3453611
Peters to Clark					32 238.615	4.5083764
Peters to Humpback					57 279.474	4.7579990
Humpback to Clark						4.9458157
Spear	54 06 29.403	+ .356	29.759	3.263	26.496	0.0914523
Humpback	101 44 04.665	− .422	04.243	3.263	00.980	9.9908287
Clark	24 09 37.367	−1.580	35.787	3.263	32.524	9.6120107
Spear to Clark					106 683.37	5.0280967
Spear to Humpback					44 594.235	4.6492787
Clark to Spear						5.0280967
Peters	163 48		05.103	0.573	04.530	0.5544424
Clark	11 21 46.995	− .124	46.871	0.574	46.297	9.2945144
Spear	4 50 09.314	+ .433	09.747	0.574	09.173	8.9258372
Peters to Spear					75 344.842	4.8770535
Peters to Clark					32 238.607	4.5083763
Spear to Fork						5.0263917
Peters	129 55		41.526	1.998	39.528	0.1152865
Spear	17 05 08.483	− .115	08.368	1.997	06.371	9.4692708
Fork	32 56 15.033	+1.065	16.098	1.997	14.101	9.7353754
Peters to Fork					40 827.142	4.6109490
Peters to Spear					75 344.860	4.8770536
Spear to Humpback						4.6492787
Peters	36 09		23.461	2.156	21.305	0.2291593
Spear	49 16 20.089	− .077	20.012	2.156	17.856	9.8795611
Humpback	94 34 23.829	− .835	22.994	2.155	20.839	9.9986156
Peters to Humpback					57 279.487	4.7579991
Peters to Spear					75 344.860	4.8770536

Adjusted primary triangles between Kent Island, Md., Washington, D. C., and Atlanta, Ga.—Continued.

Names of stations.	Observed angles.	Corrections.	Spherical angles.	Spherical excess.	Plane angles and distances.	Logarithms.
	° ′ ″	″	″	″	*Meters.*	
Humpback to Spear						4.6492787
Tobacco Row	59 27 05.418	−1.124	04.294	1.369	02.″925	0.0648995
Humpback	46 51 42.988	−1.143	41.845	1.369	40.476	9.8631442
Spear	73 41 19.078	−1.109	17.969	1.370	16.599	9.9821564
Tobacco Row to Spear					37 785.261	4.5773224
Tobacco Row to Humpback					49 697.506	4.6963346
Humpback to Spear						4.6492787
Long	30 47 57.987	−1.490	56.497	1.347	55.150	0.2907108
Humpback	28 27 31.934	+.395	32.329	1.346	30.983	9.6780845
Spear	120 44 36.003	−.789	35.214	1.347	33.867	9.9342313
Long to Spear					41 502.476	4.6180740
Long to Humpback					74 855.000	4.8742208
Humpback to Long						4.8742208
Tobacco Row	132 04 14.011	−.975	13.036	.995	12.041	0.1294050
Humpback	18 24 11.054	−1.538	09.516	.994	08.522	9.4992584
Long	29 31 39.546	+.885	40.431	.994	39.437	9.6927087
Tobacco Row to Long					31 833.485	4.5028842
Tobacco Row to Humpback					49 697.494	4.6963345
Tobacco Row to Spear						4.5773224
Long	60 19 37.533	−.605	36.928	.972	35.956	0.0610493
Tobacco Row	72 37 08.593	+.150	08.743	.972	07.771	9.9797024
Spear	47 03 16.925	+.320	17.245	.972	16.273	9.8645125
Long to Spear					41 502.486	4.6180741
Long to Tobacco Row					31 833.485	4.5028842
Spear to Long						4.6180740
Flat Top	31 44 51.536	−.040	51.496	1.555	49.941	0.2788719
Spear	37 00 48.990	−.368	48.622	1.555	47.067	9.7795945
Long	111 14 25.041	−.493	24.548	1.556	22.992	9.9694498
Flat Top to Long					47 483.242	4.6765404
Flat Top to Spear					73 518.339	4.8663957
Tobacco Row to Long						4.5028842
Flat Top	42 01 51.794	+.126	51.920	.993	50.927	0.1742298
Tobacco Row	87 03 22.607	+.833	23.440	.994	22.446	9.9994266
Long	50 54 47.508	+.112	47.620	.993	46.627	9.8899674
Flat Top to Long					47 483.264	4.6765406
Flat Top to Tobacco Row					36 904.678	4.5670814
Tobacco Row to Spear						4.5773224
Flat Top	10 17 00.258	+.166	00.424	.410	00.014	0.7483226
Tobacco Row	159 40 31.200	+.983	32.183	.410	31.773	9.5407508
Spear	10 02 27.935	+.688	28.623	.410	28 213	9.2414363
Flat Top to Spear					73 518.356	4.8663958
Flat Top to Tobacco Row					36 904.669	4.5670813
Spear to Long						4.6180740
Smith	4 48 11.728	−.248	11.480	.348	11.132	1.0771105
Spear	6 04 57.749	−.210	57.539	.349	57.190	9.0251472
Long	169 06 53.169	−1.142	52.027	.349	51.678	9.2761156
Smith to Long					52 520.843	4.7203317
Smith to Spear					93 605.234	4.9713001
Tobacco Row to Spear						4.5773224
Smith	20 50 35.048	−.369	34.679	1.963	32.716	0.4487958
Tobacco Row	118 11 11.341	+.165	11.506	1.964	09.542	9.9451824
Spear	40 58 19.176	+.530	19.706	1.964	17.742	9.8166951
Smith to Spear					93 605.234	4.9713001
Smith to Tobacco Row					69 632.629	4.8428128

Adjusted primary triangles between Kent Island, Md., Washington, D. C., and Atlanta, Ga.—Continued.

Names of stations.	Observed angles.	Corrections.	Spherical angles.	Spherical excess.	Plane angles and distances.	Logarithms.
	° ′ ″	″	″	″	*Meters.*	
Tobacco Row to Long						4.5028842
Smith	25 38 46.776	− .617	46.159	1.340	44.819	0.3637065
Tobacco Row	45 34 02.748	+ .016	02.764	1.340	01.424	9.8537410
Long	108 47 15.636	− .538	15.098	1.341	13.757	9.9762222
Smith to Long					52 520.843	4.7203317
Smith to Tobacco Row					69 632.645	4.8428129
Flat Top to Spear						4.8663958
Smith	51 03 16.151	− .428	15.723	2.995	12.728	0.1091691
Flat Top	98 01 02.127	+ .053	02.180	2.996	59.184	9.9957352
Spear	30 55 51.241	− .158	51.083	2.995	48.088	9.7109554
Smith to Spear					93 605.234	4.9713601
Smith to Flat Top					48 587.022	4.6865203
Flat Top to Tobacco Row						4.5670814
Smith	30 12 41.103	− .059	41.044	1.441	39.603	0.2962715
Flat Top	108 18 02.385	+ .219	02.604	1.442	01.162	9.9774601
Tobacco Row	41 29 19.859	+ .817	20.676	1.441	19.235	9.8211676
Smith to Tobacco Row					69 632.661	4.8428130
Smith to Flat Top					48 587.045	4.6865205
Flat Top to Long						4.6765405
Smith	55 51 27.879	− .676	27.203	1.788	25.415	0.0821586
Flat Top	66 16 10.591	+ .093	10.684	1.789	08.895	9.9616327
Long	57 52 28.128	− .650	27.478	1.788	25.690	9.9278213
Smith to Long					52 520.855	4.7203318
Smith to Flat Top					48 587.084	4.6865204
Flat Top to Long						4.6765405
Cahas	30 45 10.203	+ .726	10.929	1.960	08.969	0.2912983
Flat Top	114 08 02.461	+ .482	02.943	1.960	00.983	9.9602779
Long	35 06 51.702	+ .306	52.008	1.960	50.048	9.7598217
Cahas to Long					84 745.510	4.9281167
Cahas to Flat Top					53 414.667	4.7276605
Long to Smith						4.7203317
Cahas	29 13 42 545	−1.015	41.530	1.458	40.072	0.3118283
Long	22 45 36.426	− .956	35.470	1.458	34.012	9.5875572
Smith	128 00 47.718	− .344	47.374	1.458	45.916	9.8964566
Cahas to Smith					41 611.865	4.6192172
Cahas to Long					84 745.490	4.9281166
Flat Top to Smith						4.6865205
Cahas	59 58 52.748	− .289	52.459	1.630	50.829	0.0625535
Flat Top	47 51 51.870	+ .389	52.259	1.629	50.630	9.8701435
Smith	72 09 19.839	+ .332	20.171	1.630	18.541	9.9785867
Cahas to Smith					41 611.894	4.6192175
Cahas to Flat Top					53 414.691	4.7276607
Flat Top to Tobacco Row						4.5670814
Cahas	9 42 30.653	− .694	29.959	.674	29.285	0.7730669
Flat Top	156 09 54.255	+ .608	54.863	.675	54.188	9.6064924
Tobacco Row	14 07 35.384	+1.817	37.201	.674	36.527	9.3875123
Cahas to Tobacco Row					88 438.367	4.9466407
Cahas to Flat Top					53 414.679	4.7276606
Tobacco Row to Long						4.5028842
Cahas	21 02 39.550	+1.421	40.971	2.278	38.693	0.4448014
Tobacco Row	72 55 47.223	− .985	46.238	2.279	43.959	9.9804311
Long	86 01 39.210	+ .417	39.627	2.279	37.348	9.9989551
Cahas to Long					84 745.510	4.9281167
Cahas to Tobacco Row					88 438.367	4.9466407

Adjusted primary triangles between Kent Island, Md., Washington, D. C., and Atlanta, Ga.—Continued.

Names of stations.	Observed angles.	Corrections.	Spherical angles.	Spherical excess.	Plane angles and distances.	Logarithms.
	° ′ ″	″	″	″	*Meters.*	
Tobacco Row to Smith						4.8428129
Cahas	50 16 22.095	+.405	22.500	2.396	20.″104	0.1140228
Tobacco Row	27 21 44.475	−1.001	43.474	2.396	41.078	9.6623817
Smith	102 21 60.942	+.273	61.215	2.397	58.818	9.9898049
Cahas to Smith					41 611.885	4.6192174
Cahas to Tobacco Row					88 438.347	4.9466406
Flat Top to Smith						4.6865205
Buffalo	25 52 26.119	+.763	26.882	3.376	23.506	0.3601344
Flat Top	49 35 42.668	+.153	42.821	3.377	39.444	9.8816549
Smith	104 31 60.325	+.102	60.427	3.377	57.050	9.9858778
Buffalo to Smith					84 783.196	4.9283098
Buffalo to Flat Top					107 778.64	5.0325327
Flat Top to Cahas						4.7276606
Buffalo	1 41 55.864	+.445	56.309	.147	56.162	1.5280098
Flat Top	1 43 50.798	−.236	50.562	.147	50.415	8.4800258
Cahas	176 34 13.176	+.395	13.571	.148	13.423	8.7768621
Buffalo to Cahas					54 412.187	4.7356962
Buffalo to Flat Top					107 778.59	5.0325325
Cahas to Smith						4.6192174
Buffalo	24 10 30.255	+.318	30.573	1.599	28.974	0.3877244
Cahas	123 26 54.076	−.107	53.969	1.600	52.369	9.9213678
Smith	32 22 40.486	−.229	40.257	1.600	38.657	9.7287544
Buffalo to Smith					84 783.157	4.9283096
Buffalo to Cahas					54 412.187	4.7356962
Flat top to Smith						4.6865205
Moore	15 03 54.850	−.132	54.718	2.733	51.985	0.5851850
Flat Top	30 05 11.734	+.201	11.935	2.734	09.201	9.7000957
Smith	134 50 61.869	−.321	61.548	2.734	58.814	9.8506215
Moore to Smith					93 713.296	4.9718012
Moore to Flat Top					132 533.90	5.1223270
Cahas to Smith						4.6192174
Moore	26 21 32.836	−2.283	30.553	2.934	27.619	0.3526431
Cahas	90 56 57.568	−.696	56.872	2.934	53.938	9.9999405
Smith	62 41 42.030	−.653	41.377	2.934	38.443	9.9486913
Moore to Smith					93 713.255	4.9718010
Moore to Cahas					83 282.132	4.9205518
Cahas to Flat Top						4.7276606
Moore	11 17 37.986	−2.151	35.835	1.830	34.005	0.7081373
Cahas	150 55 50.316	−.984	49.332	1.831	47.501	9.6865290
Flat Top	17 46 40.136	+.188	40.324	1.830	38.494	9.4847539
Moore to Flat Top					132 533.87	5.1223269
Moore to Cahas					83 282.132	4.9205518
Buffalo to Flat Top						5.0325326
Moore	49 18 52.764	−.313	52.451	4.040	48.411	0.1201662
Buffalo	111 10 48.188	+.594	48.782	4.040	44.742	9.9696282
Flat Top	19 30 30.934	−.048	30.886	4.039	26.847	9.5236549
Moore to Flat Top					132 533.90	5.1223270
Moore to Buffalo					47 462.837	4.6763537
Buffalo to Cahas						4.7356962
Moore	38 01 14.778	+1.838	16.616	2.062	14.554	0.2104572
Buffalo	109 28 52.324	+.149	52.473	2.062	50.411	9.9743984
Cahas	32 29 56.508	+.588	57.096	2.061	55.035	9.7302001
Moore to Cahas					83 282.132	4.9205518
Moore to Buffalo					47 462.815	4.6763535

Adjusted primary triangles between Kent Island, Md., Washington, D. C., and Atlanta, Ga.—Continued.

Names of stations.	Observed angles.	Corrections.	Spherical angles.	Spherical excess.	Plane angles and distances.	Logarithms.
	° ′ ″	″	″	″	*Meters.*	
Buffalo to Smith						4.9283097
Moore	64 22 47.614	−.445	47.169	3.396	43.773″	0.0449510
Buffalo	85 18 22.069	−.170	21.899	3.397	18.502	9.9985404
Smith	30 18 61.544	−.423	61.121	3.396	57.725	9.7030929
Moore to Smith					93 713.277	4.9718011
Moore to Buffalo					47 462.826	4.6763536
Poore to Buffalo						5.0132543
Moore	94 42 60.412	−.594	59.818	3.514	56.304	0.0014726
Poore	27 18 39.430	−.161	39.269	3.513	35.756	9.6616268
Buffalo	57 58 31.501	−.047	31.454	3.514	27.940	9.9282993
Moore to Buffalo					47 462.837	4.6763537
Moore to Poore					87 705.367	4.9430262
Moore to Young						4.9049408
Buffalo	29 00 61.421	−.561	60.860	2.310	58.550	0.3142064
Moore	134 19 57.585	−.188	57.397	2.310	55.087	9.8544900
Young	16 39 08.829	−.156	08.673	2.310	06.363	9.4572066
Buffalo to Young					118 477.87	5.0736372
Buffalo to Moore					47 462.848	4.6763538
Young to Poore						4.7586426
Buffalo	28 57 30.080	+.513	30.593	5.008	25.585	0.3150158
Young	60 28 40.046	+.341	40.387	5.009	35.378	9.9395959
Poore	90 33 63.757	+.289	64.046	5.009	59.037	9.9999788
Buffalo to Poore					103 098.96	5.0132543
Buffalo to Young					118 477.87	5.0736372
Young to Poore						4.7586426
Moore	39 36 57.173	+.407	57.580	3.805	53.775	0.1954348
Young	77 07 48.875	+.184	49.059	3.806	45.253	9.9889489
Poore	63 15 24.327	+.450	24.777	3.805	20.972	9.9509634
Moore to Poore					87 705.388	4.9430263
Moore to Young					80 341.667	4.9049408
Young to King						4.9255300
Poore	62 05 32.977	+.096	33.073	4.042	29.031	0.0536974
Young	80 54 51.033	+1.166	52.199	4.042	48.157	9.9945154
King	36 59 44.856	+1.998	46.854	4.042	42.812	9.7794150
Poore to King					94 133.196	4.9737428
Poore to Young					57 364.395	4.7586424
Benn to Poore						4.8461656
Young	48 20 34.365	−.893	33.472	3.401	30.071	0.1266085
Benn	37 38 44.178	−1.365	42.813	3.401	39.412	9.7858687
Poore	94 00 54.057	−.139	53.918	3.401	50.517	9.9989333
Young to Poore					57 364.447	4.7586428
Young to Benn					93 693.064	4.9717074
King to Benn						4.7051366
Young	32 34 16.668	+2.059	18.727	3.599	15.128	0.2689414
King	84 01 09.511	+1.411	10.922	3.600	07.322	9.9976292
Benn	63 24 39.767	+1.382	41.149	3.599	37.550	9.9514520
Young to Benn					93 693.021	4.9717072
Young to King					84 242.255	4.9255300
King to Benn						4.7051366
Poore	31 55 21.080	−.235	20.845	2.958	17.887	0.2767423
King	47 01 24.655	−.587	24.068	2.958	21.110	9.8642867
Benn	101 03 23.945	+.017	23.962	2.959	21.003	9.9918641
Poore to Benn					70 172.279	4.8461656
Poore to King					94 133.239	4.9737430

Adjusted primary triangles between Kent Island, Md., Washington, D. C., and Atlanta, Ga.—Continued.

Names of stations.	Observed angles.	Corrections.	Spherical angles.	Spherical excess.	Plane angles and distances.	Logarithms.
	° ′ ″	″	″	″	Meters.	
King to Hogback						4.9502319
Benn	91 23 09.603	−.799	08.804	3.098	″ 05.706	0.0001269
King	53 57 57.763	−.398	57.365	3.098	54.267	9.9077652
Hogback	34 39 03.177	−.052	03.125	3.098	00.027	9.7547778
Benn to Hogback					72 131.333	4.8581240
Benn to King					50 715.023	4.7051366
King to Paris						5.0189141
Benn	83 29 09.952	+.474	10.426	4.151	06.275	0.0028136
King	67 40 23.558	+1.738	25.296	4.151	21.145	9.9661548
Paris	28 50 38.067	−1.337	36.730	4.150	32.580	9.6834089
Benn to Paris					97 248.409	4.9878825
Benn to King					50 715.023	4.7051366
King to Wofford						4.8002103
Benn	59 06 35.492	+.374	35.866	2.646	33.220	0.0664380
King	77 18 24.660	−.430	24.230	2.646	21.584	9.9892529
Wofford	43 35 05.918	+1.923	07.841	2.645	05.196	9.8384882
Benn to Wofford					71 763.100	4.8559012
Benn to King					50 715.012	4.7051365
Wofford to Paris						4.6386551
King	9 37 61.102	−2.168	58.934	.934	58.000	0.7764189
Wofford	156 19 09.654	−.054	09.600	.935	08.665	9.6038401
Paris	14 02 53.813	+.458	54.270	.935	53.335	9.3851363
King to Paris					104 451.370	5.0189141
King to Wofford					63 126.290	4.8002103
Wofford to Hogback						4.6020196
King	23 20 26.897	−.031	26.866	1.889	24.977	0.4020954
Wofford	117 57 19.445	−.282	19.163	1.890	17.273	9.9461170
Hogback	38 42 19.716	−.077	19.639	1.889	17.750	9.7960953
King to Hogback					89 172.714	4.9502320
King to Wofford					63 126.290	4.8002103
Paris to Hogback						4.4415437
King	13 42 25.795	+2.137	27.932	1.869	26.063	0.6253233
Paris	49 51 40.038	−.174	39.864	1.870	37.994	9.8833648
Hogback	116 25 57.652	+.161	57.813	1.870	55.943	9.9520470
King to Hogback					89 172.674	4.9502318
King to Paris					104 451.35	5.0189140
Paris to Hogback						4.4415437
Benn	7 53 59.651	−1.273	58.378	.816	57.562	0.8619095
Paris	21 01 01.971	+1.163	03.134	.817	02.317	9.5546708
Hogback	151 05 00.829	+.109	00.938	.817	00.121	9.6844292
Benn to Hogback					72 131.333	4.8581240
Benn to Paris					97 248.386	4.9878824
Wofford to Hogback						4.6020196
Benn	32 16 34.111	−1.172	32.939	2.341	30.598	0.2724702
Wofford	74 22 13.527	−2.205	11.322	2.342	08.980	9.9836343
Hogback	73 21 22.893	−.129	22.764	2.342	20.422	9.9814114
Benn to Hogback					72 131.350	4.8581241
Benn to Wofford					71 763.100	4.8559012
Wofford to Paris						4.6386551
Benn	24 22 34.460	+.101	34.561	2.440	32.121	0.3843483
Wofford	112 43 63.736	−1.977	61.759	2.440	59.319	9.9648791
Paris	42 53 31.879	−.879	31.000	2.440	28.560	9.8328978
Benn to Paris					97 248.409	4.9878825
Benn to Wofford					71 763.100	4.8559012

Adjusted primary triangles between Kent Island, Md., Washington, D. C., and Atlanta, Ga.—Continued.

Names of stations.	Observed angles.	Corrections.	Spherical angles.	Spherical excess.	Plane angles and distances.	Logarithms.
	° ′ ″	″	″	″	*Meters.*	
Paris to Hogback						4.4415437
Wofford	38 21 50.209	+ .228	50.437	.915	49.″522	0.2071519
Paris	63 54 33.850	+ .284	34.134	.915	33.219	9.9533239
Hogback	77 43 37.936	+ .238	38.174	.915	37.259	9.9899504
Wofford to Hogback					39 996.269	4.6020195
Wofford to Paris					43 516.606	4.6386550
Pinnacle to Hogback						4.6416975
Wofford	29 17 49.925	+1.518	51.443	1.228	50.215	0.3103888
Pinnacle	26 31 37.257	+ .250	37.507	1.228	36.279	9.6499388
Hogback	124 10 33.040	+1.695	34.735	1.229	33.506	9.9176715
Wofford to Hogback					39 996.278	4.6020196
Wofford to Pinnacle					74 089.610	4.8697573
Paris to Pinnacle						4.5032904
Wofford	9 03 60.284	−1.291	58.993	.430	58.563	0.8025080
Paris	158 30 21.824	+ .383	22.207	.431	21.776	9.5639589
Pinnacle	12 25 39.196	+ .895	40.091	.430	39.661	9.3328567
Wofford to Pinnacle					74 089.610	4.8697573
Wofford to Paris					43 516.616	4.6386551
Paris to Mauldin						4.3869655
Hogback	15 44 50.325	+ .140	50.465	.316	50.149	0.5663990
Paris	146 19 47.041	+ .774	47.815	.317	47.498	9.7438316
Mauldin	17 55 20.856	+1.813	22.669	.316	22.353	9.4881790
Hogback to Mauldin					49 796.184	4.6971961
Hogback to Paris					27 640.348	4.4415435
Paris to Pinnacle						4.5032904
Hogback	46 26 55.104	+1.457	56.561	.744	55.817	0.1398062
Paris	94 35 47.974	+ .098	48.072	.744	47.328	9.9986010
Pinnacle	38 57 16.453	+1.145	17.598	.743	16.855	9.7984473
Hogback to Pinnacle					43 822.546	4.6416976
Hogback to Paris					27 640.373	4.4415439
Mauldin to Pinnacle						4.4055863
Hogback	30 42 04.779	+1.317	06.096	.944	05.152	0.2919495
Mauldin	61 33 51.967	− .620	51.347	.944	50.403	9.9441616
Pinnacle	87 44 04.470	+ .919	05.389	.944	04.445	9.9996605
Hogback to Pinnacle					43 822.525	4.6416974
Hogback to Mauldin					49 796.207	4.6971963
Mauldin to Pinnacle						4.4055863
Paris	51 43 59.067	+ .676	59.743	.517	59.226	0.1050560
Mauldin	79 29 12.823	+1.193	14.016	.517	13.499	9.9926480
Pinnacle	48 46 48.017	− .226	47.791	.516	47.275	9.8763233
Paris to Pinnacle					31 863.265	4.5032903
Paris to Mauldin					24 376.179	4.3869656
Mauldin to Rabun						4.7982781
Paris	35 04 08.490	+1.075	09.565	.964	08.601	0.2406621
Mauldin	132 03 13.761	+1.894	15.655	.964	14.691	9.8707040
Rabun	12 52 36.622	+1.049	37.671	.963	36.708	9.3480252
Paris to Rabun					81 216.491	4.9096442
Paris to Mauldin					24 376.168	4.3869654
Mauldin to Currahee						4.8767021
Paris	5 50 41.363	− .192	41.171	.209	40.962	0.9921130
Mauldin	172 15 55.873	+1.103	56.976	.210	56.766	9.1289748
Currahee	1 53 22.197	+ .284	22.481	.209	22.272	8.5181502
Paris to Currahee					99 492.395	4.9977899
Paris to Mauldin					24 376.162	4.3869653

Adjusted primary triangles between Kent Island, Md., Washington, D. C., and Atlanta, Ga.—Continued.

Names of stations.	Observed angles.	Corrections.	Spherical angles.	Spherical excess.	Plane angles and distances.	Logarithms.
	° ′ ″	″	″	″	*Meters.*	
Rabun to Pinnacle						4.7118767
Paris	16 39 50.577	−.399	50.178	.629	″ 49.549	0.5424895
Rabun	10 13 03.436	+.767	04.203	.628	03.575	9.2489245
Pinnacle	153 07 06.908	+.597	07.505	.629	06.876	9.6552783
Paris to Pinnacle					31 863.294	4.5032907
Paris to Rabun					81 216.547	4.9096445
Currahee to Pinnacle						4.9064819
Paris	45 53 17.704	+.868	18.572	1.928	16.644	0.1438876
Currahee	16 29 02.556	+.128	02.684	1.928	00.756	9.4529205
Pinnacle	117 37 44.518	+.011	44.529	1.929	42.600	9.9474205
Paris to Pinnacle					31 863.243	4.5032900
Paris to Currahee					99 492.419	4.9977900
Currahee to Rabun						4.6892851
Paris	29 18 27.127	+1.267	28.394	3.342	25.052	0.3113848
Currahee	54 11 09.929	+.175	10.104	3.342	06.762	9.9089742
Rabun	96 35 31.659	−.131	31.528	3.342	28.186	9.9971200
Paris to Rabun					81 216.472	4.9096441
Paris to Currahee					99 492.395	4.9977899
Rabun to Pinnacle						4.7118767
Mauldin	52 34 00.938	+.700	01.638	1.076	00.562	0.1001452
Rabun	23 05 40.058	+1.816	41.874	1.075	40.799	9.5935646
Pinnacle	104 20 18.891	+.824	19.715	1.076	18.639	9.9862563
Mauldin to Pinnacle					25 444.065	4.4055865
Mauldin to Rabun					62 846.086	4.7982782
Currahee to Pinnacle						4.9064819
Mauldin	92 46 43.050	−.090	42.960	1.621	41.339	0.0005107
Currahee	18 22 24.753	+.412	25.165	1.621	23.544	9.4985935
Pinnacle	68 50 56.501	+.237	56.738	1.621	55.117	9.9697095
Mauldin to Pinnacle					25 444.041	4.4055861
Mauldin to Currahee					75 283.897	4.8767021
Currahee to Rabun						4.6892851
Mauldin	40 12 42.112	−.790	41.322	2.587	38.785	0.1900357
Currahee	56 04 32.126	+.459	32.585	2.588	29.997	9.9189571
Rabun	83 42 55.037	−1.181	53.856	2.588	51.268	9.9973812
Mauldin to Rabun					62 846.043	4.7982779
Mauldin to Currahee					75 283.879	4.8767020
Currahee to Rabun						4.6892851
Pinnacle	35 29 22.390	+.587	22.977	2.042	20.935	0.2361613
Currahee	37 42 07.373	+.047	07.420	2.043	05.377	9.7864303
Rabun	106 48 35.095	+.636	35.731	2.043	33.688	9.9810355
Pinnacle to Rabun					51 508.238	4.7118767
Pinnacle to Currahee					80 627.259	4.9064819
Skitt to Blood						4.5131465
Rabun	29 37 62.874	−1.377	61.497	1.701	59.796	0.3058805
Skitt	74 11 06.569	−.013	06.556	1.701	04.855	9.9832406
Blood	76 10 57.233	−.182	57.051	1.702	55.349	9.9872458
Rabun to Blood					63 426.043	4.8022676
Rabun to Skitt					64 013.677	4.8062728
Currahee to Skitt						4.5038010
Rabun	29 04 48.282	+1.375	49.657	1.289	48.368	0.3133352
Currahee	102 45 43.741	+.343	44.084	1.290	42.794	9.9891367
Skitt	48 09 29.511	+.616	30.127	1.289	28.838	9.8721489
Rabun to Skitt					64 013.691	4.8062729
Rabun to Currahee					48 897.326	4.6892851

Adjusted primary triangles between Kent Island, Md., Washington, D. C., and Atlanta, Ga.—Continued.

Name of stations.	Observed angles.	Corrections.	Spherical angles.	Spherical excess.	Plane angles and distances.	Logarithms.
	° ′ ″	″	″	″	Meters.	
Currahee to Blood						4. 7520745
Rabun	58 42 51. 156	— . 002	51. 154	2. 246	48. 908	0. 0682463
Currahee	73 35 40. 627	— . 751	39. 876	2. 246	37. 630	9. 9819469
Blood	47 41 35. 682	+ . 026	35. 708	2. 246	33. 462	9. 8689643
Rabun to Blood					63 426. 058	4. 8022677
Rabun to Currahee					48 897. 326	4. 6892851
Skitt to Blood						4. 5131465
Currahee	29 10 03. 114	+1. 093	04. 207	. 744	03. 463	0. 3121445
Skitt	122 20 36. 080	+ . 603	36. 683	. 745	35. 938	9. 9267835
Blood	28 29 21. 551	— . 208	21. 343	. 744	20. 599	9. 6785101
Currahee to Blood					56 503. 390	4. 7520745
Currahee to Skitt					31 900. 765	4. 5038011
Sawnee to Skitt						4. 7005389
Currahee	19 23 37. 252	+ . 120	37. 372	. 710	36. 662	0. 4787908
Sawnee	12 11 11. 855	— . 332	11. 523	. 710	10. 813	9. 3244712
Skitt	148 25 13. 755	— . 519	13. 236	. 711	12. 525	9. 7190713
Currahee to Skitt					31 900. 750	4. 5038009
Currahee to Sawnee					79 140. 909	4. 8984010
Sawnee to Blood						4. 7743119
Currahee	48 33 40. 366	+1. 218	41. 579	2. 841	38. 738	0. 1251368
Sawnee	45 25 06. 664	—1. 255	05. 409	2. 840	02. 569	9. 8526257
Blood	86 01 20. 957	+ . 577	21. 534	2. 841	18. 693	9. 9989523
Currahee to Blood					56 503. 377	4. 7520744
Currahee to Sawnee					79 140. 909	4. 8984010
Sawnee to Blood						4. 7743119
Skitt	89 14 10. 165	— . 084	10. 081	1. 386	08. 695	0. 0000386
Sawnee	33 13 54. 809	— . 924	53. 885	1. 386	52. 499	9. 7387960
Blood	57 31 59. 406	+ . 786	60. 192	1. 386	58. 806	9. 9261885
Skitt to Blood					32 594. 662	4. 5131465
Skitt to Sawnee					50 180. 965	4. 7005390
Skitt to Grassy						4. 7481618
Blood	89 29 53. 968	+ . 883	54. 851	1. 266	53. 585	0. 0000166
Skitt	54 54 23. 306	+ . 212	23. 518	1. 266	22. 252	9. 9128657
Grassy	35 35 42. 559	+2. 869	45. 428	1. 265	44. 163	9. 7649681
Blood to Grassy					45 818. 842	4. 6610441
Blood to Skitt					32 594. 662	4. 5131465
Sawnee to Grassy						4. 5027518
Blood	31 57 54. 562	+ . 097	54. 659	1. 222	53. 437	0. 2762170
Sawnee	49 39 34. 659	+ . 909	35. 568	1. 222	34. 346	9. 8820755
Grassy	98 22 32. 199	+1. 241	33. 440	1. 223	32. 217	9. 9953431
Blood to Grassy					45 818. 863	4. 6610443
Blood to Sawnee					59 471. 918	4. 7743119
Sawnee to Grassy						4. 5027518
Skitt	34 19 46. 859	— . 295	46. 564	1. 343	45. 221	0. 2487614
Sawnee	82 53 29. 468	— . 015	29. 453	1. 343	28. 110	9. 9966486
Grassy	62 46 49. 640	—1. 628	48. 012	1. 343	46. 669	9. 9490257
Skitt to Grassy					55 996. 615	4. 7481618
Skitt to Sawnee					50 180. 958	4. 7005389
Grassy to Sweat						4. 6802840 4
Kenesaw	26 33		40. 511	. 352	40. 159	0. 3495440 7
Grassy	8 09 57. 704	—1. 708	55. 996	. 352	55. 644	9. 1523867 4
Sweat	145 16 24. 035	+ . 513	24. 548	. 351	24. 197	9. 7556167 3
Kenesaw to Sweat					15 212. 990	4. 1822148 5
Kenesaw to Grassy					61 016. 156	4. 7854448 4

Adjusted primary triangles between Kent Island, Md., Washington, D. C., and Atlanta, Ga.—Continued.

Name of stations.	Observed angles.	Corrections.	Spherical angles.	Spherical excess.	Plane angles and distances.	Logarithms.
	° ′ ″	″	″	″	*Meters.*	
Pine Log to Grassy						4. 5252408 4
Kenesaw	29 48		57. 408	. 995	56.″413	0. 3034590 3
Pine Log	115 08 58. 577	+1. 142	59. 719	. 994	58. 725	9. 9567449 8
Grassy	35 02 03. 696	+2. 161	05. 857	. 995	04. 862	9. 7589665 3
Kenesaw to Grassy					61 016. 157	4. 7854448 5
Kenesaw to Pine Log					38 696. 029	4. 5876664 0
Grassy to Sawnee						4. 5027518 0
Kenesaw	31 09		48. 202	1. 290	46. 912	0. 2861106 3
Grassy	51 39 33. 335	−1. 637	31. 698	1. 290	30. 408	9. 8944969 8
Sawnee	97 10 43. 359	+ . 612	43. 971	1. 291	42. 680	9. 9965824 2
Kenesaw to Sawnee					48 234. 681	4. 6833594 1
Kenesaw to Grassy					61 016. 157	4. 7854448 5
Sawnee to Pine Log						4. 6519200 1
Grassy	86 41 37. 031	+ . 524	37. 555	. 903	36. 652	0. 0007235 7
Sawnee	48 13 27. 729	+ . 109	27. 838	. 902	26. 936	9. 8725972 6
Pine Log	45 04 56. 116	+1. 198	57. 314	. 902	56. 412	9. 8501082 1
Grassy to Pine Log					33 515. 125	4. 5252408 4
Grassy to Sawnee					31 823. 782	4. 5027517 9
Sweat to Pine Log						4. 5160840 0
Grassy	43 12 01. 400	+ . 453	01. 853	. 931	00. 922	0. 1645945 9
Sweat	44 21 28. 210	+ . 802	29. 012	. 931	28. 081	9. 8445622 4
Pine Log	92 26 31. 871	+ . 058	31. 929	. 932	30. 997	9. 9996054 4
Grassy to Pine Log					33 515. 124	4. 5252408 3
Grassy to Sweat					47 894. 322	4. 6802840 3
Sawnee to Sweat						4. 5197394 0
Grassy	43 29 35. 631	+ . 071	35. 702	. 889	34. 813	0. 1622436 9
Sawnee	95 03 54. 068	+ . 153	54. 221	. 889	53. 332	9. 9983009 6
Sweat	41 26 32. 604	+ . 140	32. 744	. 889	31. 855	9. 8207687 1
Grassy to Sweat					47 894. 324	4. 6802840 5
Grassy to Sawnee					31 823. 782	4. 5027518 0
Sawnee to Sweat						4. 5197394 0
Pine Log	47 21 35. 755	−1. 140	34. 615	. 918	33. 697	0. 1333483 6
Sawnee	46 50 26. 339	+ . 044	26. 383	. 918	25. 465	9. 8629962 4
Sweat	85 48 00. 814	+ . 942	01. 756	. 918	00. 838	9. 9988322 5
Pine Log to Sweat					32 815. 876	4. 5160840 0
Pine Log to Sawnee					44 866. 275	4. 6519200 1
Sawnee to Kenesaw						4. 6833594 1
Pine Log	70 04 02. 461	− . 056	02. 405	1. 383	01. 022	0. 0268298 1
Sawnee	48 57 15. 630	+ . 503	16. 133	1. 383	14. 750	9. 8774771 6
Kenesaw	60 58 46. 044	− . 433	45. 611	1. 383	44. 228	9. 9417307 9
Pine Log to Kenesaw					38 696. 028	4. 5876663 8
Pine Log to Sawnee					44 866. 275	4. 6519200 1
Sweat to Kenesaw						4. 1822148 7
Pine Log	22 42 26. 706	+1. 084	27. 790	. 415	27. 375	0. 4133806 4
Sweat	100 54 55. 825	− . 289	55. 536	. 416	55. 120	9. 9920708 6
Kenesaw	56 22 38. 414	− . 494	37. 920	. 515	37. 505	9. 9204884 8
Pine Log to Kenesaw					38 696. 027	4. 5876663 7
Pine Log to Sweat					32 815. 875	4. 5160839 9
Kenesaw to Sweat						4. 1822148 7
Sawnee	2 06 49. 291	+ . 460	49. 751	. 050	49. 701	1. 4331560 6
Kenesaw	4 36 07. 630	+ . 061	07. 691	. 050	07. 641	8. 9043684 6
Sweat	173 17 03. 361	− . 653	02. 708	. 050	02. 658	9. 0679884 8
Sawnee to Sweat					33 093. 248	4. 5197393 9
Sawnee to Kenesaw					48 234. 681	4. 6833594 1

Adjusted primary triangles between Kent Island, Md., Washington, D. C., and Atlanta, Ga.—Continued.

Name of stations.	Observed angles.			Corrections.	Spherical angles.	Spherical excess.	Plane angles and distances.	Logarithms.
	°	′	″	″	″	″	*Meters.*	
Northeast Base to Kenesaw								4.5066152 0
Sawnee	41	40	10.536	−.582	09.954	.935	″ 09.019	0.1772903 5
Northeast Base	92	52	24.529	−1.505	23.024	.936	22.088	9.9994538 6
Kenesaw	45	27	29.712	+.116	29.828	.935	28.893	9.8529291 7
Sawnee to Kenesaw							48 234.681	4.6833594 1
Sawnee to Northeast Base							34 421.890	4.5368347 2
Stone Mount to Kenesaw								4.6463944 5
Sawnee	54	57	07.022	−2.167	04.855	1.599	03.256	0.0868962 9
Stone Mount	63	02	55.832	+1.002	56.834	1.598	55.236	9.9500686 8
Kenesaw	62	00	04.273	−1.166	03.107	1.599	01.508	9.9459366 1
Sawnee to Kenesaw							48 234.682	4.6833594 2
Sawnee to Stone Mount							47 777.932	4.6792273 5
Stone Mount to Sweat								4.6098721 5
Sawnee	57	03	56.313	−1.707	54.606	1.125	53.481	0.0760897 4
Stone Mount	42	59	58.831	−.288	58.543	1.125	57.418	9.8337775 0
Sweat	79	56	09.977	+.248	10.225	1.124	09.101	9.9932654 6
Sawnee to Sweat							33 093.248	4.5197393 9
Sawnee to Stone Mount							47 777.932	4.6792273 5
Academy to Sweat								4.6490267 3
Sawnee	82	24	43.780	−1.244	42.536	.961	41.575	0.0038202 3
Academy	47	23	40.044	−1.106	38.938	.960	37.978	9.8668924 0
Sweat	50	11	43.207	−1.800	41.407	.960	40.447	9.8854872 2
Sawnee to Sweat							33 093.250	4.5197394 2
Sawnee to Academy							34 540.943	4.5383341 8
Northeast Base to Sweat								4.4014562 8
Sawnee	43	46	59.827	−.122	59.705	.668	59.037	0.1599379 5
Northeast Base	65	18	18.189	−.670	17.519	.668	16.851	9.9583451 6
Sweat	70	54	46.284	−1.504	44.780	.668	44.112	9.9754404 8
Sawnee to Sweat							33 093.248	4.5197393 9
Sawnee to Northeast Base							34 421.890	4.5368347 1
Academy to Northeast Base								4.3581178 5
Sawnee	38	37	43.953	−1.122	42.831	.629	42.202	0.2046297 9
Academy	70	24	16.565	−3.004	13.561	.629	12.932	9.9740870 8
Northeast Base	70	58	05.720	−.225	05.495	.629	04.866	9.9755865 2
Sawnee to Northeast Base							34 421.890	4.5368347 2
Sawnee to Academy							34 540.941	4.5383341 6
Stone Mount to Northeast Base								4.2127381 2
Sawnee	13	16	56.486	−1.586	54.900	.320	54.580	0.6387613 7
Stone Mount	28	58	56.941	+1.276	58.217	.320	57.897	9.6853352 5
Northeast Base	137	44	07.160	+.683	07.843	.320	07.523	9.8277278 8
Sawnee to Northeast Base							34 421.892	4.5368347 4
Sawnee to Stone Mount							47 777.934	4.6792273 7
Academy to Stone Mount								4.3464002 8
Sawnee	25	20	47.467	+.464	47.931	.599	47.332	0.3684637 4
Academy	112	53	42.731	−.452	42.279	.598	41.681	9.9643633 4
Stone Mount	41	45	32.095	−.509	31.586	.599	30.987	9.8234701 5
Sawnee to Stone Mount							47 777.933	4.6792273 6
Sawnee to Academy							34 540.942	4.5383341 7
Sweat to Stone Mount								4.6098721 5
Kenesaw	66	36	11.903	−1.105	10.798	.524	10.274	0.0372640 6
Sweat	93	20	53.384	−.901	52.483	.524	51.959	9.9992582 4
Stone Mount	20	02	57.001	+1.290	58.291	.524	57.767	9.5350786 7
Kenesaw to Stone Mount							44 299.054	4.6463944 5
Kenesaw to Sweat							15 213.000	4.1822148 8

Adjusted primary triangles between Kent Island, Md., Washington, D. C., and Atlanta, Ga.—Continued.

Name of stations.	Observed angles.	Corrections.	Spherical angles.	Spherical excess.	Plane angles and distances.	Logarithms.
	° ′ ″	″	″	″	*Meters.*	
Sweat to Northeast Base						4.4014562 8
Kenesaw	50 03 37.342	+ .177	37.519	.317	37.202	0.1153627 0
Sweat	102 22 17.077	+ .851	17.928	.318	17.610	9.9897962 0
Northeast Base	27 34 06.340	− .835	05.505	.317	05.188	9.6653958 7
Kenesaw to Northeast Base					32 108.142	4.5066151 8
Kenesaw to Sweat					15 212.999	4.1822148 5
Sweat to Southwest Base						4.3868156 0
Kenesaw	64 50 59.613	+ .241	59.854	.310	59.544	0.0432567 3
Sweat	80 44 21.711	+ .199	21.910	.310	21.600	9.9943023 8
Southwest Base	34 24 39.322	− .156	39.166	.310	38.856	9.7521425 4
Kenesaw to Southwest Base					26 568.970	4.4243747 1
Kenesaw to Sweat					15 213.000	4.1822148 7
Southwest Base to Stone Mount						4.2494705 0
Kenesaw	1 45 12.290	−1.346	10.944	.030	10.914	1.5144006 3
Southwest Base	175 37 26.954	− .194	26.760	.031	26.729	8.8825233 1
Stone Mount	2 37 22.258	+ .130	22.388	.031	22.357	8.6605035 9
Kenesaw to Stone Mount					44 299.054	4.6463944 4
Kenesaw to Southwest Base					26 568.970	4.4243747 2
Northeast Base to Stone Mount						4.2127381 2
Kenesaw	16 32 34.561	−1.281	33.280	.343	32.937	0.5455725 8
Northeast Base	129 23 28.311	+ .822	29.133	.344	28.789	9.8880837 5
Stone Mount	34 03 58.891	− .274	58.617	.343	58.274	9.7483045 1
Kenesaw to Stone Mount					44 299.054	4.6463944 5
Kenesaw to Northeast Base					32 108.145	4.5066152 1
Northeast Base to Southwest Base						3.9702760 2
Kenesaw	14 47 22.271	+ .064	22.335	.185	22.150	0.5930030 5
Northeast Base	46 34 26.983	+ .534	27.517	.185	27.332	9.8610956 4
Southwest Base	118 38 10.842	− .140	10.702	.184	10.518	9.9433361 5
Kenesaw to Southwest Base					26 568.970	4.4243747 1
Kenesaw to Northeast Base					32 108.145	4.5066152 2
Stone Mount to Sweat						4.6098721 5
Academy	65 30 02.687	+ .655	03.342	.763	02.579	0.0409746 2
Stone Mount	84 45 30.926	− .796	30.130	.763	29.367	9.9981799 5
Sweat	29 44 26.770	+2.047	28.817	.763	28.054	9.6955535 0
Academy to Sweat					44 568.366	4.6490267 2
Academy to Stone Mount					22 202.418	4.3464002 7
Northeast Base to Sweat						4.4014562 8
Academy	23 00 36.521	−1.897	34.624	.337	34.287	0.4079519 9
Northeast Base	136 16 23.909	− .896	23.013	.336	22.677	9.8396184 8
Sweat	20 43 03.077	+ .296	03.373	.337	03.036	9.5487095 9
Academy to Sweat					44 568.369	4.6490267 5
Academy to Northeast Base					22 809.610	4.3581178 6
Stone Mount to Northeast Base						4.2127381 2
Academy	42 29 26.166	+2.552	28.718	.290	28.428	0.1703892 1
Stone Mount	70 44 29.036	+ .768	29.804	.290	29.514	9.9749905 2
Northeast Base	66 46 01.440	+ .908	02.348	.290	02.058	9.9632729 5
Academy to Northeast Base					22 809.609	4.3581178 5
Academy to Stone Mount					22 202.418	4.3464002 8
Northeast Base to Stone Mount						4.2127381 2
Sweat	9 01 23.693	+1.752	25.445	.136	25.309	0.8045350 0
Northeast Base	156 57 34.651	− .013	34.638	.137	34.501	9.5925990 2
Stone Mount	14 01 01.890	−1.564	00.326	.136	00.190	9.3841831 5
Sweat to Stone Mount					40 726.036	4.6098721 4
Sweat to Northeast Base					25 203.234	4.4014562 7

Adjusted primary triangles between Kent Island, Md., Washington, D. C., and Atlanta, Ga.—Continued.

Name of stations.	Observed angles.	Corrections.	Spherical angles.	Spherical excess.	Plane angles and distances.	Logarithms.
	° ′ ″	″	″	″	*Meters.*	
Stone Mount to Southwest Base						4.2494705 0
Sweat	12 36 31.673	−1.099	30.574	.184	30.″390	0.6609720 8
Stone Mount	17 25 34.743	+1.160	35.903	.184	35.719	9.4763730 2
Southwest Base	149 57 53.724	+ .350	54.074	.183	53.891	9.6994295 8
Sweat to Southwest Base					24 367.760	4.3868156 0
Sweat to Stone Mount					40 726.038	4.6098721 6
Northeast Base to Southwest Base						3.9702760 2
Sweat	21 37 55.366	+ .652	56.018	.192	55.826	0.4333897 6
Northeast Base	74 08 33.323	− .301	33.022	.192	32.830	9.9831498 1
Southwest Base	84 13 31.520	+ .016	31.536	.192	31.344	9.9977905 0
Sweat to Southwest Base					24 367.759	4.3868155 9
Sweat to Northeast Base					25 203.235	4.4014562 8
Southwest Base to Northeast Base						3.9702760 2
Stone Mount	31 26 36.633	− .404	36.229	.128	36.101	0.2826162 5
Southwest Base	65 44 22.204	+ .334	22.538	.128	22.410	9.9598458 5
Northeast Base	82 49 01.328	+ .289	01.617	.128	01.489	9.9965782 2
Stone Mount to Northeast Base					16 320.675	4.2127381 2
Stone Mount to Southwest Base					17 761.126	4.2494704 9
Middle Base to Northeast Base						3.6835900 5
Stone Mount	16 56 35.265	− .335	34.930	.066	34.864	0.5354799 2
Middle Base	80 14 23.381	+ .271	23.652	.066	23.586	9.9936681 5
Northeast Base	82 49 01.328	+ .289	01.617	.067	01.550	9.9965782 4
Stone Mount to Northeast Base					16 320.675	4.2127381 2
Stone Mount to Middle Base					16 430.403	4.2156482 1
Southwest Base to Middle Base						3.6544120 2
Stone Mount	14 30 01.368	− .068	01.300	.062	01.238	0.6013902 8
Southwest Base	65 44 22.204	+ .334	22.538	.062	22.476	9.9598459 1
Middle Base	99 45 36.656	− .308	36.348	.062	36.286	9.9936682 0
Stone Mount to Middle Base					16 430.403	4.2156482 1
Stone Mount to Southwest Base					17 761.126	4.2494705 0

[PAPER No. 2.]

Estimation of the probable accuracy of a triangulation or approximate determination of the average probable error of the adjusted distances.

Since the publication of the approximate probable uncertainty in the primary triangulation between the Epping base, Maine, and the Fire Island base, New York, in Coast Survey Report of 1865 (pp. 192 and following), the subject has received further attention with a view of a closer estimation of the probable error which might be expected in adjusted triangulation.

The strict application of the method of least squares to the computation of the probable error of the adjusted distances depending upon the accuracy with which the angles and the base line have been measured—in other words, the application of the method to *conditioned* observations—is too laborious to be made, nor is it needed, since any tolerably fair estimate will answer all practical purposes.

Primarily, the accuracy of the work depends upon the accuracy of the measures of the base and of the angles, and upon the geometrical figure of the triangulation; that is, on the number of conditional equations involved. The greater the accuracy of the linear and angular measures the better the shape of the triangles, and the greater the number of checks or conditions the smaller will be the average probable error. Supposing but a single chain of triangles, and no condition as to accord of two base lines (one being a check base), we may begin the computation of probable errors with the base and first triangle and proceed to the adjacent triangle, and so on to the end or check base. This was done in 1865, with the precaution of selecting the best-shaped triangles to form the

series between the terminal lines. This procedure will necessarily assign too great a value for the probable error: first, on account of the formula there employed for the probable error of a triangle side, which supposes but 2 of the angles measured instead of 3, and consequently ignores the measure of the third and the condition of the sum of the angles; and secondly, for the reason that the triangulation is composed of various geometrical figures, notably of quadrilaterals or other more or less complicated combinations of triangles, each of which contributes something to the accuracy of the whole.

The most convenient form, and one usually employed for the computation of the probable error of a side of a triangle in which two of the angles are supposed measured and subject to small errors, viz:

$$\varepsilon_a = a\,\varepsilon \sin 1'' \sqrt{\Sigma\,(\cot^2 A + \cot^2 B)} \quad . \quad . \quad . \quad . \quad (1)$$

was given and numerically illustrated in the place cited above. If now, we take account of the fact that all the angles in each triangle are measured and adjusted to their theoretical sum, viz: two right angles + spherical excess, the above expression needs modification, and it can be made to assume the form given to it by Laplace.

The same modification should be applied to Struve's formula, which is but a different form of Equation (1). Our formula for the probable error ε_a of the deduced side a, the angle A being adjacent to the base b but opposite side a, and the angle B being at the vertex, becomes—

$$\varepsilon_a = a\,\varepsilon \sin 1'' \sqrt{\frac{2}{3}\,\Sigma\,[\cot^2 A + \cot A \cot B + \cot^2 B]} \quad . \quad . \quad (2)^*$$

where ε = the average probable error of an angle as deduced from the corrections to the observed angles, or the difference $\triangle$ between observed and adjusted angles. ε may also be deduced approximately from the average error of closing of the triangles; it may also be found roughly from $0.675\sqrt{\frac{\Sigma \triangle^2}{3n}}$, where n = the number of triangles involved; but preferably and with due regard to the number of conditional or normal equations z_b from

$$\varepsilon = 0.6745\sqrt{2\,\frac{[v^2]}{z_b}} \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where we introduce the sum of the squares of the direction-corrections or $[v^2]$.

For the theoretical considerations respecting the accuracy of functions of conditioned observations see "Gerling's Ausgleichungs-Rechnungen" (Hamburg and Gotha, 1843), chapter 8. With reference to this, and making use of his notation where needed, we have to add to the expression $A = \frac{b \sin A}{\sin B}$ the conditional equation multiplied by an indeterminate coefficient r_1 or the term r_1 $(A + B + C - 180 - e)$ before differentiating for the values $l_1 \; l_2 \; l_3 \ldots$ needed for the formation of the (so-called) transfer-equations.

$$\begin{aligned} o &= [a\,l] + [a\,a]\,r_1 + [a\,b]\,r_2 + [a\,c]\,r_3 + \ldots \\ o &= [b\,l] + [a\,b]\,r_1 + [b\,b]\,r_2 + [b\,c]\,r_3 + \ldots \\ o &= [c\,l] + [a\,c]\,r_1 + [b\,c]\,r_2 + [c\,c]\,r_3 + \ldots \end{aligned}$$

We have $d\,a = -\,(a \sin 1'' \cot B - r_1)\; d\,B + (a \sin 1'' \cot A + r_1)\; d\,A + r_1\, d\,C$

and $\left\{\begin{aligned} l_1 &= -\,a \sin 1'' \cot B \\ l_2 &= +\,a \sin 1'' \cot A \\ l_3 &= 0 \end{aligned}\right.$ | also $a_1 = a_2 = a_3 = 1$ and the transfer-equation $0 = [\,a\,l\,] + 3r_1$ or $r_1 = -\,\frac{1}{3}\,a \sin 1''\,(\cot A - \cot B)$

* It will be noticed that Formula II, on p. 194, Report of 1865, but not used there, and which was then supposed to represent Laplace's, as quoted in the British Ordnance Survey (London, 1858), p. 421, needs to be multiplied by $\sqrt{2}$. Formula 2, as now given, is also found in the work, just received, "Die geodätischen Hauptpunkte von G. Zachariae, Berlin, 1878 (p. 44).

and from $\begin{cases} L_1 = l_1 + a_1 r_1 + b_1 r_2 + c_1 r_3 + \ldots \\ L_2 = l_2 + a_2 r_1 + b_2 r_2 + c_2 r_3 + \ldots \\ L_3 = l_3 + a_3 r_1 + b_3 r_2 + c_3 r_3 + \ldots \end{cases}$ and $\varepsilon_a = \varepsilon \sqrt{[LL]}$ also $\frac{1}{P} = [LL]$

a process which leads directly* to the Equation (2), or is equivalent to it.

We may compare the results for probable error for the case of 2 angles measured and for the case of 3 angles measured by giving the numerical work for the triangle treated in the Report of 1865 (pp. 193, 194), viz:

Given $\begin{cases} b = 8715.9^{\text{m.}} \\ B = 39^\circ\ 20' \\ A = 51\ \ 37 \\ dA = dB = \pm\ 0''.48 \end{cases}$ hence $\begin{cases} \log l_1 = 8.8047_n \\ \log l_2 = 8.6170 \\ [a\,l] = -\ .02238 \\ 0 = -\ .02238 + 3r_1 \end{cases}$ and $\begin{cases} r_1 = +\ .00746 \\ L_1 = -\ .05632 \\ L_2 = +\ .04886 \\ L_3 = -\ .00746 \end{cases}$

$$[LL] = .005615 \text{ and } \varepsilon\sqrt{[LL]} = \pm\ 0.0360^{\text{m.}}$$

but in case of 2 angles measured we had .... $\pm\ 0.0416$

the difference of these results showing the gain in accuracy by the measure of the third angle. Using formula (2) directly, we have:

$\begin{cases} \cot^2 A = 1.4892 \\ \cot A \cot B = 0.9666 \\ \cot^2 B = 0.6275 \\ \text{sum} = 3.0833 \end{cases}$ and $\begin{cases} \log a = 4.03267 \\ \log \varepsilon = 9.68124 \\ \log \sin 1'' = 4.68557 \\ \log \sqrt{\tfrac{2}{3}\Sigma} = 0.15646 \end{cases}$ hence $\log \varepsilon_a = 8.55594$ and $\varepsilon_a = \pm\ 0^{\text{m}}.0360$ as before.

The advantage of the measure of the third angle in each triangle is thus apparent, and if the triangulation consisted or was made up of a string of triangles, no further remark would be needed. Suppose, however, the triangulation to consist of a string of quadrilaterals, it follows that our probable error of the last side would require to be divided by $\sqrt{2}$, since we can arrive at that side in *two independent* ways, or through 2 different sets of triangles; and, in general, it will be admissible to suppose this combination factor to be proportional to the square root of the fraction: number of angles in triangles used in computing ε_a, divided by number of angles in figure. Thus for a quadrilateral we have the factor $\sqrt{\frac{6}{12}}$ or $\sqrt{\frac{1}{2}}$; for a hexagon (hinged) $\sqrt{\frac{12}{18}}$ or $\sqrt{\frac{2}{3}}$; the factor $\sqrt{\frac{6}{15}}$ for two adjacent quadrilaterals connected by a diagonal or line, and in general for our triangulations made up principally of quadrilaterals this combination factor† will not differ much from $\sqrt{\frac{1}{2}}$; putting it equal to $\sqrt{f}$, the value of ε_a, as found by the formula (2), must be multiplied by $\sqrt{f}$. The result must be increased by the additional probable error $\pm\ \frac{a}{b}\varepsilon_b$, which is the effect of the probable error $_e$ of the measure of the base line b, which effect is propagated through the triangulation independently of the effect of the errors of the angular measures. We thus have finally,

$$\varepsilon_1 = \sqrt{f\,\varepsilon_a^2 + \frac{a^2}{b^2}\,\varepsilon_b^2} \quad . \quad . \quad . \quad . \quad . \qquad (4)$$

* If we substitute in these equations the above values of $l_1\ l_2\ l_3$ and of $a_1\ a_2\ a_3$ and the value of r_1, next square the values of $L_1\ L_2\ L_3$, and form their sum or [LL] we shall find this equal to $\frac{2}{3}\ a^2 \sin^2 1''\ (\cot^2 A + \cot A \cot B + \cot^2 B)$ —as pointed out by Mr. M. H. Doolittle, of the Computing Division.

† In connection with the use of this factor the series of primary triangles selected will comprise those of *average* shape.

If this line is a junction line where two (or more) branches of triangulations meet, and each branch depends upon its own base line, we can obtain a second (or more) value for ε_1, which values are to be combined for the final probable error of the line as depending on the whole connected triangulation. Thus, in the case of a simple junction with the respective values ε_n and ε_s the probable error ε becomes $\frac{\varepsilon_n \quad \varepsilon_s}{\sqrt{\varepsilon_n^2 + \varepsilon_s^2}}$, or we may add the respective weights and find the value of ε corresponding to the sum.

For an estimate of the average accuracy of a triangulation connecting a base line with a terminal line, we may suppose ε_b and ε expressed in parts of the length, and the average probable error of the triangulation, similarly expressed, will be given by $\varepsilon_\Delta = \sqrt{\frac{1}{2}(\varepsilon_b^2 + \varepsilon^2}$

[PAPER NO. 3.]

UNITED STATES COAST AND GEODETIC SURVEY OFFICE,
Washington, November 9, 1878.

DEAR SIR: I have the honor to present the following explanations and illustrations of the method employed in this office in the solution of normal equations and in the adjustment of a triangulation.

I.—*General method of solution of normal equations.*

Suppose given the normal equations—

$$
\begin{array}{lrcl}
1. & 0 & = & +\ 5.4237w + 2.1842x - \ \ 4.3856y + 2.3542z - 3.6584 \\
2. & 0 & = & +\ 2.1842w + 6.9241x \qquad\qquad\qquad - 1.2130z + 2.8563 \\
3. & 0 & = & -\ 4.3856w \qquad\qquad\quad + 12.8242y + 3.4695z + 8.7421 \\
4. & 0 & = & +\ 2.3542w - 1.2130x + \ \ 3.4695y + 7.1243z + 0.6847
\end{array}
$$

The solution is conducted as follows:

A.

1	2	3	4	5	6	7	8
			w	x	y	z	
1	1		+5.424	+2.184	−4.386	+2.354	−3.658
2	2	−.184	$w=$	−.401	+.807	−.433	+.6731
3	5			+6.048	+1.759	−2.157	+4.323
4	6	−.165		$x=$	−.29	+.356	−.7133
5	10				+8.77	+5.996	+4.538
6	11	−.114			$y=$	−.684	−.5173
7	16					+1.236	+0.704
8	17	−.809				$z=$	−.57

B.

1	2	3	4	5	6
		x	y	z	
1	3	+6.924		−1.213	+2.856
2	4	−.876	+1.759	−.944	+1.467
3	7		+12.82	+3.470	+8.742
4	8		−3.54	+1.900	−2.950
5	9		−.51	+.626	−1.254
6	12			+7.124	+0.685
7	13			−1.019	+1.584
8	14			−.768	+1.539
9	15			−4.101	−3.104

The first column in each of the above tables gives the number of the line, and the second the order of procedure.

The coefficients and absolute term of Equation 1 are entered in line 1, columns 4 to 8, of Table A, not proceeding in any case beyond four significant figures. The reciprocal of the leading letter w is taken from Barlow's Tables, and entered in line 2, column 3, with the negative sign prefixed. All the remaining numbers in line 1 are multiplied by this reciprocal, and the products written underneath in line 2. This gives the value of w as an explicit function of x, y, and z.

The coefficients and absolute term of Equation 2 (omitting the coefficient of w, already employed in the first equation as a coefficient of x) are now written in line 1, Table B. The coefficient of x and all the following numbers in line 1, Table A, are next multiplied by the coefficient of x in line 2, and the products are written in line 2, Table B. The algebraic sum of lines 1 and 2, Table B, is now entered in line 3, Table A, and line 4 is formed therefrom in the same manner in which line 2 was formed from line 1.

Omitting the coefficients of w and x, the remaining terms of Equation 3 are written in line 3, Table B. The coefficients of y and the following numbers in lines 1 and 3, Table A, are respectively multiplied by the coefficients of y in lines 2 and 4, and the products entered in lines 4 and 5, Table B. The algebraic sum of lines 3, 4, and 5 of the latter table is now entered in line 5, Table A, and the value of y as an explicit function of z is determined in the same manner as were similar values of w and x, and entered in the next line.

In like manner line 6, Table B, is taken from Equation 4, and lines 7, 8, and 9 are formed from the coefficients of z and following terms in lines 1, 3, and 5, Table A, multiplied by the coefficients of z in lines 2, 4, and 6. The algebraic sum is entered in line 7, Table A. If there were other equations and unknown quantities, these processes would be repeated. The last repetition gives an approximation to the value of the last unknown quantity.

It is to be observed that the numbers in Table B have but a single use, while those of Table A are used over and over; and when the number of equations is large, it is of great advantage that they should be thus tabulated by themselves in a form compact and easy of reference.

The reciprocals and coefficients of the explicit functions should be copied as in Table C below, and carefully preserved for future reference and use.

C.

	Reciprocal.	x	y	z
1	− .184	− .401	+ .807	− .433
2	− .165		− .29	+ .356
3	− .114			− .684
4	− .809			

D.

	w	x	y	z
1	+ .6731	− .7133	− .5173	− .57 }
2	+ .2468	− .2029	+ .3899	$= z_1$ }
3	− .1024	+ .0368	− .127	$= y_1$
4	+ .3524	− .879	....	$= x_1$
	+1.17	....	....	$= w_1$

The absolute terms of the explicit functions are written in the first line of Table D. The value of z, as thus far determined, is multiplied by its coefficients in the last column of Table C, and the products are written in the second line of Table D. The algebraic sum of the two numbers in column y of the latter table gives an approximate value of y written underneath in line 3, and its products by its coefficients in column y, Table C, are written to the left in the same line. Similar processes determine approximate values of the remaining unknown quantities.

These values must be substituted in the original equations, and a sufficient number of decimal places must now be used to insure the requisite degree of accuracy. The residuals are written in the first line of Table E.

E.

	1.	2.	3.	4.
1	− .0165	+ .0169	+ .0047	+ .0039
2		+ .0066	− .0133	+ .0071
3		+ .0235	− .0068	+ .0084
4			− .0154	+ .0102
				+ .0296

F.

	w	x	y	z
1	+ .0030	− .0039	+ .0018	− .0239 }
2	+ .0103	− .0085	+ .0163	$= z_2$ }
3	+ .0146	− .0052	+ .0181	$= y_2$
4	+ .0071	− .0176	....	$= x_2$
	+ .0350	....	....	$= w_2$

The first line of coefficients in Table C is now multiplied by the residual of the first equation, and the products entered in the second line of Table E. The first reciprocal in Table C is also multiplied by the same residual, and the product entered in column w, line 1, of Table F.

The algebraic sum of the two numbers in the second column of Table E is now written underneath; and line 2, Table C, is multiplied thereby, and the products written to the right of the multiplier, except the product by the reciprocal, which is entered in column x, line 1, of Table F. The remaining numbers in this line are formed in a similar manner. The product of the sum of the last column of Table E, by the last reciprocal in Table C, gives a correction to the value of z. Table F is now completed by the same process that was employed in Table D, giving $w = + 1.17 + .035 = + 1.205$; $x = - .879 - .0176 = - .8966$; $y = - .127 + .0181 = - .1089$; $z = - .57 - .0239 = - .5939$. The residuals now remaining are: 1. $- .0008$; 2. $+ .0005$; 3. $+ .0004$; 4. $+ .0001$.

As the multiplication is performed by Crelle's Tables, no multiplier is allowed to extend beyond three significant figures. Other numbers may be extended to four; but it would be a waste of time to extend any number farther, except in the process of substitution for the determination of residuals.

By this process, Mr. J. G. Porter and myself have solved in five and one-half days, or 36 working hours, with far greater than requisite accuracy, 41 equations containing 174 side coefficients counting each but once, or 430 terms in all. Each of us made a complete solution, duplicating the work, and making frequent comparisons in order to avoid errors.

For the sake of perspicuity in explanation and convenience in printing, I have here made some slight departures from actual practice. For instance, in the solution of a large number of equations, it would be inconvenient to pass the eye and hand out to a vertical column of reciprocals; and they are better written in an oblique line near the quantities from which they are derived and with which they are to be employed.

II.—*Addition of new equations.*

Suppose that after the solution of the foregoing equations and the consequent adjustment a new condition is established, resulting in the following normal equation: 5. $0 = - 2.0475w + 0.8362x + 1.8567y - 1.3149z + 8.2527u - 1.8372$; with the addition of the term $- 2.0475u$ to the first of the previous equations, $+ 0.8362u$ to the second, &c. The absolute term is supposed not to be an original discrepancy, but an outstanding residual, after the foregoing solution has fully entered into the adjustment, as is generally the case with azimuth and length equations.

C(*a*).

	Reciprocal.	x	y	z	u
1	−.184	−.401	+.807	−.433	+.377
2	−.165		−.29	+.356	−.274
3	−.114			−.684	+.0307
4	−.809				−.282
5	−.145				

G.

	w	x	y	z	u
1	−2.05	+.836	+1.866	−1.315	+8.253
2		+.822	−1.654	+.888	−.773
3		+.166	−.481	+.591	−.455
4			−.269	+.184	−.008
5				+.348	−.098
					+6.919

H.

	w	x	y	z	u
1	+.1007	−.0732	+.0082	−.0753	+.267
2	+.0326	−.0268	+.0515	$=z_2$	$=u_1$
3	+.0482	−.0173	+.0597		$=y_2$
4	+.0469	−.117			$=x_2$
5	+.228				$=w_2$

Table C(a) differs from Table C merely by the addition of the last reciprocal and the last column of coefficients. The coefficients (but not the absolute term) of the new equation are written in the first line of Table G. The mode of procedure is now very similar to that followed in the formation of Table E; but those products in which the reciprocals are factors are entered in the last column of Table C(a), and these products are again multiplied, as well as the other numbers in the same lines, by the same multipliers in the formation of the other lines of Table G. Thus the first number in the last column of Table C(a) $= - 2.05 \times - .184 = + .377$; and the second number in the last column of Table G $= - 2.05 \times - .205 \times - .184 = - 2.05 \times + .377 = - .773$. The reciprocal of the sum of the last column of Table G is entered at the end of the column of reciprocals in Table C(a); and its product by the absolute term of the new equation gives an approximate value of u. The products of this value by its coefficients in the last column of Table C(a) furnish the first line of Table H, and determine a correction to z. This table is now completed by the process employed in Tables D and F.

If the absolute term of the new equation is an original discrepancy arising from new observations, as in the case of an extension of a survey, it is simply necessary to substitute the previously

determined values of the unknown quantities in the new equation, and then treat the residual as above indicated.

In the case of several new equations, each will furnish a table resembling Table G. If we suppose a sixth equation involving t, it will be necessary to construct a table G(a) with an additional column; and if there be a seventh involving s, there will be a table G(b) with another additional column. The number of columns in Table C(a) will be correspondingly increased; but it will be best at this stage not to extend the list of reciprocals beyond those given in Table C. Columns u, t, and s will therefore all be of the same length, and will terminate with the products furnished by such factors as were introduced by the last original equation and those in the same line. The sum of the numbers in column u, Table G(a), will then form a side coefficient between u and t; the columns u and t in Table G(b) will in like manner furnish side coefficients between u and s and between t and s; and the last column in each table will furnish a corresponding diagonal coefficient. There will thus be formed a set of normal equations from which w, x, y, and z have been eliminated, and the work may now proceed, as shown in Tables A and B. Table C(a) may then be completed, and a single table like D will determine approximate values of the new unknown quantities and corrections to the old ones. As the absolute terms of the former explicit functions have already been disposed of, their places should be left blank in the first line of this table.

I regard the facility with which new conditions can be absorbed as one of the principal advantages of this method, and hope that it will largely if not entirely obviate the necessity of partial figure adjustment in the primary triangulation.

III.—*Order of solution.*

In the process of elimination it is desirable to avoid the introduction into an explicit function of an unknown quantity from which the corresponding original equation is free. In line 4, Table A, the value of x contains the unknown quantity y, which was absent from the second equation. Had the third equation changed places with the first, the terms of the original second equation might have been introduced directly into Table A, and the number of lines in Table B would have been two less and the number of columns one less.

The order of solution in a figure adjustment can be best decided by inspection of the figure. The work should commence with an angle equation from a triangle having a side (or, better, two sides) on the exterior of the figure; and no angle equation from a triangle with a new interior side should ever be introduced till after the entrance of every angle equation not thus exposed to entanglement with conditions yet untouched. A side equation should usually be postponed till after the introduction of all the angle equations that relate to the same points and no others, but should immediately follow them, so as to precede all equations that extend beyond its domain into new territory.

IV.—*Selection of angle-equations.*

While it is thus desirable to postpone entanglement with new conditions, it is still better to avoid it altogether, so far as possible, by discarding angle equations from interior triangles, and retaining in preference those that lie on the border. The formation, as well as the solution of the normal equations, is thus considerably facilitated.

It is of much less importance, but, other things being equal, it is expedient to exclude triangles with small angles, in order to avoid entanglement with side equations having large coefficients; and it is also desirable, though of still less importance, to exclude triangles that adjoin small angles, and so have a side common with them.

V.—*Treatment of small angles.*

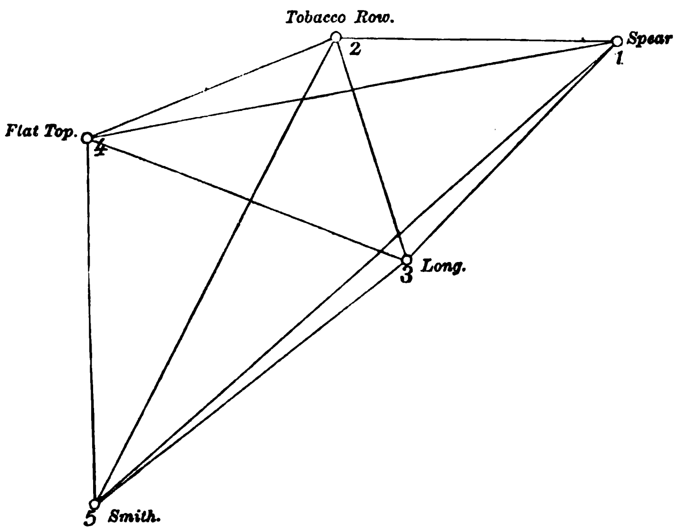
Suppose it be required to adjust the following figure with the observed directions as here tabulated:

Spear.

	°	′	″
Long	0	00	00.000
Smith	6	04	57.749
Flat Top	37	00	48.900
Tobacco Row	47	03	16.925

Tobacco Row.

	°	′	″
Spear	0	00	00.000
Long	72	37	08.593
Smith	118	11	11.341
Flat Top	159	40	31.200



Long.

	°	′	″
Smith	0	00	00.000
Flat Top	57	52	28.128
Tobacco Row	108	47	15.636
Spear	169	06	53.169

Smith.

	°	′	″
Flat Top	0	00	00.000
Tobacco Row	30	12	41.103
Spear	51	03	16.151
Long	55	51	27.879

Flat Top.

	°	′	″
Tobacco Row	0	00	00.000
Spear	10	17	00.258
Long	42	01	51.794
Smith	108	18	02.385

The figure requires three side equations. If we select those indicated by the symbols 3 5.2.1, 3 5.4.1, and 3 5.4.2, we have the following:

$$1.\quad 0 = +\,0''.12 - 0.196\tbinom{2}{1} - 1.780\tbinom{3}{1} + 1.976\tbinom{5}{1} - 0.066\tbinom{1}{2} + 0.272\tbinom{3}{2} - 0.206\tbinom{5}{2} + 2.505\tbinom{1}{5} - 0.438\tbinom{2}{5} - 2.067\tbinom{3}{5}.$$

$$2.\quad 0 = -\,0''.19 - 1.697\tbinom{3}{1} - 0.279\tbinom{4}{1} + 1.976\tbinom{5}{1} - 0.340\tbinom{1}{4} + 0.433\tbinom{3}{4} - 0.093\tbinom{5}{4} + 2.505\tbinom{1}{5} - 2.362\tbinom{3}{5} - 0.143\tbinom{4}{5}.$$

$$3.\quad 0 = -\,0''.19 - 0.195\tbinom{3}{2} - 0.011\tbinom{4}{2} + 0.206\tbinom{5}{2} - 0.233\tbinom{2}{4} + 0.326\tbinom{3}{4} - 0.093\tbinom{5}{4} + 0.438\tbinom{2}{5} - 0.295\tbinom{3}{5} - 0.143\tbinom{4}{5}.$$

In the first two equations the functions of the small angles at Spear and Smith so largely predominate over everything else, and those of corresponding terms are so nearly equal that the two equations may be considered approximately identical. The side coefficient will be about as large as the diagonal coefficients, and an attempt at solution with factors extending to but three significant figures would not be likely to furnish even an approximation to the values of the unknown quantities.

Adding the third equation to the first, and subtracting the second from the sum, the following equation results:

$$4.\quad 0 = +\,0''.12 - 0.196\tbinom{2}{1} - 0.083\tbinom{3}{1} + 0.279\tbinom{4}{1} - 0.066\tbinom{1}{2} + 0.077\tbinom{3}{2} - 0.011\tbinom{4}{2} + 0.340\tbinom{1}{4} - 0.233\tbinom{2}{4} - 0.107\tbinom{3}{4}.$$

It is evident that if the first, third, and fourth equations are satisfied, the second must also be satisfied. Hence, the fourth equation may be safely substituted for the second, or, as will easily appear, for the first, if it be preferred to retain the second.

The fourth equation corresponds to the symbol (3) 4. 2. 1, and might have been obtained directly in the usual way. The following rule may be deduced for this method of avoiding difficulty with small angles: Choose the pole so as to employ the small angles once, employ them but once, and adhere to the same pole.

If the line 2 . . 4 had not been observed, this method would have been impracticable, and, when practicable, it is sometimes inexpedient, as in this instance. It is desirable to select Tobacco Row as the pole for at least one equation, on account of the small angles in the triangle 1. 4. 2. The following method is therefore often preferable and sometimes necessary.

Subtracting the second equation from the first we have—

$$5.\quad 0 = +0''.31 - 0.196\binom{2}{1} - 0.083\binom{3}{1} + 0.279\binom{4}{1} - 0.066\binom{1}{2} + 0.272\binom{3}{2} - 0.206\binom{5}{2} + 0.340\binom{1}{4} - 0.433\binom{3}{4} + 0.093\binom{5}{4} - 0.438\binom{2}{5} + 0.295\binom{3}{5} + 0.143\binom{4}{5}.$$

This equation may be substituted for either the first or the second. It corresponds to the symbol (3) 5. 4. 1. 2, and might have been directly obtained. For this method the following rule may be given: Choose the pole so as to employ the small angles once, and with the same pole form an equation involving all the five stations and avoiding the small angles.

It may be useful to notice that the fifth equation involves neither the longest side of the small-angled triangle nor the opposite side of the quadrilateral formed by the non-polar points. If the longest side of the small-angled triangle had been a diagonal of that quadrilateral, both diagonals would have been absent from the fifth equation.

Of the two quadrilaterals (2) 1. 3. 4 and (2) 1. 5. 4, the former is somewhat to be preferred, since the latter is worse entangled with the small-angled side equation having its pole at Long. but no great difficulty may be apprehended whichever is chosen.

Respectfully submitted.

M. H. DOOLITTLE.

CHAS. A. SCHOTT,
Assistant in charge of Office.

APPENDIX 9.

ON A PHYSICAL SURVEY OF THE DELAWARE RIVER IN FRONT OF PHILADELPHIA, BY HENRY MITCHELL, ASSISTANT.

JUNE 30, 1879.

DEAR SIR: In submitting the following report upon a physical survey of the Delaware River at Philadelphia, I have felt it incumbent upon me to give something more than a compilation of the observations, because the proper understanding and use of the tables by the engineer require from him a wider view of the subject than this strictly local survey could offer. The general scheme of the survey, and the stand-point from which we have regarded the scene, are revealed in the introduction to the report, and followed by illustrations of the practical use of the tables, as far as necessary, to indicate their applications to the questions likely to present themselves.

It is an unusual privilege to deal with observations entirely trustworthy and systematically recorded, and I call your special attention to the credit due Mr. H. L. Marindin, the chief of the field party, to Mr. John B. Weir, his principal assistant, and to Mr. Charles A. Russell, the recorder. These three gentlemen, trained by experience in several different river surveys which we have been called upon to make, not only gathered the data, but performed most of the labor involved in their reduction to tabular form, &c.

The removal of Mr. Weir during the past year necessitated the employment of Mr. Edward H. Lincoln, civil engineer, temporarily. He also had been an observer formerly in our Mississippi work, and proved very competent.

I think that you will conclude from the report that our methods gradually improve and become more direct in their application to problems presented in the preservation of channels and the commercial occupation of their banks.

Appreciating the continued and intimate interest you have evinced in the work assigned to us, and the support which you have rendered and obtained for the party in the midst of *hard times*,

I remain, respectfully, yours,

HENRY MITCHELL,
Coast and Geodetic Survey.

CARLILE P. PATTERSON, LL. D.,
Superintendent Coast and Geodetic Survey.

REPORT.

The channel.—There are no elements in the questions of riparian rights and privileges, which give rise to more discussion and contest than the proper location and form of the channel. All are agreed that a common pathway for navigation must be preserved, and all are agreed that the essential drain capacity for river waters or tides must be retained. But how far may opposite shore-owners extend their wharves? To a certain stated distance from the original shore line; to within a certain distance of the line of deepest water; or to that distance which shall exclude a given volume? How shall encroachment be limited to that which will not involve injury at times of floods or great tides? In fine, where is the *essential channel*, and what is its *essential form?* This is the question we have to consider.

The actual shore line is among the most uncertain contours in nature, varying with every change in the elevation of the stream, especially in low countries, and rarely returning to its previous position after inundation. The *thalweg*, or line of greatest depression, is also an uncertain contour, often difficult of determination by soundings because of the general flatness of the river bed, and always disposed to shift where the bed is alluvial. To locate the channel properly all its contours must be taken into the account; it must be treated as a whole and not defined by its salients only.

The method reached in our yet very imperfect study may be stated thus:

From the soundings the ordinates of many cross-sections are determined, and each profile is reduced to a smooth figure by a formula, based upon type forms, and fitted to each particular case by coefficients. The shore lines are thus corrected or reconstructed by eliminating accidental features or mere anomalies, and the normal position of the *thalweg* is fixed.

From the original soundings, or from the ordinates of the corrected cross-sections, the line of *mid-area*, or the locus of the center of gravity, is determined and its radii of curvature computed; so that this too may be plotted in smoother curves if necessary. As a rule, however, in alluvial bottoms, the line of *mid-area* actually observed presents smooth curves, which appear upon the plan in striking contrast with the ragged outlines of the shore, the contorted profiles of sections, and the meandering course of the *thalweg.*

After treating the river as an inert mass, and determining the shape of its mould, as above indicated, it is taken up again as a living stream, and the profile of its forces and the locus of its flow are determined. The use of the word *profile* as applied to forces may not appear to have the same significance as when applied to the transverse section, which has a real existence in nature; but it is not *imaginary*, since it may be called into transient existence as a visible form. If a number of apples were to be simultaneously dropped upon a stream along a transverse line, the figure which would be given by a line drawn through these apples at the end of one second would be what we have called the profile of the forces, and between this profile of forces and the profile of the section there exists, in alluvial bottoms, the relation of cause and effect. The transverse profile of forces will be found to resemble the transverse profile of the bottom, especially if both are observed at the time of ordinary floods, when the flowing water is constructing its channel. In low-water seasons or at great floods the stream is often subservient to or out of register with it; at such times the loci of *mid-area* and *mid-volume* present lunes at all sharp bends and coincide only in straight reaches or at points of reversion.

There are also intervals between the loci of *mid-area* and *mid-volume* where the river, as a living body, has recently shifted its bed and not supplied the material for filling up its old channel. In a case like this the line of *mid-area* would evidently not represent the course of the channel in any abiding sense, and the line of *mid-volume* might not yet be firmly located. We have here the great source of trouble in meeting local questions of jurisdiction and riparian title.

Our observations of currents have not been made during floods nor made simultaneously in all the sections at any time; so that while we design to study both cause and effect we inevitably discover that our active causes are not those which produced the entire effects, nor do they faithfully represent in miniature those causes. For instance, when a stream goes around a bend it is disposed to press along the concave shore, but in those seasons when the stream is strong and at work, this tendency is greater than at seasons of sluggish movement. The line of greatest motion will lie beyond the line of greatest depth where the bend is increasing, and by a careful study of the relative positions of the loci of *mid-area* and *mid-volume* one may ascertain whether the destructive or constructive disposition of the river predominates at any point.

The form of the cross-section is dependent upon the curvature in the course of the stream, and where a stream has a tendency to increase its bend, the center of the cross-section and the apex of its profile will move toward the concave shore at a rate which will vary with the inverse radius of curvature of the bend.

Form of cross-section.—The most casual observer may notice that, as a rule, deeper water is found at a bend than in the straight portion of a stream, and that while the greatest depth lies midway between the shores in the straight reach, at bends the greatest depth lies nearer the concave shore, and he naturally associates the depression and the shift of the *thalweg* with the curvature of the river.

In the practical study of channels it has become our way to seek first the fundamental form at straight reaches, and then introduce into its equation elements of change calculated to convert the curve of the straight reach into one representing the form of section observed at bends, &c.

The fundamental section varies in different soils or where there is a difference in the vegetation upon the shore, but its profile oscillates, I am satisfied, only from the curve of sines to the semi-ellipse—from the curve of sines whose arc is 1.57 times the mean width observed (from surface to bottom), and whose maximum ordinate is 1.57 times the mean depth (from shore to shore) to the ellipse, whose axial ratio is that of mean depth to half mean width.

With central origin, the expressions for these profiles would be simply (1), $y = 1.57\, d \cos x$ with limit of $x = 1.57\, c$, and (2) $y = \frac{d}{c} \sqrt{\text{C}^2 - x^2}$, in which d equals the observed mean depth and c equals the half mean chord observed, while C in the second expression is the surface chord or width from shore to shore. The circular arc, lying between these two curves, may be *essentially* represented by (3) $y = \text{D}\, \frac{(\text{C}^2 - x^2)}{\text{C}^2}$, in which D is the maximum ordinate at origin.

In practice, the first of these expressions—the *sinusoid*—is the most convenient, because no successive trials are necessary; the surface chord and the maximum ordinate being computed directly from the mean depth and mean chord observed; and in (3) the values of D and C may be similarly computed from the mean elements observed for *first trial*, the observed surface width being doubtful.

The main object in view is to *replace the observed fundamental section by one preserving all its essential features without its accidental ones.* In practice we find that where there is no curvature in the course of the stream, a symmetrical curve, given by one of these expressions, fits very nicely, except at and very near the surface where banks have been swept away or caved in.

Now, the centrifugal force, which would press the current upon the concave shore in going round a bend, would cause the center of gravity to move toward this shore in the inverse ratio of the radius of curvature; and, as the volume passing is the same, there is no diminution of area.*

To produce this change of the center of gravity, without diminution of area, an element must be inserted, which will reduce one part of the figure and increase the other equally; in other words, on one side of the origin shoaling must be represented, and on the other deepening.

This is very simply done by adding to (1), $\psi \sin 2\, x$; to (2), $\psi\, \frac{\sqrt{\text{C}^2 - x^2}}{\text{C}^2}\, 2\, x$ (*i. e.*, the sine of twice the eccentric angle), and to (3), $\psi_3 \left(\frac{\text{C}^2 - x^2}{\text{C}^2}\right) 2\, x$.†

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* Critically speaking there must be an increase of sectional area to balance the greater resistance which would otherwise occur.

† To avoid imaginary quantities where the curves require shifting, the ellipse may be replaced by a curve of a *higher* (even) *degree.*

In using the first or third of the above expressions, special cases may present themselves in which the capacity for change under the influence of the second (bend) element is too limited; but for *ordinary bends the second coefficient varies very nearly with the shoreward movement of the thalweg and the center of gravity, and consequently with the inverse of the radius of curvature of the bend.*

Where there is a tendency of the stream to split, so as to form two semi-detached channels, a third element must be added (co sin $3x$, or the square of the second element in (2) and (3) with a constant for equating areas and other constants to shift positions), and many devices become necessary in fitting the computed to the observed curve, till no recurrent curves appear in the residuals.

The foregoing is not presented as a *theory* of the cross-section, but as a simple and convenient method of generalizing the forms presented. The supposition that there are, in the absence of any split, only two elements in the formula for the profile of cross-section, retaining the same relative position, presumes that there is a midway point that does not alter its depth, and that the inclination of the bottom, at this point, is similar (although in reverse sense) to that of a railroad going round a bend at uniform grade.

In Diagram A the type forms are furnished for the cross-sections, as given by the simple formulæ above mentioned, and a glance at the "Dynamic Charts" (not furnished for publication with this report) will satisfy you that every case presented among the forty-seven sections measured in the Delaware falls under one of these theoretical types.

It remains to state what practical advantage results immediately from the application of these or better formulæ which generalize the forms of the sections.

1st. To determine the normal positions of the shore lines and the thalweg by *deducing them from all the measures of each cross-section.* Our earlier method was to smooth out the observed profile of the section by sweeping a line through the irregularities by the eye—the method of "*graphical correction*" that Dr. Whewell applied to tidal observations; but a comparison of this old method with the one now proposed or a comparison of the curves drawn by any two experts with the old method, shows that personal prejudice is a very considerable element rendering the result by the graphical method almost useless. The mean of many cross-sections ought logically to give a curve from which mere errors of observation are eliminated, but it cannot furnish the *virtual shore-line* which is an element in the proper location and allotment of the *Port-Warden line.**

2d. To decide what feature in any cross-section has an artificial origin or is so recent that the stream has not yet accommodated itself to the change.

3d. To decide what portion of any cross-section can be improved permanently by dredging. A shoal which is an obstruction to navigation may not be an obstruction to the river, but the result of a deposit by the river itself in a space which it has relinquished as it has shifted its bed. In such a case the artificial removal would only be a temporary relief to navigation. The application of our formulæ to the section as a test, and a comparison with the profile of forces, would in almost every case seem to be entirely satisfactory in determining the character of an obstruction, whether *forced* or *suffered.*

4th. To measure in advance the degree to which any proposed structure would injure the channel or induce change elsewhere.

In the second part of this report the result of some efforts to determine the *necessary* form of the cross-section at several points on the Delaware, before ascertaining the fundamental section, will be found. Some very ugly cases present themselves, in a stream in which ice has played a part which is yet very obscure to us; but, on the whole, we have been (I submit) as successful as ought to be expected in a first essay.

It may be objected to the elliptical figure that it gives a vertical bank immediately at the shore line, which is unnatural in material that takes a long slope of repose, but when it is considered

*The statutes fixing the position of Port-Warden lines make no provision for their division as frontages among riparian proprietors, each of whom is entitled to a right of way to the sea from every foot of his shore. Expensive litigation, ending usually in very unequal justice, has necessarily attended the attempt to determine a numerical relation between an irregular shore and a geometrical curve lying far from it. I proposed, some years ago, and still insist, that legislatures should provide for the establishment, first, of a "*conventional*" (I now say *virtual*) shore-line in stated curves and tangents, and that the harbor-line should be so drawn, subsequently, that each section should contain the interior shore-line curves as factors, direct or indirect.

A

DELAWARE RIVER.
Types of the Cross Section.

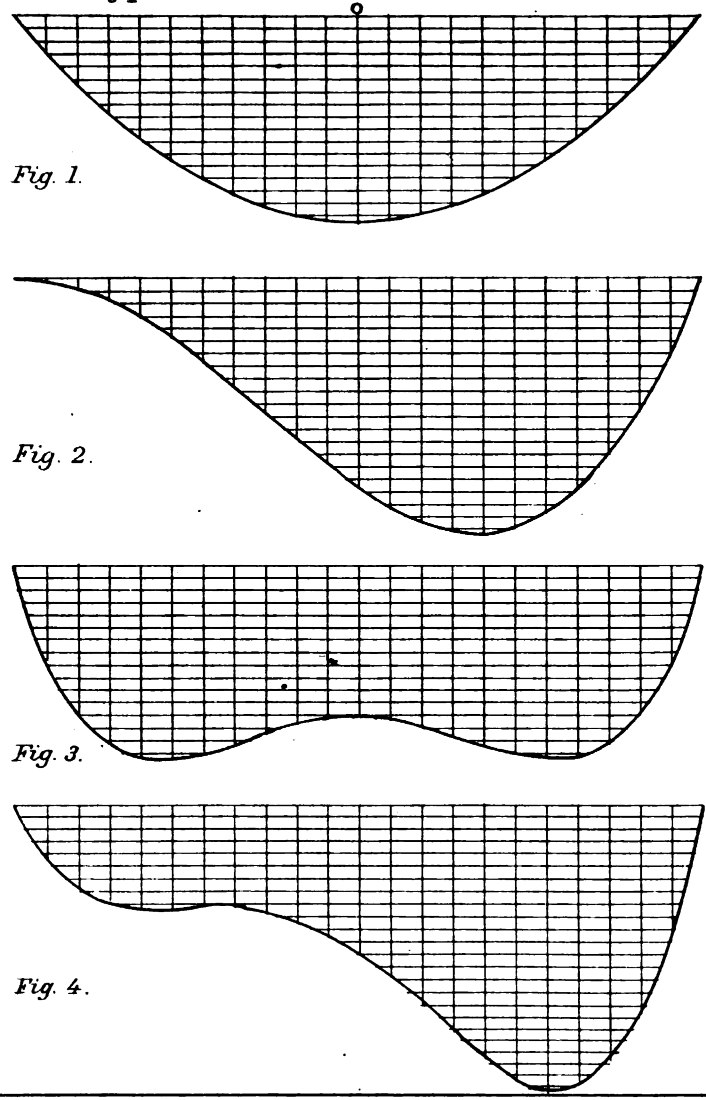


Fig. 1.
Fig. 2.
Fig. 3.
Fig. 4.

S W. PASS, MISSISSIPPI RIVER.
Section 7½.

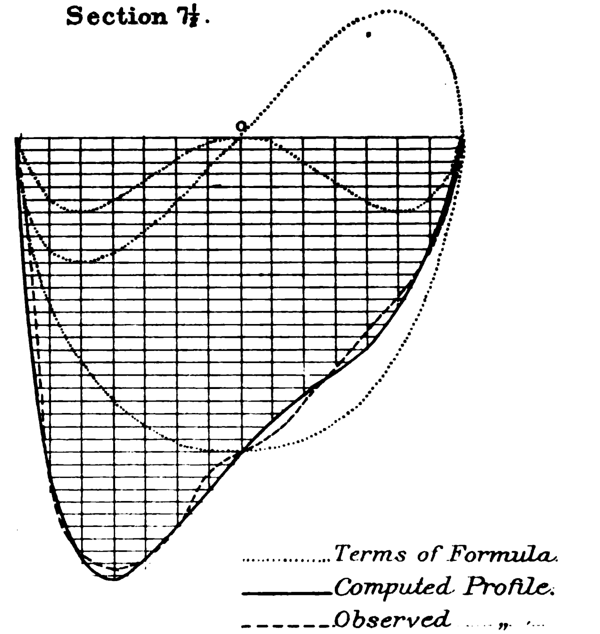


Fig. 1 Straight Reach.
" 2 Simple Bend.
" 3 Bifurcation, or shift of bed in a straight reach
" 4 " " " bend of the River.

that this long slope is not an essential condition to the proper flow of the stream, the objection, from our point of view, disappears.

The general scheme of the cross-section, stated above, was incorporated in a foot-note to my report on New York Harbor (Ann. Rep. Coast Survey, 1876), since which I have found, in the *Annales des Ponts et Chaussées*, 1868, an article, by M. Farque, in which he has attempted to show the relations between the course of the thalweg and the bend of the artificial banks of a river, and to connect them with the radius of curvature. The objection to his method lies in his reliance upon single contours instead of using all the soundings and treating the river as a whole. His paper involves, necessarily, much mathematical ingenuity; and while his ultimate aim seems to be much the same as my own, his line of argument is very different.

Upon Diagram A I have plotted the profile of one of the cross-sections observed in the Southwest Pass of the Mississippi River, showing my conception of the three elements of which it was composed, viz: The fundamental ellipse, the twist due to the bend, and the enlargement. The observed curve might be expressed in ascending powers thus:

$$y = 52 + 0.05173x - (0.0028633x)^5 - (0.0026833x)^6$$

but here the terms, individually, have no physical meaning.

SECTION 7½.—SOUTHWEST PASS, MISSISSIPPI RIVER.

Distance. x	a	b	c	d	e	Difference $e-d$	Remarks.
	$\frac{50}{720}\sqrt{720^2-x^2}$	$20\ \frac{\sqrt{720^2-x^2}}{720^2}\ 2x$	$12\left(\frac{720^2-x^2}{720^4}\right)4x^2$	$a+b+c$ Computed depth y	Observed depth.		
Feet.	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	
720	0	0	0	0	1.0	+ 1.0	Formula of computed curve, $y=\frac{50}{720}\sqrt{720^2-x^2}+20\frac{\sqrt{720^2-x^2}}{720^2}2x+12\left(\frac{720^2-x^2}{720^4}\right)4x^2$
660	20.0	14.6	6.4	41.0	27.0	—14.0	
600	27.6	18.4	10.2	56.2	57.9	+ 1.7	
540	33.0	19.8	11.8	64.6	65.5	+ 0.9	
480	37.3	19.9	11.9	69.1	67.8	— 1.3	
420	40.6	19.0	10.8	70.4	68.5	— 1.9	
360	43.3	17.4	9.0	69.7	68.5	— 1.2	
300	45.4	15.2	6.9	67.5	67.6	+ 0.1	
240	47.1	12.6	4.7	64.4	65.5	+ 1.1	
180	48.4	9.6	2.8	60.8	61.4	+ 0.6	
120	49.3	6.6	1.3	57.2	55.0	— 2.2	
60	49.8	3.4	0.3	53.5	51.6	— 1.9	
0	50.0	0	0	50.0	49.4	— 0.6	
60	49.8	3.4	0.3	46.7	48.6	+ 1.9	
120	49.3	— 6.6	1.3	44.0	46.4	+ 2.4	
180	48.4	- 9.6	2.8	41.6	43.6	+ 2.0	
240	47.1	--12.6	4.7	39.2	40.0	+ 0.8	
300	45.4	—15.2	6.9	37.1	37.0	— 0.1	
360	43.3	-17.4	9.0	34.9	33.6	— 1.3	
420	40.6	—10.0	10.8	32.4	30.5	— 1.9	
480	37.3	—19.9	11.9	29.3	27.2	— 2.1	
540	33.0	—19.8	11.8	25.0	24.0	— 1.0	
600	27.6	—18.4	10.2	19.4	20.4	+ 1.0	
660	20.0	—14.6	6.4	11.8	13.1	+ 1.3	
720	0	0	0	0	0	0	

Mean depth of observed section	44.6 feet.	Area of observed section	64,266 sq. feet.
Mean depth of computed section	45.2 feet.	Area of computed section	65,148 sq. feet.
Mean chord of computed section	918 feet.	Mid-area of computed section	— 171 feet.
Mean chord of observed section	892 feet.	Mid-area of observed section	— 165 feet.

The formula for the cross-section of the Delaware in front of Philadelphia, supposing it perfectly canalized, so as to be reduced to the minimum width, I make

$$y = 33\,\frac{1100^2 - x^2}{1100^2} + \frac{289}{r}\left(\frac{1100^2 - x^2}{1100^2}\right)2x$$

in which "r" is the radius of curvature of the bend, varying from 16200 to infinity.

THE DELAWARE.

A glance at the general chart of "Delaware Bay and River," issued by the Coast Survey, will suffice to sanction the following propositions, which, as hints to our inquiries, have been found valuable:

1st. The water-way augments as it approaches the sea, although no important tributary rivers exist.

2d. The river, at first tortuous, unwinds as it approaches the sea.

3d. The *channel* (thus distinguishing the main artery) threads its way among shoals, with a disposition, as it approaches the sea, to separate into *passes* like delta rivers.

1st and 2d. The expansion of the water-way in the absence of fresh-water tributaries, and the increase of the radii of curvature in the course of the river as it approaches the sea, are both due to the action of tides mainly. Other things equal, the size of a tidal channel through alluviums may be said to vary directly as the amount of service which it is called upon to perform as a conduit, and the increase of tidal volume, between the mouths of the Schuylkill and Christiana Creek for instance, fully accounts for the increase of the average section.

Again, the range of the tide being about six feet, a *potential head* to this amount may be said to exist at every point. Any obstruction which the tide encounters calls forth its measure of active force within this limit of six feet (which an actual stoppage would induce), and in approaching the sea the increasing volume has greater and greater power to overcome resistances. Bends of small radius are resistances that must therefore diminish, and the tendency of the water-way as it approaches the sea is to straighten as well as augment its section. The bend which a stream makes on meeting an obstruction is not symmetrical—the deflection above is more abrupt than the reflection below; the only form of channel, then, that will harmonize flood and ebb streams, whose directions are opposed, will be the straight one. We are continually witnesses of the unwillingness of flood and ebb streams in tidal rivers to submit to the same bend; the fact that they have different velocities, and, therefore, centrifugal forces differing as the squares of the velocities, renders entire harmony out of the question. The introduction of tides into a river must have a tendency to straighten the water-way.

3d. The Delaware occupies only a part of the antecedent depression through which it flows, and it has formed—perhaps is still forming—a sort of delta with the materials brought down from the interior—somewhat as the Alabama River is building its delta at the head of Mobile Bay or the Atrato in the Gulf of Darien. It will be seen on a close examination of the chart that this river presents great irregularity of section, indicating that the volume of water in motion does not everywhere occupy the entire water-way; that it has shifted its pathway at many points without yet filling up its old avenue, so that the cross-section of the water-way is not in each instance a cross-section of the true stream.

The river proper as a *physical force* steadily declines as it approaches the sea, for the volume of fresh water remains essentially constant while the volume of the tide increases. This would be true *comparatively* even if the water-way did not enlarge, but where such enlargement takes place, as in the Delaware, an actual loss of power, almost to the annihilation of this element as an active one, must occur. Nevertheless, as we have seen, the river outflow modifies the form of channel-section nearly to the threshold of the ocean.

Location of the channel.—In the studies that we have made in various harbors with a view to the proper location of Port-Warden lines limiting the extension of wharves, &c., we have narrowed the vexed questions of channel location down to two alternative propositions, viz:

1st. The locus of a stream is that of its center of gravity, *i. e.*, the line of *mid-area* of section.

2d. The locus of a stream is that of its axis of motion, *i. e.*, the line of *mid-volume.*

In straight reaches these two loci concur at all times; and at symmetrical bends they might be presumed to concur when the stream is in its average *working* condition. In the tables appended to this report the numerical values of the co-ordinates of forty-seven cross-sections are given representing the mold of the river from Five-Mile Point to the mouth of the Schuylkill, with the posi-

B

Note.

The locations of transverse sections are indicated on the map by broken lines.

The locus of the centre of gravity (mid-area) from observation is represented by a full line on the map and in Fig. 2, where the Curves are enlarged for the space between Sect. 37 and Sect. 46.

The locus of the centre of flow (mid-volume) of ebb is represented in Fig. 2 by a heavy broken line.

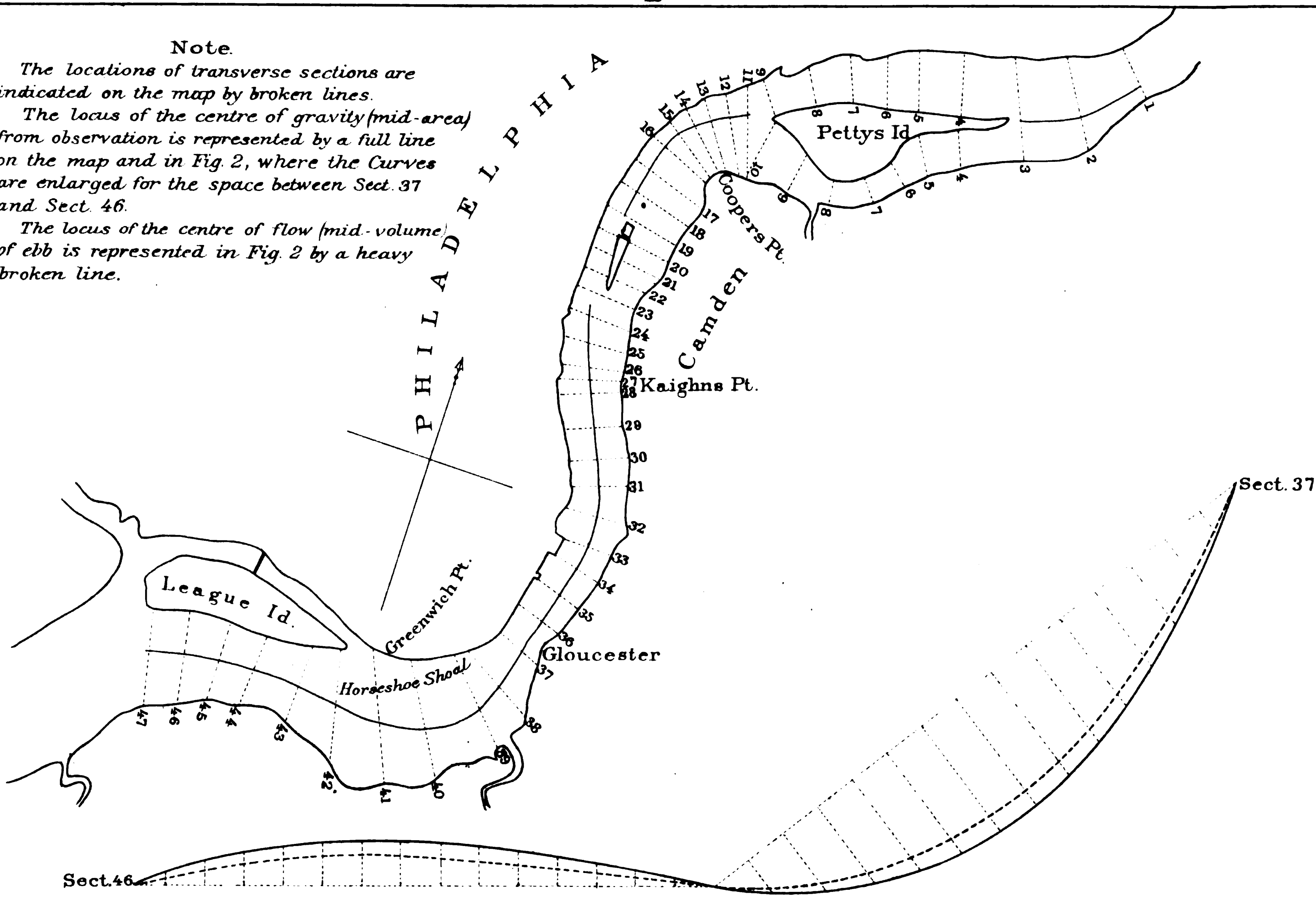


Fig. 2.

tions of *mid-area* and *mid-volume* for each section, and in Table No. 48 the radii of curvature are furnished for the locus of *mid-area*, rudely sketched upon Diagram B.

The appearance of lunes between the curves of *mid-area* and *mid-volume* when we came to plot them satisfied us that we had not made our current observations at the average *working* stage of the stream.

Below Gloucester Point the river remains essentially in a state of nature, and here we find the loci of *mid-area* and *mid-volume* describing very smooth curves (see Diagram B, Fig. 2), but leaving considerable lunes between them except at the straight reach near Gloucester Point and the place of reversion below. The radii of curvature for the upper bend, that having its concavity turned to the northwest, are 6,205 feet for *mid-area* and 6,835 feet for *mid-volume* of ebb.

Notwithstanding the existence of these lunes, if a treatment of this portion of the river for location of Port-Warden lines were proposed, it would seem patent that, acting exclusively upon physical indications, we should simply construct a water-way diverging in due proportion to the increase of tidal volume, and provide that it should be so placed as to be everywhere balanced about the *mid-area* line observed. The width at the initial point would be the most serious question to settle. There is a nearly straight pass above Gloucester Point where, as might have been expected, the smallest section, in proportion to discharges, is found; but this section is not in the form most characteristic of the river, even for a straight reach, and would be found too narrow elsewhere.

It is above the Gloucester reach, in the portion of the river whose shores are occupied for commercial purposes, that the greatest perplexity in regard to the location of the stream exists. We have given the actual places of *mid-area* and *mid-volume*, but where there are islands or where the stream partially bifurcates we cannot bring the points into accord. It is evident that we must more intimately examine the transverse sections before we can trace the progress of the characteristic form down the river and determine correctly the points of bifurcation and reunion.

It may be properly stated here that at bends one often finds upon the convex side a "*slough channel*" which has been created during floods and served as a *waste-weir*. The double channel, in a case like this, must not be confounded with bifurcation, although in time of flood the "*slough channel*," because of its great slope, exhibits the more rapid current.

Cross-section.—The gradual augmentation of the water-way, referred to at an earlier portion of this report as characteristic of the Delaware in its approach to the sea, is scarcely recognizable in the portion of the stream lying along the frontage of Philadelphia. It will be observed from a glance at Diagram C, Fig. 1, that there is no sustained increase of sectional area from the upper end of Petty's Island to Gloucester Point, notwithstanding that in this distance the increase of flowage is full 27 per cent.

Before entering upon the inquiry to what extent changes in the natural order may have been wrought by artificial means—an inquiry reserved for a future report when the soundings upon old and new surveys shall have been compared—it behooves us to consider a little more critically to what extent the natural order makes uniformity in the increase of section necessary.

Were the river straight and all its sections symmetrical and similar, the magnitude of the water-way would increase nearly in the same ratio as the discharge; but where a stream is compelled to meander among obstructions, even if through a generally yielding bed, the sections must have variations due to the changes of course, or, in other words, to the change in the radius of curvature from point to point. Theoretically the least depth and the least width in the course of a constant stream should be found in the straightest reach, because here the least resistance and the most economical form of section is found.

In the earlier part of this report it was stated that the scheme of our inquiry was based upon a broader experience than this local survey would have furnished, and in the study of actual sections in the portion of the Delaware under consideration, the fundamental form with simple variations at bends and voluntary bifurcations, are often only obscurely recognized, and in fitting the computed to the observed curves many devices are resorted to.

The bed of the river is not molded by the land-waters only, but modified by the tides which not only cause an increase of discharge, as we go down, but present antagonisms at every bend

which is not symmetrical. It will be an important purpose with the engineer who may project improvements of this river to reduce the bends to symmetry as far as possible.

In a tideless river one may conceive of a fundamental section as sustained in the reach between any two tributaries, but where there are tides a continual augmentation of sectional area is necessary. This report cannot pretend to do more than cross the threshold of the Delaware problem.

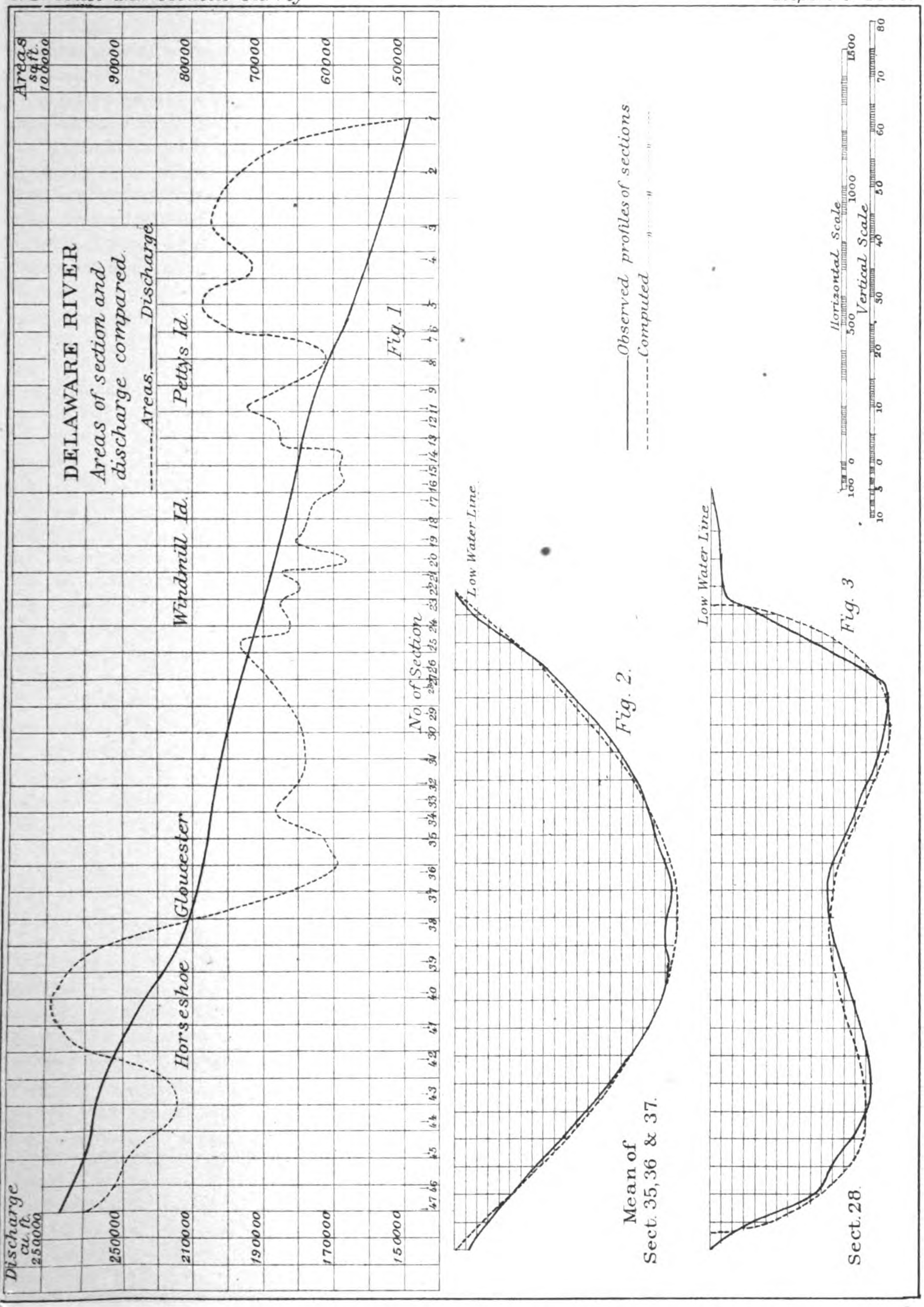
We are presented in this survey with no straight reach of sufficient length to permit the complete exhibition of the symmetrical section which can be properly recognized as a fundametal form. By grouping the measures of Sections XXXV, XXXVI, and XXXVII together and taking a mean, we obtain, it is true, a symmetrical form, but one that seems rather to result from a confluence of streams heretofore separated than from an orderly and sustained flow. Here we have a sectional profile nearly corresponding to the arc of a circle whose radius is 725 times the mean depth—or still more nearly with a sinusoid whose mean ordinates and mean chord correspond with the mean depth and mean width observed.

MEAN OF SECTIONS XXXV, XXXVI, AND XXXVII.

Distance x	Observed depth.	Computed depth $y=40 \cos x$	Difference.	Remarks.
Feet.	*Feet.*	*Feet.*	*Feet.*	
1,200	0.0	0.0	0.0	Pennsylvania side.
1,100	2.0	5.2	−3.2	
1,000	9.5	10.4	−0.9	
900	14.5	15.3	−0.8	
800	21.5	20.0	+1.5	*Feet.*
700	25.7	24.4	+1.3	Mean depth of observed section.... 25.3
600	28.0	28.3	−0.3	Mean depth of computed section ... 25.5
500	32.0	31.7	+0.3	Mean chord of computed section ... 1,560.
400	35.7	34.6	+1.1	Mean chord of observed section .... 1,530.
300	38.7	37.0	+1.7	*Sq. feet.*
200	39.5	38.6	+0.9	Area of observed section 60,730
100	38.5	39.6	−1.1	Area of computed section 61,114
0	38.0	40.0	−2.0	Mid-area of computed section 0
100	38.0	39.6	−1.6	Mid-area of observed section .... − 2
200	38.5	38.6	−0.1	
300	37.2	37.0	+0.2	
400	34.5	34.6	−0.1	N. B.—The plane of reference is here 4 feet
500	31.5	31.7	−0.2	above mean low water
600	27.5	28.3	−0.8	
700	24.0	24.4	−0.4	$y = 36 \cos. x$ at low water (nearly).
800	19.5	20.0	−0.5	
900	14.5	15.3	−0.8	
1,000	10.5	10.4	+0.1	
1,100	6.5	5.2	+1.3	
1,200	3.0	0.0	+3.0	New Jersey side.

The second figure of Diagram C illustrates the foregoing table. The area of the section, 61,000 square feet, is that for the stage of maximum ebb current; reduced to mean low-tide, it falls to about 51,000 square feet. While it must be admitted, from the near approach of this profile to the arc of a circle that the section is one of economy from a physical point of view, in the interest of navigation a greater width with less maximum depth would be preferable and seems from an inspection of other sections to be admissible without incurring the danger of a *middle ground.* In using our formulæ for changing widths we have simply to regard x and y as undergoing reverse variations, the area remaining the same.

As a rule the surface width of rivers is greater than the law of the curvature of the sectional profile would give, because the waves and rains break down the shore. But in alluvial beds the river at time of freshets often overflows its banks and by this relief suffers the lower portion of the profile of section to retain the form impressed upon it under more ordinary conditions, so that the permanent portion of the water-way does not represent the extreme demand. It would not seem wise to restrict a river to the dimensions of its minimum section, however perfectly this section may realize the law of economy.


DELAWARE RIVER
Areas of section and discharge compared.
Areas. Discharge.
Areas sq. ft. 100000
90000
80000
70000
60000
50000
Discharge cu. ft. 250000
250000
210000
190000
170000
150000
Pettys Id.
Windmill Id.
Gloucester
Horseshoe
No of Section
Fig 1
Low Water Line
Fig. 2.
Mean of Sect. 35, 36 & 37.
Low Water Line
Fig. 3
Sect. 28.
Observed profiles of sections
Computed
Horizontal Scale
Vertical Scale

Mr. Marindin has computed for me the average gauge from his entire field of work, extending from Five-Mile Point to the mouth of the Schuylkill, and furnishes the following:

Surface width, 3,645 feet.
Mean depth, 19.43 feet.
Thalweg depth, 39.50 feet.
Sectional area, 70,822 square feet.
Plane of reference, 4 feet above mean low-water.

Comparing these elements with those given in the previous table for the midway section, we find that the width is 50 per cent. greater, while the mean depth is 24 per cent. less, the thalweg depth much the same, and the area only 13 per cent. greater. One must infer from these figures that the straight section given in the table has too small surface width and too great mean depth, while admitting that the average surface width is considerably greater than it need be. An inspection of the chart shows that the majority of the sections have shallows on one side or the other which augment the surface width without anything like a proportional increase of section.

It is evident that Mr. Marindin's gauge for the grand average would not yield a symmetrical profile of section, because neither the sine curve nor the ellipse would admit of the coexistence of such elements; but they yield, as might have been expected, the form characteristic of a winding river, produced by adding to the sine curve the sine of double the angle, or to the semi-ellipse the sine of twice the eccentric angle. The average section, then, is *essentially* represented by Fig. 2 of Diagram A, but actually by Fig. 4 of the same diagram, since a split in the channel is characteristic of the present state of things, although *not essential.*

The average surface width of the river for the portion (within the limit of our survey) above Gloucester Point is over 3,000 feet, while at the lower end of the course, *i. e.*, at Gloucester Point itself, the width seems to have been naturally less than any other at half tide.

We are not now prepared to state what would be an adequate mean width of this portion of Delaware River under a system of artificial embankments or improved shores, because a very important consideration is to be given to the question, How far can the islands in the river be reduced and to what extent can the shoals be removed? To narrow the river where now excessively broad, and leave the shoals to the mercy of the temporarily increased flow, would be simply to remove the shoals from their present locations to points lower down in the pathway of navigation, where they might be greater sources of trouble than now.

Of course it will be understood that the thalweg depth given by the great average does not imply a continuity of deep water, but it may be said that the average depth of 19½ feet implies the practicability of securing a thalweg depth of not less than 30 feet at half tide, and the possibility, by retrenchment and by removal of obstructions, of maintaining everywhere, by the action of the currents, a continuous channel depth as great as the heaviest merchant-ships may require.

The next tolerably straight reach lies between Sections XXXI and XXVIII, and we may take these terminal sections as samples for the reach. A very marked feature in this reach is a middle ground, which, for the upper section, seems to be a continuation of Windmill Island. From the tables that follow, it will be seen that, with a fundamental circular curve whose maximum ordinate is 32 to 33 and whose surface chord is 2,200 to 2,400 feet, we have been able to insert a term representing very closely the middle ground. A slight bend can be traced in Section XXVIII, which becomes marked in XXXI.

Between the sections just commented upon occurs a 17 feet shoal, which, I think, must be an obstacle which resists the scouring action of the stream. This, and the recently extended solid structures, embankments, &c., on the Pennsylvania shore, render the region quite unmanageable under our method of analysis—the effect of the bend being much obscured. I give a computation by my method of Section XXXIV, and a theoretical curve by general formula for the mean of Sections XXXII, XXXIII, and XXXIV.

SECTION XXXIV.

Distance x	Observed depth.	Computed depth.	Difference.	Remarks.
Feet.	*Feet.*	*Feet.*	*Feet.*	
1, 100	0. 0	0. 0	....	Pennsylvania side.
1, 000	5. 0	11. 8	− 6. 8	
900	20. 0	19. 5	+ 0. 5	
800	24. 9	23. 9	+ 1. 0	
700	26. 0	25. 9	+ 0. 1	*Feet.*
600	25. 5	26. 3	− 0. 8	Mean depth of observed section 22. 6
500	27. 5	25. 7	+ 1. 8	Mean depth of computed section ... 25. 1
400	25. 5	24. 7	+ 0. 8	Mean chord of computed section ... 1, 540
300	24. 0	23. 8	+ 0. 2	Mean chord of observed section 1, 624
200	23. 5	23. 2	+ 0. 3	*Sq. feet.*
100	21. 2	23. 2	− 2. 0	Area of observed section 56, 500
0	21. 0	24. 0	− 3. 0	Area of computed section 55, 596
100	23. 2	25. 4	− 2. 2	*Feet.*
200	28. 2	27. 5	+ 0. 7	Mid-area of observed section .... + 204
300	31. 6	30. 0	+ 1. 6	Mid-area of computed section ...... + 150
400	34. 8	32. 5	+ 2. 3	
500	34. 8	34. 7	+ 0. 1	
600	34. 2	35. 9	− 1. 7	
700	33. 7	35. 6	− 1. 9	The plane of reference is low-water.
800	31. 6	33. 0	− 1. 4	
900	28. 0	27. 2	+ 0. 8	
1, 000	18. 0	17. 2	+ 0. 8	
1, 100	12. 7	2. 0	+10. 7	New Jersey side.
1, 200	6. 2	....	....	
1, 300	6. 0	....	....	
1, 400	0. 0	....	....	

$$y = 32\left(\frac{1150^2 - x^2}{1150^2}\right) + 0.00355\left(\frac{1150^2 - x^2}{1150^3}\right)2x + \left[20\,\frac{4x^2(1150^2 - x^2)}{1150^4} - 8\right]$$

MEAN OF SECTIONS XXXII, XXXIII, AND XXXIV.

Distance x	Observed depth.	Computed depth.	Difference.	Remarks.
Feet.	*Feet.*	*Feet.*	*Feet.*	
1, 200	2. 5	6. 0	−3. 5	Pennsylvania side.
1, 100	9. 1	17. 2	−8. 1	
1, 000	23. 1	23. 8	−0. 7	
900	28. 3	27. 6	+0. 7	
800	30. 0	29. 0	+1. 0	*Feet.*
700	29. 2	29. 6	−0. 4	Mean depth of observed section 21.2
600	27. 3	28. 5	−1. 2	Mean depth of computed section...... 26.0
500	25. 5	27. 2	−1. 7	Mean chord of computed section...... 1.656
400	24. 5	26. 2	−1. 7	Mean chord of observed section 1,644
300	24. 5	25. 5	−1. 0	*Sq. feet.*
200	25. 1	25. 2	−0. 1	Area of observed section.............. 65,790
100	27. 0	25. 8	+1. 2	Area of computed section............. 62,471
0	29. 3	27. 3	+2. 0	
100	31. 9	29. 2	+2. 7	*Feet.*
200	35. 1	32. 0	+3. 1	Mid-area of computed section......... + 33
300	37. 8	34. 9	+2. 9	Mid-area of observed section.......... + 22
400	39. 4	37. 0	+2. 4	
500	40. 0	38. 2	+1. 8	
600	39. 6	38. 5	+1. 1	The plane of reference is low-water.
700	37. 4	36. 8	+0. 6	
800	32. 3	31. 6	+0. 7	
900	21. 6	22. 5	−0. 9	
1, 000	13. 9	9. 7	+4. 2	New Jersey side.
1, 100	7. 4	−10. 9	....	
1, 200	6. 6	....	....	
1, 300	3. 4	....	....	
1, 400	2. 8	....	....	

For the simple fitting of a smooth curve to the observed profile the following formula is used:

$$y = 27.2 + \frac{25689x + 48.487x^2 - 0.08578x^3 - 0.00006389x^4}{1200^2}$$

SECTION XXXI.

Distance x	a $32\frac{1100^2-x^2}{1100^2}$	b $25\frac{4x^2(1100^2-x^2)}{1100^4}-11$	c $.00593\frac{1100^2-x^2}{1100^2}2x$	d $a+b+c$ Computed depth y	e Observed depth.	Difference $e-d$	Remarks.
Feet.	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	*Feet.*	
1,100	0.0	−11.0	0.0	−11.0	....	....	Pennsylvania side.
1,050	2.9	− 2.9	−1.1	− 4.0	0.0	+ 4.0	
1,000	5.6	+ 3.3	−2.1	+ 6.8	9.1	+ 2.3	
900	10.6	11.1	−3.5	18.2	21.0	+ 2.8	
800	15.1	13.9	−4.5	24.5	25.0	+ 0.5	
700	19.0	13.1	−4.9	27.2	26.5	− 0.7	
600	22.5	9.9	−5.0	27.4	27.2	− 0.2	
500	25.4	5.4	−4.7	26.1	24.8	− 1.3	
400	27.8	0.5	−4.1	24.2	22.8	− 1.4	
300	29.6	− 4.1	−3.3	22.2	22.6	+ 0.4	
200	30.9	− 7.8	−2.3	20.8	22.0	+ 1.2	
100	31.7	10.2	−1.2	20.3	21.7	+ 1.4	
0	32.0	−11.0	0.0	21.0	21.7	+ 0.7	Formula of computed curve, $y=32\frac{1100^2-x^2}{1100^2}+25\frac{4x^2(1100^2-x^2)}{1100^4}-11+.00593\frac{1100^2-x^2}{1100^2}2x$
100	31.7	−10.2	+1.2	22.7	23.5	+ 0.8	
200	30.9	− 7.8	+2.3	25.4	25.0	− 0.4	
300	29.6	− 4.1	+3.3	28.8	26.3	− 2.5	
400	27.8	+ 0.5	+4.1	32.4	29.5	− 2.9	
500	25.4	5.4	+4.7	35.5	32.1	− 3.4	
600	22.5	9.9	+5.0	37.4	37.0	− 0.4	
700	19.0	13.1	+4.9	37.0	38.0	+ 1.0	
800	15.1	13.9	+4.5	33.5	36.0	− 2.5	
900	10.6	11.1	+3.5	25.2	21.5	− 3.7	
1,000	5.6	3.3	+2.1	11.0	5.4	− 5.6	
1,100	0.0	−11.0	0.0	−11.0	2.1	+13.1	New Jersey side.

Mean depth of observed section (at mean low-water) .. 23.5 feet.
Mean depth of computed section .... 23.8 feet.
Mean chord of computed section .... 1,145 feet.
Mean chord of observed section .... 1,108 feet.
Area of observed section .... 51,760 sq. feet.
Area of computed section .... 52,315 sq. feet
Mid-area of computed section .... +164 feet.
Mid-area of observed section .... +121 feet.

SECTION XXVIII.

FIRST COMPUTATION.

Distance x	Observed depth.	Computed depth y	Difference.	Remarks.	
Feet.	*Feet.*	*Feet.*	*Feet.*		
1,150	2.7	0.0	+ 2.7	Pennsylvania side.	
1,100	7.8	9.6	− 1.8		
1,000	18.4	19.2	− 0.8		
900	21.5	24.0	− 3.1		*Feet.*
800	25.0	27.4	− 2.4	Mean depth of observed section	22.23
700	28.2	28.0	+ 0.2	Mean depth of computed section	22.24
600	28.5	27.6	+ 0.9	Mean chord of computed section	1,841
500	28.2	26.4	+ 1.8	Mean chord of observed section	1,743
400	27.0	24.7	+ 2.3		*Sq. feet.*
300	25.6	23.4	+ 2.2	Area of observed section	54,707
200	24.1	22.0	+ 2.1	Area of computed section	55,555
100	22.3	21.5	+ 0.8		*Feet.*
0	21.6	21.5	+ 0.1	Mid-area of computed section	+ 57
100	20.8	22.1	− 1.3	Mid-area of observed section	+ 34
200	22.5	23.4	− 0.9	Formula for computed curve:	
300	25.0	25.4	− 0.4	$y=\frac{21.5}{1150}\sqrt{1150^2-x^2}+14\frac{4x^2(1150^2-x^2)}{1150^4}+2\frac{2x\sqrt{1150^2-x^2}}{1150^2}$	
400	26.4	27.3	− 0.9		
500	29.0	29.4	− 0.4		
600	30.6	31.2	− 0.6		
700	31.6	32.0	− 0.4		
800	31.6	31.4	+ 0.2		
900	27.3	28.4	− 1.1		
1,000	18.0	22.6	− 4.6		
1,100	8.7	11.8	− 3.1		
1,150	3.5	0.0	+ 3.5	New Jersey side.	

SECTION XXVIII—Continued.

SECOND COMPUTATION.

Distance x	Observed depth.	Computed depth y	Difference.	Remarks.
Feet.	*Feet.*	*Feet.*	*Feet.*	
1,200	2.7	0.0	+ 2.7	New Jersey shore.
1,100	9.0	8.2	+ 0.8	
1,000	18.0	15.9	+ 2.1	
900	27.5	22.2	+ 5.3	
800	31.5	26.8	+ 4.7	Formula for computed depth:
700	31.5	29.7	+ 1.8	$y = 32 \cos x + \sin 2x - 10 \cos 3x$
600	30.6	30.7	− 0.1	
500	29.0	30.2	− 1.2	
400	26.4	28.5	− 2.1	
300	25.0	26.5	− 1.5	
200	22.5	24.3	− 1.8	*Feet.*
100	20.8	22.7	− 1.9	Mean depth of observed section 23.02
0	21.6	22.0	− 0.4	Mean depth of computed section.......................... 22.45
100	22.3	22.3	0.0	Mean chord of computed section.......................... 1,753
200	24.0	23.3	+ 0.7	Mean chord of observed section 1,793
300	25.7	25.1	+ 0.6	*Sq. feet.*
400	27.1	26.9	+ 0.2	Area, at low-water, of observed section 55,260
500	28.2	28.3	− 0.1	Area, at low-water, of computed section.................. 53,880
600	28.5	28.7	− 0.2	*Feet.*
700	28.2	27.8	+ 0.4	Mid-area of computed section............................. − 93
800	25.0	25.2	− 0.2	Mid-area of observed section............................. − 98
900	21.5	20.8	+ 0.7	
1,000	18.3	14.9	+ 3.4	
1,100	9.0	7.8	+ 1.2	
1,200	0.0	0.0	0.0	Pennsylvania shore.

THIRD COMPUTATION.

1,200	2.5	−11.0	13.5	New Jersey side.
1,100	9.0	6.2	+ 2.8	
1,000	18.0	17.8	+ 0.2	
900	27.5	25.4	+ 2.1	Formula for computed curve:
800	31.6	29.1	+ 2.5	$y = 33 \frac{1200^2 - x^2}{1200^2} + 22 \frac{4x^2 (1200^2 - x^2)}{1200^4} - 11$
700	31.6	30.6	+ 1.0	
600	30.6	30.3	+ 0.3	
500	29.0	28.6	+ 0.4	
400	26.5	26.9	− 0.4	
300	25.0	25.0	0.0	
200	22.4	23.5	− 1.1	
100	20.8	22.4	− 1.6	
0	21.7	22.0	− 0.3	
100	22.3	22.4	− 0.1	
200	24.0	23.5	+ 0.5	
300	25.6	25.0	+ 0.6	
400	27.1	26.9	+ 0.2	
500	28.2	28.6	− 0.4	
600	28.4	30.3	− 1.9	
700	28.2	30.6	− 2.4	
800	25.0	29.1	− 4.1	
900	21.5	25.4	− 3.9	
1,000	18.3	17.8	+ 0.5	
1,100	10.0	6.2	+ 3.8	
1,200	0.0	−11.0	+11.0	Pennsylvania side.

The frontage of Philadelphia from Section XXX (just above Greenwich Point Salt-works) to a point near Section XIII, opposite Cooper's Point, lies on the arc of a circle of 16,200 feet radius for a chord distance 16,498 feet. Just at the end of this curve a sharper turn occurs, which is succeeded for some distance beyond by a nearly straight line of frontage. In the neighborhood where the greatest bend occurs we have large values for our coefficient of the second term in our formula. Sections XIV and XVI lie in this neighborhood, and each gives 15 as the bend coefficient, which is the largest that we have used. From each of these sections considerable portions have been cut off by wharves; but it will be readily admitted that the computed curve must restore something like the original slopes over which the wharves were built, if these structures have not greatly changed the channel.

Although these sections are about 1,400 feet apart, the formulas that satisfy them are nearly the same, viz: $y = 29 \sin a + 15 \sin 2a + 10 \sin 3a - 3$ for Section XIV, and $y = 29 \sin a + 15 \sin 2a + 15 \sin 3a - 2$ for Section XVI; the third term, due to the shoal extending from Windmill Island, being larger as that island is approached. For each of these expressions, $x = 1250 \cos. a$; but the resulting surface width is less than 2,500 feet.

It is not a purpose of this report to comment upon what structures may be charged with being encroachments upon the stream, because, at the present stage of the inquiry, we are not fully prepared to distinguish them individually, but examples are given to indicate the manner of this kind of inquiry. Until the new soundings over the whole river bed are compared with those of earlier surveys, we cannot properly connect cause and effect.

From the examples which follow it will be seen that the depth of the fundamental curve has been found to be considerably smaller at Sections XIV and XVI than at the straight reach near Gloucester.

SECTION XVI.

Distance x	Observed depth.	Computed depth.	Difference.	Remarks.
Feet.	*Feet.*	*Feet.*	*Feet.*	
1,250	Wharf.	− 2.0	....	
1,200	Wharf.	25.2	....	Formula for computed curve: $29 \sin a + 15 \sin 2a + 15 \sin 3a - 2$
1,100	39.5	39.4	+0.1	$\cos a = \frac{x}{1250}$
1,000	46.8	44.5	+2.3	
900	46.0	44.6	+1.4	
800	47.0	42.0	+5.0	
700	42.0	39.0	+3.0	
600	36.0	35.0	+1.0	
500	30.0	30.6	−0.6	
400	26.0	25.6	+0.4	
300	22.0	22.2	−0.2	
200	19.5	18.4	+1.1	*Feet.*
100	14.0	14.6	−0.6	Mean depth of computed curve...... 20.5
0	4.0	12.0	−8.0	Mean chord of computed curve...... 1,172
100	3.5	10.0	−6.5	*Sq. feet.*
200	5.0	9.0	−4.0	Area of computed curve............. 51,390
300	6.0	8.0	−2.0	*Feet.*
400	8.5	8.5	0	Mid-area of computed curve......... − 581
500	10.3	8.6	+1.7	
600	11.8	9.8	+2.0	
700	12.6	11.0	+1.6	
800	13.0	12.6	+0.4	
900	15.5	14.0	+1.5	
1,000	13.6	15.7	−2.1	
1,100	15.0	14.2	+0.8	
1,180	Wharf.	....	....	
1,200	....	9.0	....	
1,250	....	− 2.0	....	

SECTION XIV.

Distance x	Observed depth.	Computed depth.	Difference.	Remarks.
Feet.	*Feet.*	*Feet.*	*Feet.*	
1,250	Wharf.	− 3.0	....	
1,200	26.3	20.0	+6.3	Formula for computed curve:
1,100	32.0	33.0	−1.0	$y = 29 \sin a + 15 \sin 2a + 10 \sin 3a - 3$
1,000	37.0	39.0	−2.0	
900	41.0	39.0	+2.0	
800	41.5	39.0	+2.5	
700	36.0	37.0	−1.0	
600	30.5	34.0	−3.5	
500	25.0	31.5	−6.5	*Feet.*
400	22.7	27.0	−4.3	Mean depth of computed curve...... 19.4
300	22.7	25.0	−2.3	Mean chord of computed curve...... 1,256
200	23.6	21.0	+2.6	
100	19.2	18.0	+1.2	*Sq. feet.*
0	19.5	16.0	+3.5	Area of computed curve............. 48,590
100	18.6	13.5	+5.1	Mid area of computed curve........525 feet
200	13.2	11.5	+1.7	
300	7.8	11.0	−3.2	
400	8.0	9.5	−1.5	
500	8.7	9.5	−0.8	
600	9.0	8.8	+0.2	
700	8.4	9.0	−0.6	
800	7.3	9.6	−2.3	
900	6.5	9.0	−2.5	
1,000	4.0	10.0	−6.0	
1,100	4.2	8.0	−3.8	
1,200	4.0	4.0	0.0	
1,250	2.8	− 3.0	+5.8	

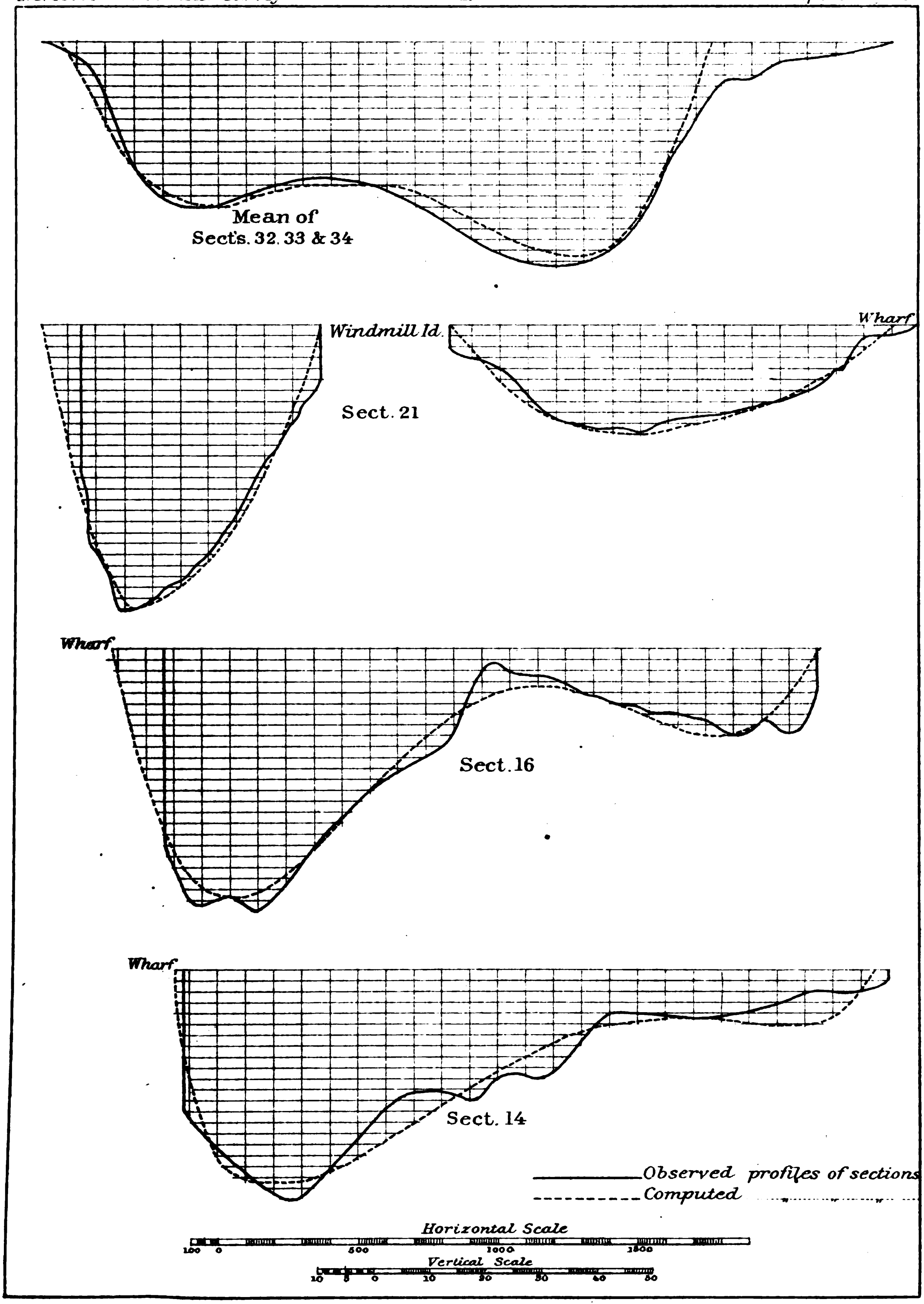
One conclusion from the foregoing must be that the fundamental curve, which had a depth of thirty-six feet at Gloucester, has shrunk to one of thirty feet at Kensington.

At Windmill Island we have the river completely separated into two channels while preserving something like its characteristic form of section as a whole. One can easily see how Section XVI degenerates into Section XXI as the middle ground rises to be an island. It is worthy of notice that in the eastern channel, between Windmill Island and the New Jersey shore, the deep water is nearer the western shore, although the local curvature is concave to the eastward; showing that these two channels reflect in their mold the centrifugal force of the great curve just referred to. (See Diagram D.)

SECTION XXI.

EAST CHANNEL.

Distance x	Observed depth L. W.	Computed depth y	Difference.	Remarks.
Feet.	*Feet.*	*Feet.*	*Feet.*	
....	4.0	0.0	+4.0	Windmill Island side.
700	6.0	4.6	+1.4	
600	7.5	9.8	−2.3	Formula for computed curve:
500	12.6	14.0	−1.4	$y = 21.7 \cos x - 4 \cos 2(x - 100) + 1$
400	17.2	17.0	+0.2	
300	18.2	18.7	−0.5	Mean depth of observed section... 12.2 feet.
200	19.6	19.7	−0.1	Mean depth of computed section.. 12.3 feet.
100	19.2	18.6	+0.4	Area of computed section......... 20,922 square feet.
0	17.3	19.1	−1.8	Area of observed section.......... 20,395 square feet.
100	17.0	18.1	−1.1	Mean chord of observed section... 1,064 feet.
200	16.3	16.9	−0.6	Mean chord of computed section.. 1,086 feet.
300	15.0	15.4	−0.4	Mid-area of computed section..... — 36 feet.
400	13.6	13.7	−0.1	Mid-area of observed section...... — 45 feet.
500	12.0	11.2	+0.8	
600	8.2	8.3	−0.1	
700	2.7	4.6	−1.9	
800	1.7	0.1	+1.6	New Jersey side.


Mean of
Sect's. 32. 33 & 34
Windmill Id.
Wharf
Sect. 21
Wharf
Sect. 16
Wharf
Sect. 14
Observed profiles of sections
Computed
Horizontal Scale
100 0 500 1000 1500
Vertical Scale
10 5 0 10 20 30 40 50

SECTION XXI—Continued.

WEST CHANNEL.

Distance. x	Observed depth L. W.	Computed depth y	Difference.	Remarks.
Feet.	*Feet.*	*Feet.*	*Feet.*	
500	17.0	0.0	+17.0	Pennsylvania side in slip.
400	20.6	23.3	− 2.7	Formula for computed curve:
300	41.0	40.1	+ 0.9	$y = 50.5 \cos x + 7 \sin 2x - 4 \cos 3x$
200	51.2	48.6	+ 2.6	Mean depth of observed section... 30.5 feet.
100	48.7	49.8	− 1.1	Mean depth of computed section.. 29.8 feet.
0	45.0	46.5	− 1.5	Area of computed section......... 32,750 square feet.
100	40.3	41.6	− 1.3	Area of observed section.......... 33,530 square feet.
200	33.0	35.4	− 2.4	Mean chord of observed section... 677 feet.
300	25.0	26.9	− 1.9	Mean chord of computed section.. 631 feet.
400	17.0	15.3	+ 1.7	Mid-area of computed section..... + 46 feet.
500	10.0	0.0	+10.0	Mid-area of observed section...... − 55 feet.
				Windmill Island side.

Windmill Island has evidently been an obstruction to which the river has accommodated itself, and is not a middle ground deposited voluntarily by the stream. It would be possible, from the elements of this and other sections, to predict the form of section that would accommodate the stream in the absence of this obstruction and one that the stream would preserve.

Other examples are also given of the manner of smoothing out the observed profile by determining a formula for it. And here I beg leave to repeat that aside from any theory of the *essential* form of section as represented by formulæ, the advantage of ascertaining a fitting curve for each section lies in determining the essential elements under *existing* conditions, *i. e.*, the essential surface width, the positions of the *virtual shore lines* and thalweg, the mean depth and thalweg depth, from all the soundings made, while at the same time eliminating all mere accidents and errors of observations. The formula furnishes the type form about which any sectional profile is oscillating and toward which it tends.

SECTION XLIV.

Distance x	Observed depth ½ tide.	Computed depth y	Difference.	Remarks.
Feet.	*Feet.*	*Feet.*	*Feet.*	
1,350	3.5	0.0	—3.5	Pennsylvania side.
1,300	7.8	13.7	—5.9	
1,200	18.5	22.0	—3.5	
1,100	25.9	26.2	—0.3	
1,000	27.5	28.7	—1.2	
900	29.5	30.2	—0.7	
800	30.2	31.1	—0.9	Observed curve from original plotting.
700	30.6	31.4	—0.8	Fomula for computed curve:
600	31.1	31.5	—0.4	$y = \frac{31}{1350}\sqrt{1350^2 - x^2} + \frac{0.004\sqrt{1350^2 - x^2}}{1350^2} 4x^2$
500	31.5	31.5	0.0	
400	30.6	31.4	—0.8	*Feet.*
300	29.7	31.2	—1.5	Mean depth of observed section...... 22.5
200	29.7	31.0	—1.3	Mean depth of computed section..... 28.6
100	30.5	31.0	—0.5	Mean chord of computed section..... 2,443
0	31.5	31.0	+0.5	Mean chord of observed section...... 2,301
100	32.0	31.0	+1.0	*Sq. feet.*
200	32.9	31.0	+1.9	Area of observed section.............. 78,690
300	33.5	31.2	+2.3	Area of computed section............. 77,280
400	33.5	31.4	+2.1	*Feet.*
500	33.6	31.5	+2.1	Mid area of computed section......... 0
600	33.5	31.5	+2.0	Mid area of observed section.......... + 72
700	33.5	31.4	+2.1	
800	32.0	31.1	+0.9	
900	30.6	30.2	+0.4	
1,000	28.4	28.7	—0.3	
1,100	21.6	26.2	—4.6	
1,200	13.0	22.0	—9.0	
1,300	8.7	13.7	—5.0	
1,350	8.0	0.0	—8.0	
1,400	7.5	....	....	
1,500	6.2	....	....	
1,600	4.6	....	....	
1,700	3.5	....	....	
1,800	3.3	....	....	
1,900	3.0	....	....	
2,000	1.0	....	....	New Jersey side.

In a future report I hope to show a codification of the pheuomena revealed by the current observations, which present much confusion at present, but which will doubtless be more comprehensible after the hydrographical sheets are compared.

I beg leave to call your attention to a distinction among the tables of this report, which is intentional. Those which appear within the text are illustrative of the new method of treatment, which is an attempted step beyond the original plan of our work represented by the tables placed after the text.

Among the last of the appended tables will be found one entitled "Locus of the center of the cross-section," in which I have had the line passing through the center of gravity (if I may so term it) of every section referred to rectangular co-ordinates and the radii computed by successive differences.

I have purposely avoided making any suggestion, touching *improvements*, in this preliminary report.

This report is accompanied by four "Dynamic charts" (on too elaborate a scale for publication), showing profiles of sections, &c. (from the same data that appear in the annexed tables), plotted upon plans of the river furnished from the surveys of Messrs. McCorkle and Bache, of the Coast and Geodetic Survey.

HENRY MITCHELL,
Coast and Geodetic Survey.

DELAWARE RIVER.

Transverse curves of velocity, and perimeters.

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 1..........	0	0.00	0.55	0.00	− 0.20	Position of origin of section (mean low-water, Pennsylvania shore): Latitude, 39° 59′ 06″.1; longitude, 75° 04′ 24″.2.
	100	1.34	6.15	1.44	+ 5.40	
	200	1.83	11.55	2.06	10.80	
	300	2.11	17.55	2.35	16.80	
	400	2.33	16.25	2.42	15.50	
	500	2.53	13.75	2.48	13.00	
	600	2.74	12.35	2.52	11.60	
	700	2.87	9.65	2.62	8.90	
	800	2.87	9.25	2.76	8.50	
	900	2.77	10.25	2.92	9.50	Area of section: Flood, 51,825 square feet; ebb, 49,922 square feet.
	1,000	2.72	11.85	3.31	11.10	
	1,100	2.67	14.85	3.54	14.10	Mid-area (or distance, from the origin, of a vertical line which divides the section into two equal areas) = 1,590 feet.
	1,200	2.64	23.65	3.57	22.90	
	1,300	2.64	26.05	3.57	25.30	Mid-volume (or distance, from the origin, of a vertical line which divides the volume passing through the section into two equal parts) = flood, 1,613 feet; ebb, 1,575 feet.
	1,400	2.72	28.35	3.55	27.60	
	1,500	2.83	32.65	3.55	31.90	
	1,600	2.78	37.15	3.45	36.40	
	1,700	2.56	38.95	3.22	38.20	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,800	2.67	40.25	3.16	39.50	
	1,900	3.05	42.25	3.09	41.50	
	2,000	3.11	37.05	2.95	36.30	
	2,100	3.08	29.25	2.77	28.50	
	2,200	2.96	21.55	2.53	20.80	
	2,300	2.68	16.05	2.14	15.30	
	2,400	2.05	8.85	1.53	8.10	
	2,500	0.00	3.05	0.00	2.30	
	2,550	....	0.75	....	0.00	Position of end of section: Latitude, 39° 58′ 48″.9; longitude, 75° 04′ 00″.4.
Section 2..........	19	0.00	0.00	0.00	0.00	Position of origin of section: Latitude, 39° 58′ 53″.6; longitude, 75° 04′ 52″.6.
	20	0.00	2.15	0.00	1.40	
	100	1.77	5.15	0.46	4.40	
	200	2.25	5.45	1.19	4.70	
	300	2.36	6.75	1.65	6.00	
	400	2.42	8.95	1.70	8.20	
	500	2.48	10.35	1.89	9.60	
	600	2.55	10.35	1.95	9.60	
	700	2.60	9.95	2.03	9.20	
	800	2.76	11.25	2.09	10.50	
	900	2.92	16.25	2.20	15.50	
	1,000	3.08	21.25	2.31	20.50	
	1,100	3.15	23.25	2.43	22.50	
	1,200	3.17	23.05	2.51	22.30	
	1,300	3.17	21.75	2.51	21.00	
	1,400	3.13	19.55	2.45	18.80	
	1,500	3.01	17.15	2.35	16.40	
	1,600	2.87	16.55	2.31	15.80	
	1,700	2.67	15.05	2.25	14.30	
	1,800	2.50	11.15	2.20	10.40	
	1,900	1.93	7.35	2.56	6.60	
	2,000	1.76	7.75	2.88	7.00	
	2,100	1.79	11.45	2.93	10.70	
	2,200	2.15	16.55	3.02	15.80	
	2,300	2.39	23.05	3.10	22.30	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 2—Cont'd.	2, 400	2. 35	26. 15	3. 13	25. 40	
	2, 500	2. 20	28. 35	3. 10	27. 60	
	2, 600	2. 11	25. 25	3. 06	24. 50	
	2, 700	2. 05	20. 75	3. 05	20. 00	
	2, 800	2. 02	17. 95	3. 02	17. 20	
	2, 900	2. 01	17. 25	3. 00	16. 50	
	3, 000	1. 99	17. 05	3. 00	16. 30	
	3, 100	1. 95	17. 15	2. 90	16. 40	
	3, 200	1. 84	17. 25	2. 69	16. 50	
	3, 300	1. 70	17. 25	2. 44	16. 50	
	3, 400	1. 59	17. 05	2. 22	16. 30	
	3, 500	1. 45	17. 05	2. 06	16 30	Area of section: Flood, 76, 172 square feet; ebb, 71,822
	3, 600	1. 32	16. 85	1. 90	16. 10	square feet.
	3, 700	1. 17	16. 55	1. 75	15. 80	
	3, 800	1. 04	17. 75	1. 58	17. 00	Mid-area = flood, 2,575 feet; ebb, 2,585 feet.
	3, 900	0. 90	16. 85	1. 38	16. 10	
	4, 000	0. 74	15. 85	1. 19	15. 10	
	4, 100	0. 58	14. 75	0. 95	14. 00	Mid-volume: Flood, 1,841 feet; ebb, 2,404 feet.
	4, 200	0. 41	10. 75	0. 63	10. 00	
	4, 300	0. 23	6. 65	0. 33	5. 90	
	4, 400	0. 00	6. 25	0. 00	5. 50	Plane of reference for depths above mean low-water:
	4, 500	0. 00	6. 05	0. 00	5. 30	Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	4, 600	0. 00	5. 95	0. 00	5. 20	
	4, 700	0. 00	5. 75	0. 00	5. 00	
	4, 800	0. 00	5. 85	0, 00	5. 10	
	4, 900	0. 00	5. 85	0. 00	5. 10	
	5, 000	0. 00	5. 85	0, 00	5. 10	
	5, 100	0. 00	5. 85	0. 00	5. 10	
	5, 200	0. 00	6. 05	0. 00	5. 30	
	5, 300	0. 00	6. 15	0. 00	5. 40	
	5, 400	0. 00	6. 15	0. 00	5. 40	
	5, 500	0. 00	6. 55	0. 00	5. 80	
	5, 600	0. 00	6. 65	0. 00	5. 90	
	5, 700	0. 00	6. 55	0. 00	5. 80	
	5, 800	0. 00	2. 75	0. 00	2. 00	Position of end of section: Latitude, 39° 58′ 04″.8; longi-
	5, 820	0. 00	0. 75	0. 00	0. 00	tude, 75° 04′ 12″.6.
Section 3	14	0. 00	0. 00	0. 00	0. 00	Position of origin of section: Latitude, 39° 58′ 46″.1; longi-
	15	0. 00	4. 10	0. 00	3. 35	tude, 75° 05′ 22″.9.
	100	0. 00	4. 25	0. 00	3. 50	
	200	0. 45	4. 65	0. 00	3. 90	
	300	0. 82	5. 55	0. 25	4. 80	
	400	1. 13	6. 55	0. 52	5. 80	
	500	1. 41	7. 25	0. 82	6. 50	
	600	1. 61	7. 85	1. 05	7. 10	
	700	1. 74	8. 85	1. 32	8. 10	Mid-area = flood, 2,664 feet; ebb, 2,649 feet.
	800	1. 86	10. 95	1. 60	10. 20	
	900	1. 97	15. 55	2. 12	14. 80	
	1, 000	2. 15	25. 25	2. 52	24. 50	Mid-volume: Flood, 2,254 feet; ebb, 2,625 feet.
	1, 100	2. 36	29. 95	2. 72	29. 20	
	1, 200	2. 60	31. 05	2. 81	30. 30	
	1, 300	2. 78	30. 65	2. 81	29. 90	
	1, 400	2. 83	29. 15	2. 42	28. 40	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood.		Ebb.		Remarks.
		Maximum velocity (reduced to mean tide).	Depth (at time of maximum velocity).	Maximum velocity (reduced to mean tide).	Depth (at time of maximum velocity).	
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 3—Cont'd.	1, 500	2. 85	27. 65	1. 85	26. 90	
	1, 600	2. 82	24. 15	1. 79	26. 40	
	1, 700	2. 71	15. 75	1. 82	15. 00	
	1, 800	2. 51	6. 55	1. 85	5. 80	
	1, 900	2. 26	6. 65	1. 89	5. 90	
	2, 000	1. 99	7. 35	1. 94	6. 60	
	2, 100	1. 79	9. 25	2. 02	8. 50	
	2, 200	1. 76	11. 35	2. 20	10. 60	
	2, 300	1. 82	14. 25	2. 38	13. 50	
	2, 400	1. 94	15. 85	2. 56	15. 10	
	2, 500	2. 08	15. 95	2. 71	15. 20	
	2, 600	2. 17	13. 45	2. 79	12. 70	
	2, 700	2. 17	13. 25	2. 79	12. 50	
	2, 800	2. 17	11. 45	2. 79	10. 70	
	2, 900	2. 17	8. 95	2. 79	8. 20	
	3, 000	2. 17	9. 25	2. 79	8. 50	
	3, 100	2. 17	11. 25	2. 79	10. 50	
	3, 200	2. 17	13. 35	2. 79	12. 60	
	3, 300	2. 18	14. 15	2. 79	13. 40	
	3, 400	2. 18	14. 65	2. 65	13. 90	Area of section: Flood, 77,806 square feet; ebb, 73,447 square feet.
	3, 500	2. 19	15. 25	2. 53	14. 50	
	3, 600	2. 19	16. 95	2. 48	16. 20	
	3, 700	2. 19	18. 35	2. 48	17. 60	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	3, 800	2. 19	19. 05	2. 49	18. 30	
	3, 900	2. 19	20. 55	2. 52	19. 80	
	4, 000	2. 17	21. 35	2. 60	20. 60	
	4, 100	2. 11	21. 05	2. 62	20. 30	
	4, 200	1. 98	20. 95	2. 60	20. 20	
	4, 300	1. 85	19. 95	2. 55	19. 20	
	4, 400	1. 65	17. 25	2. 41	16. 50	
	4, 500	1. 35	14. 85	2. 24	14. 10	
	4, 600	1. 01	9. 35	1. 92	8. 60	
	4, 700	0. 63	6. 55	1. 35	5. 80	
	4, 800	0. 32	6. 25	0. 77	5. 50	
	4, 900	0. 00	6. 25	0. 00	5. 50	
	5, 000	0. 00	6. 25	0. 00	5. 50	
	5, 100	0. 00	6. 25	0. 00	5. 50	
	5, 200	0. 00	6. 25	0. 00	5. 50	
	5, 300	0. 00	6. 65	0. 00	5. 90	
	5, 400	0. 00	7. 05	0. 00	6. 30	
	5, 500	0. 00	6. 95	0. 00	6. 20	
	5, 600	0. 00	6. 95	0. 00	6. 20	
	5, 700	0. 00	6. 85	0. 00	6. 10	
	5, 800	0. 00	5. 85	0. 00	5. 10	
	5, 900	0. 00	3. 25	0. 00	2. 50	Position of end of section: Latitude, 39° 57′ 50″.7; longitude, 75° 04′ 58″.4.
Section 4 (north channel).	19	0. 00	0. 00	0. 00	0. 00	Position of origin of section: Latitude, 39° 58′ 41″.5; longitude, 75° 05′ 50″.0.
	30	0. 00	3. 25	0. 00	2. 50	
	100	0. 00	3. 25	0. 00	2. 50	
	200	0. 00	3. 25	0. 00	2. 50	
	300	0. 00	3. 25	0. 00	2. 50	
	400	0. 00	3. 75	0. 00	3. 00	
	500	0. 60	5. 25	0. 80	4. 50	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 4 (north channel)—Continued.	600	1.48	14.55	1.43	13.80	
	700	1.92	17.65	1.68	16.90	
	800	2.22	16.75	1.95	16.00	
	900	2.41	17.55	2.26	16.80	
	1,000	2.49	17.85	2.73	17.10	
	1,100	2.43	20.15	3.01	19.40	Area of section: Flood, 40,925 square feet; ebb, 38,795 square feet.
	1,200	2.42	21.75	3.20	21.00	
	1,300	2.46	22.35	3.27	21.60	
	1,400	2.52	25.25	3.30	24.50	Mid-area = flood, 1,488; ebb, 1,489.
	1,500	2.61	28.95	3.09	28.20	
	1,600	2.65	28.95	2.66	28.20	
	1,700	2.49	24.05	2.11	23.30	Mid-volume: Flood, 1,516 feet; ebb, 1,423 feet.
	1,800	2.12	14.05	1.40	13.30	
	1,900	2.06	9.25	1.12	8.50	
	2,000	2.11	8.95	1.09	8.20	
	2,100	2.18	9.65	1.15	8.90	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	2,200	2.23	11.25	1.26	10.50	
	2,300	2.24	14.85	1.59	14.10	
	2,400	2.32	16.55	2.01	15.80	
	2,500	2.49	17.95	2.10	17.20	
	2,600	2.47	16.45	1.87	15.70	
	2,700	1.92	9.45	1.36	8.70	
	2,800	0.00	5.05	0.00	4.30	[tude, 75° 05′ 38″.4.
	2,860	0.00	2.75	0.00	2.00	Position of end of section: Latitude, 39° 58′ 14″.5; longi-
Section 5 (north channel.)	89	0.00	0.00	0.00	0.00	Position of origin of section: Latitude, 39° 58′ 35″.7; longitude, 75° 06′ 17″.9.
	90	0.00	3.35	0.00	2.60	
	100	0.00	3.85	0.00	3.10	
	200	0.00	7.85	0.00	7.10	
	300	0.76	10.95	0.66	10.20	
	400	1.41	15.25	1.17	14.50	
	500	1.71	15.25	1.51	14.50	
	600	1.77	14.75	1.67	14.00	
	700	1.78	14.95	1.73	14.20	Area of section: Flood, 46,251 square feet; ebb, 44,256 square feet.
	800	1.83	16.95	1.76	16.20	
	900	1.99	20.25	1.88	19.50	
	1,000	2.06	23.15	2.01	22.40	
	1,100	2.01	23.25	2.16	22.50	Mid-area = flood, 1,425 feet; ebb, 1,425 feet.
	1,200	2.09	22.85	2.34	22.10	
	1,300	2.19	25.95	2.57	25.20	
	1,400	2.27	23.45	2.64	22.70	
	1,500	2.30	21.55	2.67	20.80	Mid-volume: Flood, 1,512 feet; ebb, 1,462 feet.
	1,600	2.32	20.95	2.61	20.20	
	1,700	2.30	20.05	2.45	19.30	
	1,800	2.28	19.75	2.22	19.00	
	1,900	2.29	18.75	1.95	18.00	
	2,000	2.30	17.65	1.75	16.90	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	2,100	2.32	18.65	1.71	17.90	
	2,200	2.31	20.25	1.88	19.50	
	2 300	2.25	21.25	1.89	20.50	
	2,400	2.11	20.35	1.87	19.60	
	2,500	1.92	17.25	1.75	16.50	
	2,600	1.39	5.45	1.36	4.70	
	2,700	0.00	3.55	0.00	2.80	[tude, 75° 06′ 06″.4.
	2,750	0.00	3.25	0.00	2.50	Position of end of section: Latitude, 39° 58′ 09″.7; longi-

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 6 (north channel).	0	0.00	17.6	0.00	16.8	Position of origin of section (corner of wharf, Pennsylvania side): Latitude, 39° 58′ 24″.7; longitude, 75° 06′ 32″.7.
	100	1.45	19.2	1.88	18.4	
	200	1.76	18.9	1.97	18.1	
	300	1.98	18.8	2.08	18.0	Area of section: Flood, 41,350 square feet; ebb, 39,830 square feet.
	400	2.29	23.4	2.38	22.6	
	500	2.47	25.0	2.34	24.2	
	600	2.41	27.0	2.04	26.2	
	700	2.34	29.8	1.98	29.0	Mid-area = 871 feet.
	800	2.55	29.3	2.13	28.5	
	900	2.80	27.7	2.48	26.9	
	1,000	2.76	26.6	2.61	25.8	
	1,100	2.55	26.5	2.50	25.7	Mid-volume: Flood, 895 feet; ebb, 916 feet.
	1,200	2.34	26.6	2.40	25.8	
	1,300	2.21	26.1	2.36	25.3	
	1,400	2.31	24.2	2.44	23.4	
	1,500	2.35	21.3	2.47	20.5	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,600	2.03	18.5	2.18	17.7	
	1,700	1.53	8.8	1.34	8.0	
	1,800	1.01	3.7	0.23	2.9	
	1,900	0.00	1.7	0.00	0.9	Position of end of section: Latitude, 39° 58′ 07″.2; longitude, 75° 06′ 24″.6.
Section 7 (north channel).	0	0.06	17.4	0.02	16.6	Position of origin of section (end of wharf, Pennsylvania side): Latitude, 39° 58′ 19″.1; longitude, 75° 06′ 50″.5.
	100	0.24	16.8	0.19	16.0	
	200	1.44	19.0	1.25	18.2	
	300	2.23	29.5	2.01	28.7	
	400	2.56	29.4	2.39	28.6	Area of section: Flood, 41,619 square feet; ebb, 40,235 square feet.
	500	2.72	30.7	2.53	29.9	
	600	2.82	30.3	2.33	29.5	
	700	2.63	29.8	2.41	29.0	Mid-area = 792 feet.
	800	2.64	30.4	2.61	29.6	
	900	2.79	30.9	2.66	30.1	
	1,000	2.80	31.4	2.54	30.6	Mid-volume: Flood, 817 feet; ebb, 849 feet.
	1,100	2.60	31.2	2.51	30.4	
	1,200	2.57	30.8	2.66	30.0	
	1,300	2.37	27.8	2.71	27.0	
	1,400	1.83	23.0	2.29	22.2	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,500	0.60	6.8	0.83	6.0	
	1,550	0.00	....	0.00	....	
	1,600	....	4.1	....	3.3	
	1,700	....	3.4	....	2.6	
	1,730	....	3.3	....	2.5	Position of end of section: Latitude, 39° 58′ 03″.5; longitude, 75° 06′ 41″.5.
Section 8 (north channel).	0	0.00	34.6	0.00	33.8	Position of origin of section (entrance of slip, Pennsylvania side): Latitude, 39° 58′ 11″.9; longitude, 75° 07′ 05″.8.
	100	2.38	37.0	2.42	36.2	
	200	2.94	33.4	2.71	32.6	
	300	3.23	31.9	2.95	31.1	
	400	3.38	31.3	2.94	30.5	Area of section: Flood, 33,865 square feet; ebb, 32,785 square feet.
	500	3.37	32.7	3.00	31.9	
	600	3.29	33.3	3.11	32.5	
	700	3.15	30.1	3.18	29.3	Mid-area = 504 feet.
	800	3.08	27.5	3.24	26.7	
	900	3.04	26.8	3.30	26.0	Mid-volume: Flood, 520 feet; ebb, 542 feet.
	1,000	2.87	21.7	3.30	20.9	
	1,100	2.07	6.1	1.68	5.3	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 8 (north channel)—Cont'd.	1,150	0.00	....	0.00	....	Plane of reference for depths above mean low-water:
	1,190	....	4.8	....	4.0	Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,200	....	4.6	....	3.8	
	1,300	....	2.7	....	1.9	[tude, 75° 06′ 57″.3.
	1,350	....	2.6	....	1.8	Position of end of section: Latitude, 39° 57′ 58″.6; longi-
Section 9 (north channel).	0	0.00	22.8	0.00	22.0	Position of origin of section (end of wharf, Pennsylvania side): Latitude, 39° 58′ 02″.8; longitude, 75° 07′ 22″.4.
	100	2.24	37.5	2.37	36.7	
	200	2.63	40.1	2.93	39.3	
	300	3.02	41.0	2.91	40.2	Area of section: Flood, 36,610 square feet; ebb, 35,626 square feet.
	400	3.27	40.2	2.84	39.4	
	500	3.22	38.6	2.90	37.8	Mid-area — 481 feet.
	600	3.09	37.5	2.92	36.7	
	700	2.91	42.8	2.85	42.0	Mid-volume: Flood, 479 feet; ebb, 469 feet.
	800	2.63	27.7	2.74	26.9	
	900	2.39	24.0	2.59	23.2	Plane of reference for depths above mean low-water:
	1,000	2.25	16.3	2.01	15.5	Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,100	1.51	7.6	0.00	6.8	
	1,200	0.31	5.1	0.00	4.3	[tude, 75° 07′ 14″.5.
	1,230	0.00	4.8	0.00	4.0	Position of end of section: Latitude, 39° 57′ 52″.2; longi-
Section 4 (south channel).	0	0.00	4.8	0.00	4.0	Position of origin of section (marsh, Petty's Island): Latitude, 39° 58′ 08″.5; longitude, 75° 05′ 40″.3.
	100	1.66	14.8	2.03	14.0	
	200	2.23	22.4	2.57	21.6	
	300	2.36	22.2	2.53	21.4	
	400	2.48	19.6	2.17	18.8	
	500	2.47	19.7	2.47	18.9	
	600	2.38	22.0	3.06	21.2	
	700	2.23	25.7	3.63	24.9	
	800	2.17	26.9	3.79	26.1	Area of section: Flood, 31,200 square feet; ebb, 29,440 square feet.
	900	2.35	26.0	3.79	25.2	
	1,000	2.33	22.8	3.69	22.0	Mid-area = 777 feet.
	1,100	2.13	18.5	3.30	17.7	
	1,200	1.90	14.8	2.56	14.0	
	1,300	1.75	11.5	1.79	10.7	Mid-volume: Flood, 720 feet; ebb, 758 feet.
	1,400	1.55	9.0	0.83	8.2	
	1,480	....	....	0.00	....	
	1,500	1.28	6.5	....	5.7	Plane of reference for depths above mean low-water:
	1,600	0.98	5.8	....	5.0	Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,700	0.72	5.4	....	4.6	
	1,800	0.49	5.2	....	4.4	
	1,900	0.12	4.3	....	3.5	
	1,980	0.00	....	....	....	
	2,000	....	3.4	....	2.6	
	2,100	....	2.3	....	1.5	
	2,200	....	1.0	....	0.2	[tude, 75° 05′ 27″.7.
	2,230	....	0.0	....	− 0.8	Position of end of section: Latitude, 39° 57′ 48″ 9; longi-
Section 5 (south channel).	0	....	2.0	....	1.2	Position of origin of section (levee, Petty's Island): Latitude, 39° 58′ 00″.8; longitude, 75° 05′ 57″.1.
	100	....	2.8	....	2.0	
	200	....	3.6	....	2.8	
	300	....	4.0	....	3.2	
	400	....	4.3	....	3.5	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood.		Ebb.		Remarks.
		Maximum velocity (reduced to mean tide).	Depth (at time of maximum velocity).	Maximum velocity (reduced to mean tide).	Depth (at time of maximum velocity).	
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 5 (south channel)—Continued.	500	....	4.4	....	3.6	Area of section: Flood, 33,425 square feet; ebb, 31,305
	580	0.00	....	0.00	....	square feet.
	600	0.21	11.8	1.56	11.0	
	700	1.14	16.1	2.53	15.3	
	800	1.57	14.6	2.49	13.8	
	900	1.77	14.6	2.39	13.8	
	1,000	1.95	15.6	2.47	14.8	
	1,100	2.05	19.9	2.79	19.1	
	1,200	2.10	24.3	3.31	23.5	
	1,300	2.22	27.6	3.54	26.8	
	1,400	2.36	26.4	3.51	25.6	Mid-area = 1,361 feet.
	1,500	2.56	23.8	3.36	23.0	
	1,600	2.61	20.8	3.19	20.0	Mid-volume: Flood, 1,449 feet; ebb, 1,358 feet.
	1,700	2.53	20.1	3.02	19.3	
	1,800	2.33	17.4	2.73	16.6	
	1,900	2.23	15.4	2.15	14.6	
	2,000	2.16	12.1	1.29	11.3	Plane of reference for depths above mean low-water:
	2,100	1.95	10.9	0.73	10.1	Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	2,200	1.67	7.6	0.32	6.8	
	2,280	....	....	0.00	....	
	2,300	1.43	5.2	....	4.4	
	2,400	1.17	4.8	....	4.0	
	2,500	0.74	3.5	....	2.7	
	2,580	0.00	....	....	....	
	2,600	....	1.5	....	0.7	
	2,650	....	0.8	....	0.0	
	2,700	....	0.8	....	0.0	[tude, 75° 05′ 39″.6.
	2,740	....	0.0	....	—0.8	Position of end of section: Latitude, 39° 57′ 44″.3; longi-
Section 6 (south channel).	0	....	3.3	....	2.5	Position of origin of section (levee on Petty's Island):
	100	....	3.7	....	2.9	Latitude, 39° 57′ 55″.3; longitude, 75° 06′ 12″.6.
	200	....	4.1	....	3.3	
	300	....	4.4	....	3.6	
	400	....	5.0	....	4.2	
	440	0.00	....	0.00	....	
	450	0.10	8.1	1.48	7.3	
	500	0.46	14.2	2.60	13.4	
	600	0.95	10.8	2.72	10.0	
	700	1.29	9.4	2.56	8.6	
	800	1.52	9.3	2.57	8.5	
	900	1.70	10.4	2.76	9.6	
	1,000	1.90	15.2	3.38	14.4	
	1,100	2.09	23.4	3.67	22.6	
	1,200	2.27	28.8	3.37	28.0	
	1,300	2.53	29.8	2.77	29.0	Area of section: Flood, 32,950 square feet; ebb, 31,070
	1,400	2.70	30.5	2.63	29.7	square feet.
	1,500	2.67	31.4	2.61	30.6	Mid-area = 1,354 feet.
	1,600	2.46	25.2	2.61	24.4	
	1,700	2.17	20.8	2.48	20.0	
	1,800	2.06	18.4	2.24	17.6	Mid-volume: Flood, 1,413 feet; ebb, 1,302 feet.
	1,900	1.97	15.0	1.77	14.2	
	2,000	1.75	12.0	1.17	11.2	
	2,100	1.45	6.1	0.74	5.3	Plane of reference for depths above mean low-water:
	2,200	0.92	4.4	0.06	3.6	Maximum flood, 4.75 feet; maximum ebb, 4 feet.

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood.		Ebb.		Remarks.
		Maximum velocity (reduced to mean tide).	Depth (at time of maximum velocity).	Maximum velocity (reduced to mean tide).	Depth (at time of maximum velocity).	
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 6 (south channel) — Continued.	2,240	0.00	....	0.00	....	
	2,250	....	3.1	....	2.3	
	2,300	....	1.8	....	1.0	[tude, 75° 05' 50".7.
	2,380	....	0.0	....	− 0.8	Position of end of section: Latitude, 39° 57' 39".1; longi-
Section 7 (south channel.)	0	....	0.8	....	0.0	Position of origin of section (levee on Petty's Island):
	100	....	1.8	....	1.0	Latitude, 39° 57' 40".6; longitude, 75° 06' 22".8.
	160	....	....	0.00	....	
	200	0.00	7.2	1.58	6.4	
	300	0.64	7.8	3.14	7.0	Area of section: Flood, 26,562 square feet; ebb, 25,354
	400	1.18	9.0	3.15	8.2	square feet.
	500	1.63	17.1	3.29	16.3	Mid-area — 756 feet.
	600	2.06	33.1	3.91	32.3	
	700	2.74	34.1	3.96	33.3	
	800	3.49	29.0	3.58	28.2	Mid-volume: Flood, 875 feet; ebb, 776 feet.
	900	3.47	24.5	3.20	23.7	
	1,000	2.96	20.4	3.14	19.6	
	1,100	2.75	17.9	3.09	17.1	Plane of reference for depths above mean low-water:
	1,200	2.58	16.8	2.86	16.0	Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,300	2.58	17.4	2.46	16.6	
	1,400	2.53	18.0	1.98	17.2	
	1,500	2.28	16.4	1.52	15.6	[tude, 75° 06' 10".9.
	1,510	2.24	16.1	1.49	15.3	Position of end of section: Latitude, 39° 57' 28".6; longi-
Section 8 (south channel).	0	0.00	3.0	....	2.2	Position of origin of section (levee on Petty's Island):
	100	1.95	4.3	....	3.5	Latitude, 39° 57' 36".4; longitude, 75° 06' 33".9.
	200	2.32	6.1	....	5.3	
	300	2.41	6.4	....	5.6	
	400	2.48	13.8	0.00	13.0	
	500	2.69	35.3	3.49	34.5	
	600	2.88	36.1	4.22	35.3	Area of section: Flood, 28,485 square feet; ebb, 27,045
	700	2.87	29.8	4.52	29.0	square feet.
	800	2.67	23.8	4.06	23.0	Mid-area — 790 feet.
	900	2.46	20.8	3.53	20.0	
	1,000	2.34	23.1	3.11	22.3	Mid-volume: Flood, 733 feet; ebb, 783 feet.
	1,100	2.26	23.7	2.85	22.9	
	1,200	2.13	23.5	2.61	22.7	
	1,300	1.97	18.3	2.37	17.5	
	1,400	1.51	7.8	1.83	7.0	Plane of reference for depths above mean low-water:
	1,470	0.00	....	0.00	....	Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,500	0.00	4.0	....	3.2	
	1,600	0.00	2.4	....	1.6	
	1,700	0.00	1.6	....	0.8	
	1,800	0.00	0.9	....	0.0	[tude, 75° 06' 32".1.
	1,900	0.00	0.0	....	....	Position of end of section: Latitude, 39° 57' 17".6; longi-
Section 9 (south channel).	0	0.00	2.7	....	1.9	Position of origin of section (levee on Petty's Island):
	100	0.09	3.1	....	2.3	Latitude, 39° 57' 44".1; longitude, 75° 06' 51".6.
	200	1.02	3.6	....	2.8	
	300	1.92	5.5	....	4.7	
	400	1.94	8.3	....	7.5	
	500	1.87	7.8	....	7.0	
	600	1.86	6.8	....	6.0	
	700	1.94	7.0	0.00	6.2	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 9 (south channel) — Continued.	759	....	....	0.08	....	
	800	2.09	8.9	0.27	8.1	
	900	2.19	13.9	1.28	13.1	
	1,000	2.22	20.1	2.60	19.3	
	1,100	2.20	25.1	3.37	24.3	
	1,200	2.18	26.5	3.71	25.7	Area of section: Flood, 32,685 square feet; ebb, 30,625
	1,300	2.11	24.5	3.88	23.7	square feet.
	1,400	2.03	22.6	3.86	21.8	Mid-area = 1,351 feet.
	1,500	2.01	20.5	3.76	19.7	
	1,600	2.01	18.6	3.66	17.8	Mid-volume: Flood, 1,292 feet; ebb, 1,439 feet.
	1,700	1.98	17.0	3.55	16.2	
	1,800	1.90	16.1	3.43	15.3	
	1,900	1.77	15.7	3.25	14.9	
	2,000	1.57	16.2	2.96	15.4	Plane of reference for depths above mean low-water:
	2,100	1.31	15.0	2.55	14.2	Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	2,200	0.97	11.2	2.00	10.4	
	2,300	0.38	5.1	0.91	4.3	
	2,340	0.00	....	0.00	....	
	2,350	....	4.3	....	3.5	
	2,400	....	3.5	....	2.7	
	2,500	....	1.8	....	1.0	
	2,600	....	0.3	....	− 0.5	[tude, 75° 07′ 03″.6.
	2,620	....	0.0	....	− 0.8	Position of end of section: Latitude, 39° 57′ 19″.8; longi-
Section 10 (south channel).	0	0.00	3.4	....	2.6	Position of origin of section (at bulkhead Petty's Island):
	100	1.26	3.5	....	2.7	Latitude, 39° 57′ 50″.9; longitude, 75° 07′ 14″.2.
	200	1.46	3.5	....	2.7	
	300	1.57	4.1	0.00	3.3	
	400	1.65	6.0	0.26	5.2	
	500	1.74	8.7	0.44	7.9	
	600	1.82	9.8	0.53	9.0	
	700	1.87	10.3	0.53	9.5	
	800	1.95	10.4	0.57	9.6	
	900	2.03	10.8	0.61	10.0	
	1,000	2.06	12.0	0.82	11.2	
	1,100	2.07	14.3	1.17	13.5	Area of section: Flood, 38,554 square feet; ebb, 36,194
	1,200	2.07	15.2	1.61	14.4	square feet.
	1,300	2.14	16.1	2.13	15.3	Mid-area = 1,687 feet.
	1,400	2.17	20.4	2.81	19.6	
	1,500	2.15	19.4	3.42	18.6	
	1,600	2.15	20.4	3.53	19.6	Mid-volume: Flood, 1,609 feet; ebb, 1,841 feet.
	1,700	2.17	20.3	3.56	19.5	
	1,800	2.18	21.2	3.58	20.4	
	1,900	2.18	20.8	3.52	20.0	Plane of reference for depths above mean low-water:
	2,000	2.16	20.3	3.32	19.5	Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	2,100	2.12	19.9	3.14	19.1	
	2,200	2.03	17.5	3.26	16.7	
	2,300	1.85	14.8	3.25	14.0	
	2,400	1.57	14.0	3.25	13.2	
	2,500	1.21	13.6	3.15	12.8	
	2,600	0.71	13.8	2.68	13.0	
	2,700	0.00	12.8	0.00	12.0	
	2,800	0.00	8.0	0.00	7.2	
	2,875	0.00	4.8	0.00	4.0	
	2,900	0.00	4.6	0.00	3.8	[tude, 75° 07′ 23″.7.
	2,960	0.00	− 0.7	0.00	1.5	Position of end of section: Latitude, 39° 57′ 22″.5; longi-

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 11	0	0.00	34.7	0.00	33.9	Position of origin of section (corner of wharf, Pennsylvania side): Latitude, 39° 57′ 57″.1; longitude, 75° 07′ 33″.5.
	100	2.64	39.0	2.36	38.2	
	200	2.86	39.2	2.92	38.4	
	300	2.81	40.6	2.97	39.8	
	400	2.80	40.9	2.93	40.1	
	500	2.89	37.6	2.81	36.8	
	600	2.91	32.9	2.73	32.1	
	700	2.86	29.1	2.75	28.3	
	800	2.77	24.2	2.55	23.4	
	900	2.65	20.8	2.41	20.0	
	1,000	2.44	16.9	2.42	16.1	
	1,100	2.24	14.1	1.84	13.3	
	1,200	2.22	12.3	1.52	11.5	
	1,300	2.27	12.0	1.35	11.2	
	1,400	2.40	13.0	1.14	12.2	
	1,500	2.53	14.1	0.92	13.3	Area of section: Flood, 73,620 square feet; ebb, 70,740 square feet.
	1,600	2.54	14.4	0.87	13.6	
	1,700	2.51	15.2	1.14	14.4	Mid-area = 1,299 feet.
	1,800	2.43	15.3	1.53	14.5	
	1,900	2.37	15.8	1.96	15.0	
	2,000	2.35	16.6	2.42	15.8	Mid-volume: Flood, 983 feet; ebb, 1,197 feet.
	2,100	2.37	18.3	2.85	17.5	
	2,200	2.41	20.6	3.13	19.8	
	2,300	2.55	21.5	3.29	20.7	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	2,400	2.73	22.6	3.33	21.8	
	2,500	2.86	22.0	3.34	21.2	
	2,600	2.88	21.6	3.34	20.8	
	2,700	2.56	21.0	3.28	20.2	
	2,800	1.80	18.6	3.14	17.8	
	2,900	1.17	16.7	3.02	15.9	
	3,000	0.58	14.3	2.92	13.5	
	3,100	0.00	13.8	2.80	13.0	
	3,200	0.00	14.0	2.65	13.2	
	3,300	0.00	13.8	2.43	13.0	
	3,400	0.00	9.3	1.25	8.5	
	3,500	0.00	5.8	0.00	5.0	
	3,600	0.00	3.8	0.00	3.0	Position of end of section: Latitude, 39° 57′ 22″.7; longitude, 75° 07′ 22″.5.
Section 12	0	0.00	35.5	0.00	34.7	Position of origin of section (corner of wharf, Pennsylvania side): Latitude, 39° 57′ 51″.9; longitude, 75° 07′ 43″.9.
	100	2.52	37.6	2.40	36.8	
	200	2.85	40.0	2.95	39.2	
	300	3.14	41.3	3.30	40.5	
	400	3.31	40.3	3.56	39.5	
	500	3.29	38.4	3.47	37.6	
	600	3.22	38.8	3.15	38.0	
	700	3.17	28.1	3.04	27.3	
	800	2.98	24.8	3.04	24.0	
	900	2.73	21.8	2.79	21.0	
	1,000	2.62	18.5	1.78	17.7	
	1,100	2.60	18.0	1.79	17.2	
	1,200	2.60	18.1	1.99	17.3	Area of section: Flood, 69,407 square feet; ebb, 67,031 square feet.
	1,300	2.57	18.6	2.14	17.8	
	1,400	2.50	19.1	1.92	18.3	Mid-area = 1,063 feet.
	1,500	2.47	18.3	1.95	17.5	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood.		Ebb.		Remarks.
		Maximum velocity (reduced to mean tide).	Depth (at time of maximum velocity).	Maximum velocity (reduced to mean tide).	Depth (at time of maximum velocity).	
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 12—Continued.	1,600	2.44	18.1	2.30	17.3	Mid-volume: Flood, 808 feet; ebb, 879 feet.
	1,700	2.41	18.2	2.67	17.4	
	1,800	2.33	19.6	3.14	18.8	
	1,900	2.17	21.7	3.14	20.9	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	2,000	1.93	22.6	3.15	21.8	
	2,100	2.26	23.8	3.12	23.0	
	2,200	2.65	22.6	3.01	21.8	
	2,300	2.50	21.4	2.92	20.6	
	2,400	2.31	15.8	2.82	15.0	
	2,500	2.00	14.3	2.56	13.5	
	2,600	1.38	15.6	2.38	14.8	
	2,700	0.00	17.0	2.19	16.2	
	2,800	....	14.3	1.60	13.5	
	2,900	....	12.0	0.00	11.2	
	2,970	....	7.1	....	6.3	Position of end of section: Latitude, 39° 57′ 24″.5; longitude, 75° 07′ 30″.4.
Section 13	0	....	11.2	....	10.4	Position of origin of section (end of wharf, Pennsylvania side): Latitude, 39° 57′ 46″.3; longitude, 75° 07′ 57″.3.
	60	....	....	0.00	....	
	100	0.00	13.8	0.50	13.0	
	170	0.00	....	....	....	
	200	0.58	23.9	1.41	23.1	
	300	2.27	36.1	2.19	35.3	
	400	2.95	38.2	2.69	37.4	
	500	3.34	39.8	2.97	39.0	
	600	3.42	40.2	3.21	39.4	
	700	3.55	39.0	3.04	38.2	
	800	3.40	36.5	2.84	35.7	
	900	2.83	32.9	2.73	32.1	
	1,000	2.88	29.0	2.62	28.2	Area of section: Flood, 69,302 square feet; ebb, 66,914 square feet.
	1,100	2.88	24.8	2.50	24.0	
	1,200	2.78	22.8	2.38	22.0	
	1,300	2.69	23.1	2.31	22.3	Mid-area=1,111 feet.
	1,400	2.62	22.8	2.28	22.0	
	1,500	2.56	22.2	2.44	21.4	Mid-volume: Flood, 980 feet; ebb, 1,152 feet.
	1,600	2.52	22.0	2.69	21.2	
	1,700	2.37	22.5	2.92	21.7	
	1,800	2.24	23.9	3.11	23.1	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,900	2.18	24.6	3.23	23.8	
	2,000	2.32	23.1	3.25	22.3	
	2,100	2.55	18.8	3.21	18.0	
	2,200	3.36	17.1	3.14	16.3	
	2,300	3.42	16.3	3.04	15.5	
	2,400	2.58	16.1	2.91	15.3	
	2,500	1.37	15.5	2.72	14.7	
	2,590	0.00	....	....	....	
	2,600	....	13.2	2.28	12.4	
	2,700	....	12.8	1.52	12.0	
	2,800	....	9.6	0.00	8.8	
	2,900	....	4.7	....	3.9	
	2,985	....	2.5	....	1.7	Position of end of section: Latitude, 39° 57′ 22″; longitude, 75° 07′ 35″.8.

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 14	0	0. 00	29. 8	0. 00	29. 0	Position of origin of section (entrance of slip, Pennsylvania side): Latitude, 39° 57′ 37″.1; longitude, 75° 08′ 05″.2.
	100	2. 51	34. 6	3. 15	33. 8	
	200	3. 20	40. 6	3. 78	39. 8	
	300	3. 29	45. 7	3. 64	44. 9	
	400	3. 47	46. 6	2. 99	45. 8	
	500	3. 30	42. 4	2. 75	41. 6	
	600	3. 04	36. 8	2. 67	36. 0	
	700	2. 58	31. 1	2. 65	30. 3	
	800	2. 56	28. 1	2. 84	27. 3	
	900	2. 76	26. 8	3. 15	26. 0	
	1, 000	2. 60	27. 6	3. 39	26. 8	Area of section: Flood, 59,945 square feet; ebb, 57,745 square feet.
	1, 100	2. 33	24. 8	3. 48	24. 0	
	1, 200	2. 38	24. 0	3. 41	23. 2	Mid-area = 773 feet.
	1, 300	2. 54	23. 6	3. 31	22. 8	
	1, 400	2. 73	19. 7	3. 33	18. 9	
	1, 500	3. 04	12. 8	3. 51	12. 0	Mid-volume: Flood, 719 feet; ebb, 801 feet.
	1, 600	3. 36	12. 6	3. 80	11. 8	
	1, 700	3. 61	13. 3	3. 99	12. 5	
	1, 800	4. 10	13. 1	4. 12	12. 3	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1, 900	4. 60	13. 5	4. 18	12. 7	
	2, 000	4. 33	12. 4	4. 15	11. 6	
	2, 100	3. 52	11. 8	3. 96	11. 0	
	2, 200	2. 56	8. 6	3. 48	7. 8	
	2, 300	1. 61	8. 6	2. 41	7. 8	
	2, 400	0. 00	9. 5	0. 00	8. 7	
	2, 500	....	6. 8	....	6. 0	
	2, 600	....	5. 0	....	4. 2	
	2, 700	....	2. 0	....	1. 2	
	2, 750	....	0. 0	....	−0. 8	[Latitude, 39° 57′ 18″.9; longitude, 75° 07′ 38″.4.
	2, 770	....	−1. 8	....	−2. 6	Position of end of section (at bulkhead, New Jersey side):
Section 15	0	0. 00	23. 2	0. 00	22. 4	Position of origin of section (end of wharf, Pennsylvania side): Latitude, 39° 57′ 27″.9: longitude, 75° 08′ 14″.2.
	60	0. 00	....	....	....	
	100	2. 62	43. 6	3. 38	42. 8	
	200	3. 12	51. 5	3. 53	50. 7	
	300	3. 18	53. 8	3. 37	53. 0	
	400	3. 09	48. 3	3. 43	47. 5	
	500	3. 04	43. 0	3. 61	42. 2	
	600	3. 22	37. 2	3. 82	36. 4	
	700	3. 10	33. 3	3. 65	32. 5	
	800	3. 01	28. 8	3. 80	28. 0	Area of section: Flood, 60,067 square feet; ebb, 58,167 square feet.
	900	2. 91	27. 7	4. 22	26. 9	
	1, 000	2. 66	27. 0	4. 20	26. 2	Mid-area = 674 feet.
	1, 100	2. 59	21. 2	4. 07	20. 4	
	1, 200	2. 67	12. 8	3. 79	12. 0	
	1, 300	2. 89	8. 6	3. 02	7. 8	Mid-volume: Flood, 604 feet; ebb, 610 feet.
	1, 400	3. 23	9. 8	2. 37	9. 0	
	1, 500	3. 36	12. 3	2. 40	11. 5	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1, 600	3. 29	14. 5	2. 51	13. 7	
	1, 700	3. 23	16. 4	2. 50	15. 6	
	1, 800	3. 36	18. 3	2. 97	17. 5	
	1, 900	3. 56	19. 5	3. 45	18. 7	
	2, 000	3. 75	21. 1	3. 43	20. 3	
	2, 100	3. 59	17. 6	2. 95	16. 8	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 15—Cont'd.	2,200	0.60	13.9	0.46	13.1	
	2,215	0.00	....	0.00	....	
	2,300	....	5.8	....	5.0	[side): Latitude, 39° 57′ 18″.4; longitude, 75° 07′ 48″.5.
	2,375	....	3.0	....	2.2	Position of end of section (at corner of wharf, New Jersey
Section 16........	0	....	37.8	....	37.0	Position of origin of section (at entrance of slip, Pennsylvania side): Latitude, 39° 57′ 20″.8; longitude, 75° 08′ 18″.8.
	70	0.00	....	0.00	....	
	100	2.74	51.6	3.45	50.8	
	200	2.96	50.9	3.77	50.1	
	300	3.11	51.8	3.81	51.0	
	400	3.26	49.6	3.59	48.8	
	500	3.21	42.8	3.59	42.0	
	600	3.11	37.5	3.69	36.7	
	700	2.91	32.4	3.77	31.6	
	800	2.62	28.2	3.84	27.4	Area of section: Flood, 59,514 square feet; ebb, 57,658 square feet.
	900	2.32	25.1	3.90	24.3	
	1,000	2.17	22.5	3.88	21.7	Mid-area=635 feet.
	1,100	2.54	11.8	3.79	11.0	
	1,200	2.99	7.3	3.43	6.5	Mid-volume: Flood, 697 feet; ebb, 615 feet.
	1,300	3.35	9.4	2.84	8.6	
	1,400	3.64	10.3	2.55	9.5	
	1,500	3.79	12.3	2.49	11.5	
	1,600	3.76	14.9	2.52	14.1	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,700	3.58	15.2	2.63	14.4	
	1,800	3.40	16.5	3.07	15.7	
	1,900	3.28	17.2	3.47	16.4	
	2,000	3.45	19.9	3.46	19.1	
	2,100	3.92	19.0	2.46	18.2	
	2,200	3.92	19.5	1.13	18.7	
	2,280	....	....	0.00	....	
	2,300	1.68	15.2	....	14.5	[side): Latitude, 39° 57′ 15″; longitude, 75° 07′ 50″.
	2,320	0.00	13.2	....	12.4	Position of end of section (at entrance of slip, New Jersey
Section 17........	0	0.00	29.3	0.00	28.5	Position of origin of section (in slip, Pennsylvania side): Latitude, 39° 57′ 09″.9; longitude, 75° 08′ 23″.5.
	100	1.71	60.7	2.71	59.9	
	200	2.82	59.2	4.01	58.4	
	300	3.30	51.3	4.07	50.5	
	400	3.64	44.4	4.07	43.6	
	500	3.82	35.6	3.83	34.8	
	600	3.71	30.5	3.41	29.7	
	700	3.36	25.9	3.75	25.1	
	800	2.37	24.0	3.97	23.2	
	900	1.75	23.7	3.56	22.9	
	1,000	1.82	21.2	3.35	20.4	
	1,100	2.13	12.7	3.09	11.9	
	1,200	2.56	5.3	2.18	4.5	
	1,300	2.77	4.8	1.15	4.0	Area of section: Flood, 64,025 square feet; ebb, 61,185 square feet.
	1,400	2.93	9.1	1.32	8.3	
	1,500	3.07	13.2	2.63	12.4	Mid-area=721 feet.
	1,600	3.23	16.6	2.78	15.8	
	1,700	3.31	18.2	2.81	17.4	
	1,800	3.33	18.7	3.02	17.9	Mid-volume: Flood, 673 feet; ebb, 569 feet.
	1,900	3.32	18.8	3.48	18.0	
	2,000	3.30	17.1	3.44	16.3	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 17—Cont'd.	2,100	3.24	14.8	2.86	14.0	Plane of reference for depths above mean low-water:
	2,200	3.16	13.0	2.27	12.2	Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	2,300	3.05	14.0	1.79	13.2	
	2,400	2.95	13.8	1.36	13.0	
	2,500	2.75	12.7	0.90	11.9	
	2,600	2.40	11.6	0.40	10.8	
	2,680	....	....	0.00	....	
	2,700	1.76	7.8	....	7.0	
	2,780	0.00	....	....	....	
	2,800	....	4.7	....	3.9	
	2,900	....	3.9	....	3.1	
	3,000	....	3.3	....	2.5	
	3,100	....	2.6	....	1.8	
	3,200	....	2.3	....	1.5	
	3,300	....	1.9	....	1.1	
	3,400	....	1.9	....	1.1	
	3,500	....	1.6	....	0.8	[Latitude, 39° 57′ 04″.1; longitude, 75° 07′ 38″.2.
	3,570	....	−0.2	....	−1.0	Position of end of section (at shore, New Jersey side):
Section 18	0	0.00	53.8	0.00	53.0	Position of origin of section (end of pier, Pennsylvania side): Latitude, 39° 56′ 59″.5; longitude, 75° 08′ 24″.8.
	100	1.60	59.8	3.16	59.0	
	200	2.61	55.4	4.59	54.6	
	300	3.37	48.2	4.63	47.4	
	400	3.50	39.0	4.26	38.2	
	500	3.41	32.4	3.77	31.6	
	600	3.27	28.1	3.46	27.3	
	700	3.10	23.8	3.31	23.0	
	800	2.77	20.5	3.25	19.7	
	900	2.99	18.1	3.92	17.3	
	1,000	2.55	15.5	3.77	14.7	
	1,100	2.27	12.3	3.55	11.5	
	1,200	2.32	9.9	3.11	9.1	
	1,300	2.59	8.0	2.55	7.2	
	1,400	2.95	6.7	2.04	5.9	Area of section: Flood, 64,772 square feet; ebb, 61,802 square feet.
	1,500	3.25	9.9	1.58	9.1	
	1,600	3.43	18.3	1.49	17.5	Mid-area = 790 feet.
	1,700	3.57	20.8	2.61	20.0	
	1,800	3.65	21.5	2.98	20.7	Mid-volume: Flood, 831 feet; ebb, 525 feet.
	1,900	3.65	20.2	2.77	19.4	
	2,000	3.56	19.1	2.49	18.3	
	2,100	3.46	18.1	2.31	17.3	
	2,200	3.29	17.9	2.18	17.1	Plane of reference for depths above mean low-water:
	2,300	3.13	17.9	2.06	17.1	Maximum flood, 475 feet; maximum ebb, 4 feet.
	2,400	3.02	17.7	1.89	16.9	
	2,500	2.79	15.4	1.53	14.6	
	2,600	1.93	12.6	0.00	11.8	
	2,660	0.00	....	....	....	
	2,700	....	6.2	....	5.4	
	2,800	....	4.6	....	3.8	
	2,900	....	4.0	....	3.2	
	3,000	....	3.5	....	2.7	
	3,100	....	2.9	....	2.1	
	3,200	....	2.3	....	1.5	
	3,300	....	2.4	....	1.6	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 18—Cont'd.	3,400	....	2.1	....	1.3	
	3,500	....	1.8	....	1.0	
	3,600	....	0.8	....	0.0	
	3,630	....	0.0	....	....	Position of end of section: Latitude, 39° 56′ 56″.5; longitude, 75° 07′ 39″.1.
Section 19.........	0	....	14.6	....	13.8	Position of origin of section (end of wharf, Pennsylvania side): Latitude, 39° 56′ 50″.4; longitude, 75° 08′ 30″.
	45	....	22.6	....	21.8	
	145	....	53.9	....	53.1	
	165	0.00	....	0.00	....	
	245	2.80	58.5	3.04	57.7	
	345	3.36	56.8	4.17	56.0	
	445	3.52	52.0	4.44	51.2	
	545	3.53	46.9	4.39	46.1	
	645	3.27	39.1	4.15	38.3	
	745	2.82	28.8	3.72	28.0	
	845	2.10	17.3	3.15	16.5	
	945	1.17	13.1	2.77	12.3	Area of section: Flood, 67,264 square feet; ebb, 64,960 square feet.
	1,045	1.06	11.3	2.54	10.5	
	1,145	1.17	10.4	2.39	9.6	Mid-area=725 feet.
	1,245	1.38	11.2	2.27	10.4	
	1,345	1.69	11.0	2.22	10.2	
	1,445	2.06	7.6	2.14	6.8	Mid-volume: Flood, 730 feet; ebb, 648 feet.
	1,545	2.42	8.1	2.06	7.3	
	1,645	2.75	9.3	1.96	8.5	
	1,745	3.11	14.5	2.08	13.7	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,845	3.54	20.8	2.45	20.0	
	1,945	3.69	23.4	2.90	22.6	
	2,045	3.55	21.8	3.06	21.0	
	2,145	3.28	20.2	2.97	19.4	
	2,245	3.12	19.9	2.89	19.1	
	2,345	3.09	19.8	2.81	19.0	
	2,445	3.01	19.2	2.67	18.4	
	2,545	2.90	19.3	2.77	18.5	
	2,645	2.65	18.1	2.17	17.3	
	2,745	2.28	13.5	1.29	12.7	
	2,845	1.47	11.4	0.00	10.6	
	2,855	1.34	11.0	....	10.2	Position of end of section: Latitude, 39° 56′ 48″.5; longitude, 75° 07′ 53″.
Section 20 (east channel).	0	0.00	6.6	....	5.8	Position of origin of section (bulkhead on Windmill Island): Latitude, 39° 56′ 38″.3; longitude, 75° 08′ 14″.1.
	100	2.11	7.9	....	7.1	
	150	....	....	0.00	....	
	200	2.49	10.4	1.04	9.6	
	300	2.82	16.4	2.06	15.6	Area of section: Flood, 26,205 square feet, ebb, 25,065 square feet.
	400	3.25	20.8	2.63	20.0	
	500	3.50	22.8	2.81	22.0	Mid-area=761 feet.
	600	3.54	23.3	2.87	22.5	
	700	3.49	22.6	2.99	21.8	Mid-volume: Flood, 755 feet; ebb, 800 feet.
	800	3.43	21.6	3.09	20.8	
	900	3.28	21.8	3.15	21.0	
	1,000	3.14	20.1	3.15	19.3	
	1,100	3.29	19.6	3.01	18.8	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,200	3.31	19.4	2.71	18.6	
	1,300	2.34	19.1	2.08	18.3	
	1,400	1.19	16.1	0.95	15.3	
	1,425	0.00	15.3	0.00	14.5	Position of end of section: Latitude, 39° 56′ 37″.4; longitude, 75° 07′ 55″.9.

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 20 (west channel).	0	0.00	43.3	0.00	42.5	Position of origin of section (wharf on Pennsylvania side): Latitude, 39° 56′ 40″.2; longitude, 75° 08′ 36″.1.
	40	1.69	49.8	1.96	49.0	
	140	3.43	56.1	4.16	55.3	Area of section: Flood, 32,996 square feet; ebb, 32,348 square feet.
	240	3.39	55.3	4.55	54.5	Mid-area = 305 feet.
	340	3.32	51.3	4.37	50.5	
	440	3.32	43.8	4.18	43.0	Mid-volume: Flood, 318 feet; ebb, 315 feet.
	540	3.24	33.4	3.97	32.6	
	640	2.95	26.0	2.51	25.2	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	740	2.29	17.4	2.58	16.6	
	810	1.19	9.6	1.57	8.8	Position of end of section: Latitude, 39° 56′ 39″.6; longitude, 75° 08′ 20″.3.
Section 21 (east channel).	0	....	8.8	....	8.0	Position of origin of section (bulkhead on Windmill Island): Latitude, 39° 56′ 28″.5; longitude, 75° 08′ 15″.
	15	0.00	10.0	....	9.2	
	75	....	....	0.00	....	
	115	2.49	10.7	0.65	9.9	
	215	2.75	14.1	1.37	13.3	
	315	2.90	20.2	2.44	19.4	
	415	3.04	22.7	2.94	21.9	
	515	3.15	24.0	2.92	23.2	Area of section: Flood, 29,073 square feet; ebb, 27,652 square feet.
	615	3.15	24.2	2.83	23.4	
	715	3.10	23.3	2.83	22.5	Mid-area = 766 feet.
	815	2.95	21.8	2.78	21.0	
	915	2.85	21.6	2.71	20.8	Mid-volume: Flood, 699 feet; ebb, 730 feet.
	1,015	3.01	20.2	2.72	19.4	
	1,115	2.96	18.9	2.66	18.1	
	1,215	2.54	17.6	2.35	16.8	
	1,315	1.80	16.3	1.63	15.5	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,415	0.57	12.6	0.44	11.8	
	1,435	0.00	....	0.00	....	
	1,515	....	6.8	....	6.0	
	1,615	....	5.7	....	4.9	
	1,715	....	4.1	....	3.3	
	1,765	....	3.8	....	3.0	
	1,775	....	3.6	....	2.8	Position of end of section: Latitude, 39° 56′ 27″.9; longitude, 75° 07′ 52″.3.
Section 21 (west channel).	0	....	14.8	....	14.0	Position of origin of section (bulkhead in slip, Pennsylvania side): Latitude, 39° 56′ 30″.4; longitude, 75° 08′ 34″.5.
	100	....	21.8	....	21.0	
	200	....	28.0	....	27.2	Area of section: Flood, 39,677 square feet; ebb, 38,813 square feet.
	235	0.00	....	0.00	....	
	300	1.89	48.2	2.81	47.4	Mid-area = 516 feet.
	400	3.48	56.0	4.43	55.2	
	500	3.45	53.7	4.45	52.9	Mid-volume: Flood, 592 feet; ebb, 573 feet.
	600	3.47	49.2	4.37	48.4	
	700	3.51	44.1	4.23	43.3	
	800	3.53	36.4	3.99	35.6	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	900	3.41	28.4	3.62	27.6	
	1,000	2.94	29.6	2.88	19.8	
	1,080	0.00	13.8	0.00	13.0	Position of end of section: Latitude, 39° 56′ 29″.6; longitude, 75° 08′ 20″.8.
Section 22 (west channel).	0	0.00	40.8	0.00	40.0	Position of origin of section (at wharf on Pennsylvania side): Latitude, 39° 56′ 22″.2; longitude, 75° 08′ 32″.9.
	100	2.62	50.4	2.60	49.6	
	200	2.76	53.3	4.32	52.5	Area of section: Flood, 36,910 square feet; ebb, 36,134 square feet.
	300	2.85	52.5	4.41	51.7	Mid-area = 368 feet.
	400	2.98	47.2	4.28	46.4	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood.		Ebb.		Remarks.
		Maximum velocity (reduced to mean tide).	Depth (at time of maximum velocity).	Maximum velocity (reduced to mean tide).	Depth (at time of maximum velocity).	
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 22 (west channel)—Cont'd.	500	3.01	43.2	4.11	42.4	Mid-volume: Flood, 377 feet; ebb, 380 feet.
	600	2.88	37.3	3.97	36.5	
	700	2.67	29.6	3.74	28.8	Plane of reference for depths above mean low-water:
	800	2.45	21.6	3.08	20.8	Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	900	2.11	10.8	1.36	10.0	[tude, 75° 08′ 18″.2.
	970	0.00	2.1	0.00	1.3	Position of end of section: Latitude, 39° 56′ 22″.1; longi-
Section 22 (east channel).	0	0.00	4.8	....	4.0	Position of origin of section (at bulkhead on Windmill Isl-
	70	....	....	0.00	....	and): Latitude, 39° 56′ 22″.2; longitude, 75° 08′ 16″.3.
	100	2.36	12.1	0.41	11.3	
	200	2.83	13.4	1.73	12.6	
	300	3.06	16.8	2.34	16.0	
	400	3.15	21.8	2.74	21.0	
	500	3.21	22.5	3.08	21.7	Area of section: Flood, 27,984 square feet; ebb, 26,536
	600	3.28	23.2	3.23	22.4	square feet.
	700	3.31	23.8	3.26	23.0	Mid-area = 771 feet.
	800	3.30	23.3	3.25	22.4	
	900	3.24	22.1	3.18	21.3	
	1,000	3.11	21.6	3.16	20.8	Mid-volume: Flood, 744 feet; ebb, 767 feet.
	1,100	3.20	20.8	3.14	20.0	
	1,200	3.30	20.0	2.96	19.2	
	1,300	3.02	13.0	2.36	12.2	Plane of reference for depths above mean low-water:
	1,400	2.20	6.8	1.17	6.0	Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,470	0.00	....	0.00	....	
	1,500	....	6.2	....	5.4	
	1,600	....	5.2	....	4.4	
	1,700	....	4.1	....	3.3	
	1,800	....	3.0	....	2.2	[tude, 75° 07′ 53″.1.
	1,810	....	2.8	....	2.0	Position of end of section: Latitude, 39° 56′ 22″; longi-
Section 23.........	0	....	17.8	....	17.0	Position of origin of section (at end of wharf, Pennsylvania
	70	0.00	....	0.00	....	side): Latitude 39° 56′ 12″; longitude, 75° 08′ 31″.3.
	100	0.99	38.1	0.94	37.3	
	200	3.10	47.6	3.54	46.8	
	300	3.02	50.0	3.99	49.2	
	400	3.06	48.9	4.07	48.1	
	500	3.13	45.5	4.15	44.7	
	600	3.12	41.0	3.90	40.2	
	700	3.06	36.1	3.59	35.3	
	800	2.90	28.0	3.43	27.2	
	900	2.56	21.1	3.19	20.3	
	1,000	2.25	11.4	2.51	10.6	Area of section: Flood, 68,037 square feet; ebb, 65,913
	1,100	2.20	9.1	1.80	8.3	square feet.
	1,200	2.32	10.2	1.24	9.4	Mid-area = 843 feet.
	1,300	2.53	13.8	1.07	13.0	
	1,400	2.67	16.3	1.83	15.5	Mid-volume: Flood, 806 feet; ebb, 667 feet.
	1,500	2.77	21.2	2.54	20.4	
	1,600	2.87	23.3	2.75	22.5	
	1,700	2.96	23.4	2.81	22.6	Plane of reference for depths above mean low-water:
	1,800	2.98	24.4	2.82	23.6	Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,900	2.79	24.4	2.89	23.6	
	2,000	2.65	23.0	2.98	22.2	
	2,100	2.67	22.2	2.95	21.4	
	2,200	2.80	20.7	2.73	19.9	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood.		Ebb.		Remarks.
		Maximum velocity (reduced to mean tide).	Depth (at time of maximum velocity).	Maximum velocity (reduced to mean tide).	Depth (at time of maximum velocity).	
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 23—Continued.	2,300	2.97	21.2	2.44	20.4	
	2,400	2.82	21.3	1.97	20.5	
	2,500	2.23	19.6	0.98	18.8	
	2,530	....	....	0.00	....	
	2,590	0.00	....	....	....	
	2,600	....	19.8	....	19.0	[tude, 75° 07′ 57″.4.
	2,655	....	5.3	....	4.5	Position of end of section: Latitude, 39° 56′ 12″.4; longi-
Section 24.........	0	0.00	22.6	0.00	21.8	Position of origin of section (in slip, Pennsylvania side): Latitude, 39° 56′ 01″.3; longitude, 75° 08′ 29″.6.
	100	1.41	28.5	1.83	27.7	
	200	2.62	33.0	3.06	32.2	
	300	3.30	43.3	3.99	42.5	
	400	3.19	46.8	4.10	46.0	
	500	2.97	45.0	4.02	44.2	
	600	2.96	41.9	3.87	41.1	
	700	2.97	37.3	3.64	36.5	
	800	2.76	31.2	3.46	30.4	
	900	2.70	24.5	3.33	23.7	Area of section: Flood, 67,104 square feet; ebb, 65,096 square feet.
	1,000	2.69	17.0	3.22	16.2	
	1,100	2.69	14.3	2.75	13.5	Mid-area = 911 feet.
	1,200	2.71	14.0	1.70	13.2	
	1,300	2.70	15.6	1.34	14.8	Mid-volume: Flood, 896 feet; ebb, 731 feet.
	1,400	2.62	17.6	1.62	16.8	
	1,500	2.94	24.5	2.54	23.7	
	1,600	3.06	28.8	2.86	28.0	
	1,700	2.94	28.0	2.80	27.2	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,800	2.79	28.0	2.73	27.2	
	1,900	2.71	27.3	2.62	26.5	
	2,000	2.70	25.5	2.64	24.7	
	2,100	2.65	26.1	2.73	25.3	
	2,200	2.49	23.8	2.41	23.0	
	2,300	2.04	19.1	1.55	18.3	
	2,400	1.23	13.2	0.77	12.4	
	2,485	0.00	....	0.00	....	
	2,500	....	6.2	....	5.4	[tude, 75° 07′ 57″.5.
	2,510	....	5.7	....	4.9	Position of end of section: Latitude, 39° 55′ 59″.7; longi-
Section 25.........	0	....	10.0	....	9.2	Position of origin of section (end of wharf, Pennsylvania side): Latitude, 39° 55′ 49″.9; longitude, 75° 08′ 31″.1.
	40	....	13.3	....	12.5	
	140	....	14.4	....	13.6	
	220	0.00	....	0.00	....	
	240	0.94	22.8	0.44	22.0	
	340	2.19	30.2	1.71	29.4	
	440	2.79	31.4	2.48	30.6	
	540	2.83	37.1	3.20	36.3	
	640	2.62	41.8	3.92	41.0	
	740	2.53	43.1	4.01	42.3	
	840	2.60	42.1	3.91	41.3	
	940	2.78	38.4	3.68	37.6	
	1,040	2.87	34.8	3.39	34.0	
	1,140	2.87	30.7	3.21	29.9	
	1,240	2.83	25.3	3.07	24.5	Area of section: Flood, 73,923 square feet; ebb, 71,795 square feet.
	1,340	3.75	20.7	3.01	19.9	
	1,440	2.67	18.3	2.88	17.5	Mid-area = 1,161 feet.

DALAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections	Distance from origin.	Flood.		Ebb.		Remarks.
		Maximum velocity (reduced to mean tide).	Depth (at time of maximum velocity).	Maximum velocity (reduced to mean tide).	Depth (at time of maximum velocity).	
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 25—Cont'd.	1,540	2.64	17.2	2.50	16.4	
	1,640	2.71	19.0	2.16	18.2	
	1,740	2.81	22.8	2.06	22.0	
	1,840	2.82	29.8	2.30	29.0	Mid-volume: Flood, 1,234 feet; ebb, 1,063 feet.
	1,940	2.78	32.1	2.64	31.3	
	2,040	2.68	32.3	2.79	31.5	
	2,140	2.66	30.8	2.82	30.0	Plane of reference for depths above mean low-water:
	2,240	2.81	27.8	2.72	27.0	Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	2,340	2.76	27.1	2.41	26.3	
	2,440	2.53	24.1	1.99	23.3	
	2,540	2.10	20.4	0.82	19.6	
	2,550	....	....	0.00	....	
	2,640	0.97	16.1	....	15.3	[tude, 75° 07′ 57″.1.
	2,660	0.00	12.2	....	11.4	Position of end of section: Latitude, 39° 55′ 50″.7; longi-
Section 26.........	0	0.00	14.5	0.00	13.7	Position of origin of section (entrance of slip, Pennsylva-
	100	2.54	27.9	0.91	27.1	nia side): Latitude, 39° 55′ 37″.7; longitude, 75° 08′ 26″.1.
	200	2.70	29.2	1.82	28.4	
	300	2.71	33.1	2.67	32.3	
	400	2.71	36.3	3.51	35.5	
	500	2.67	36.6	3.91	35.8	
	600	2.60	36.7	3.97	35.9	
	700	2.55	36.6	3.96	35.8	
	800	2.52	35.5	3.93	34.7	
	900	2.56	33.9	3.71	33.1	Area of section: Flood, 71,810 square feet; ebb, 69,846
	1,000	2.65	30.1	3.48	29.3	square feet.
	1,100	2.71	26.7	3.25	25.9	Mid-area = 1,104 feet.
	1,200	2.73	24.2	3.01	23.4	
	1,300	2.73	22.0	2.67	21.2	Mid volume: Flood, 1,115 feet; ebb, 950 feet.
	1,400	2.77	21.7	2.24	20.9	
	1,500	2.82	21.9	2.08	21.1	
	1,600	2.83	28.6	2.20	27.8	
	1,700	2.83	33.1	2.32	32.3	Plane of reference for depths above mean low-water:
	1,800	2.83	37.0	2.49	36.2	Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,900	2.82	87.1	2.61	36.3	
	2,000	2.81	35.8	2.68	35.0	
	2,100	2.74	31.5	2.69	30.7	
	2,200	2.39	28.3	2.28	27.5	
	2,300	1.42	19.2	0.00	18.4	
	2,350	0.00	....	....	....	
	2,400	....	6.0	....	5.2	[tude, 75° 07′ 54″.9.
	2,455	....	5.2	....	4.4	Position of end of section: Latitude, 39° 55′ 40″; longi-
Section 27.........	0	....	4.8	....	4.0	Position of origin of section (bulkhead, Pennsylvania side):
	100	....	5.6	....	4.8	Latitude, 39° 55′ 29″.5; longitude, 75° 08′ 26″.
	200	0.00	10.6	0.00	9.8	
	300	2.30	24.0	1.42	23.2	
	400	2.76	25.0	2.36	24.2	
	500	2.75	29.6	2.96	28.8	
	600	2.63	83.0	3.43	32.2	
	700	2.66	35.4	3.56	34.6	
	800	2.63	35.5	3.62	34.7	
	900	2.59	35.1	3.95	34.3	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide.)	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 27—Cont'd.	1,000	2.62	33.8	3.92	33.0	
	1,100	2.67	30.8	3.73	30.0	Area of section: Flood, 70,100 square feet; ebb, 68,100 square feet.
	1,200	2.70	30.0	3.42	29.2	
	1,300	2.77	26.3	3.11	25.5	
	1,400	2.81	24.9	2.94	24.1	Mid-area = 1,324 feet.
	1,500	2.85	23.5	2.88	22.7	
	1,600	2.89	22.8	2.88	22.0	Mid-volume: Flood, 1,401 feet; ebb, 1,211 feet.
	1,700	2.89	25.2	2.88	24.4	
	1,800	2.80	29.5	2.86	28.7	
	1,900	2.75	34.3	2.83	33.5	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	2,000	3.00	37.0	2.83	36.2	
	2,100	3.25	37.7	2.75	36.9	
	2,200	3.12	36.2	2.47	35.4	
	2,300	2.80	32.4	2.21	31.6	
	2,400	2.55	29.4	1.66	28.6	
	2,500	0.00	19.8	0.00	19.0	Position of end of section: Latitude, 39° 55′ 35″.9; longitude, 75° 07′ 55″.2.
Section 28	0	....	3.8	....	3.0	Position of origion of section (marsh line, Pennsylvania side): Latitude, 39° 55′ 21″.4; longitude, 75° 08′ 21″.9.
	100	....	4.1	....	3.3	
	200	....	6.8	....	6.0	
	210	0.00	....	0.00	....	
	300	2.00	20.6	1.84	19.8	
	400	2.68	23.9	2.63	23.1	
	500	2.77	27.8	2.95	27.0	
	600	2.72	31.8	3.09	31.0	
	700	2.65	33.8	3.14	33.0	
	800	2.62	33.3	3.27	32.5	
	900	2.62	32.6	3.67	31.8	
	1,000	2.65	31.2	3.96	30.4	Area of section: Flood, 69,445 square feet; ebb, 67,125 square feet.
	1,100	2.72	29.8	3.92	29.0	
	1,200	2.74	28.0	3.75	27.2	Mid-area = 1,411 feet.
	1,300	2.79	27.2	3.57	26.4	
	1,400	2.89	25.7	3.36	24.9	
	1,500	2.99	26.1	3.26	25.3	Mid-volume: Flood, 1,476 feet; ebb, 1,309 feet.
	1,600	3.05	28.1	3.16	27.3	
	1,700	3.10	30.6	3.03	29.8	
	1,800	3.11	32.1	2.89	31.3	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,900	3.11	35.0	2.95	34.2	
	2,000	3.11	35.8	3.13	35.0	
	2,100	3.13	35.9	3.08	35.1	
	2,200	3.08	34.5	2.78	33.7	
	2,300	2.68	28.3	2.24	27.5	
	2,400	2.10	19.3	1.73	18.5	
	2,500	1.57	9.1	0.41	8.2	
	2,510	....	....	0.00	....	
	2,600	1.29	7.0	....	6.2	
	2,700	1.16	6.5	....	5.7	
	2,800	0.83	5.8	....	5.0	
	2,900	0.00	5.0	....	4.2	Position of end of section: Latitnde, 39° 55′ 31″.6; longitude, 75° 07′ 47″.2.
Section 29	0	0.00	2.5	....	1.7	Position of origin of section (on marsh, Pennsylvania side): Latitude, 39° 55′ 09″.3; longitude, 75° 08′ 12″.7.
	100	1.80	3.4	0.00	2.6	
	200	2.65	10.6	1.82	9.8	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 29—Cont'd.	300	2.69	25.5	3.03	24.7	
	400	2.73	28.8	3.47	28.0	
	500	2.78	31.3	3.75	30.5	
	600	2.91	33.1	3.86	32.3	
	700	2.99	31.9	3.81	31.1	
	800	2.92	30.2	3.72	29.4	
	900	2.87	28.2	3.64	27.4	
	1,000	2.90	28.3	3.56	27.5	
	1,100	3.04	28.1	3.49	27.3	
	1,200	3.13	28.5	3.44	27.7	
	1,300	3.13	31.6	3.39	30.8	
	1,400	3.08	32.8	3.35	32.0	
	1,500	3.17	34.0	3.40	33.2	Area of section: Flood, 67,060 square feet; ebb, 64,212 square feet.
	1,600	3.42	37.7	3.51	36.9	
	1,700	3.61	38.0	3.62	37.2	Mid-area = 1,337 feet.
	1,800	3.66	32.5	3.41	31.7	
	1,900	3.64	26.1	3.02	25.3	Mid-volume: Flood, 1,314 feet; ebb, 1,194 feet.
	2,000	3.53	22.1	2.66	21.3	
	2,100	3.29	18.6	2.36	17.8	
	2,200	2.81	16.1	2.08	15.3	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	2,300	2.23	12.6	1.74	11.8	
	2,400	0.26	8.2	0.27	7.4	
	2,405	0.00	....	0.00	....	
	2,500	....	10.2	....	9.4	
	2,600	....	7.0	....	6.2	
	2,700	....	5.8	....	5.0	
	2,800	....	4.8	....	4.0	
	2,900	....	4.2	....	3.4	
	3,000	....	3.8	....	3.0	
	3,100	....	3.8	....	3.0	
	3,200	....	3.4	....	2.6	
	3,300	....	3.0	....	2.2	
	3,400	....	2.5	....	1.7	
	3,500	....	2.7	....	1.9	
	3,600	....	0.0	....	− 0.8	Position of end of section: Latitude, 39° 55′ 23″.8; longitude, 75° 07′ 30″.7.
Section 30	0	....	3.3	....	2.5	Position of origin of section (on marsh, Pennsylvania side): Latitude, 39° 55′ 00″.4; longitude, 75° 08′ 07″.2.
	50	0.00	....	0.00	....	
	100	2.28	6.3	1.36	5.5	
	200	3.21	25.6	2.75	24.8	
	300	3.32	29.2	3.47	28.4	
	400	3.05	31.0	3.88	30.2	
	500	2.89	30.0	3.93	29.2	
	600	2.80	29.1	3.76	28.8	
	700	2.81	28.7	3.64	27.9	
	800	2.80	28.1	3.69	27.3	
	900	2.95	28.6	3.83	27.8	Area of section: Flood, 65,282 square feet; ebb, 63,382 square feet.
	1,000	2.96	29.4	3.50	28.6	
	1,100	2.93	30.5	3.24	29.7	Mid-area = 1,239 feet.
	1,200	2.73	31.7	3.22	30.9	
	1,300	2.63	30.4	3.51	29.6	Mid-volume: Flood, 1,245 feet; ebb, 1,133 feet.
	1,400	2.76	32.6	3.60	31.8	
	1,500	3.22	34.8	3.36	34.0	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood.		Ebb.		Remarks.
		Maximum velocity (reduced to mean tide).	Depth (at time of maximum velocity).	Maximum velocity (reduced to mean tide).	Depth (at time of maximum velocity).	
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 30—Cont'd.	1,600	3.43	38.4	3.28	37.6	Plane of reference for depths above mean low-water:
	1,700	3.52	40.1	3.51	39.3	Maximum flood, 4.75 feet: maximum ebb, 4 feet.
	1,800	3.55	39.2	3.33	38.4	
	1,900	3.44	32.6	2.68	31.8	
	2,000	3.07	26.3	2.02	25.5	
	2,100	2.57	19.4	1.24	18.6	
	2,200	1.22	14.8	0.29	14.0	
	2,230	0.00	....	0.00	....	
	2,300	....	11.8	....	11.0	[tude, 75° 07′ 38″.5.
	2,375	....	9.0	....	8.2	Position of end of section: Latitude, 39° 55′ 08″.6; longi-
Section 31........	0	0.00	5.8	....	5.0	Position of origin of section (entrance of slip, Pennsylva-
	70	....	....	0.00	....	nia side): Latitude, 39° 54′ 46″.4; longitude, 75° 08′ 00″.9.
	100	2.47	22.0	1.80	21.2	
	200	3.31	27.8	3.43	27.0	
	300	3.50	31.2	4.04	30.4	
	400	3.55	32.5	4.26	31.7	
	500	3.39	30.9	4.19	30.1	
	600	3.09	28.6	3.92	27.8	
	700	2.87	26.6	3.79	25.8	
	800	2.82	27.4	3.73	26.6	
	900	2.81	26.6	3.71	25.8	
	1,000	2.81	26.6	3.71	25.8	Area of section: Flood, 65,090 square feet; ebb, 62,850
	1,100	2.79	26.8	3.70	26.0	square feet.
	1,200	2.80	29.4	3.58	28.6	Mid-area = 1,203 feet.
	1,300	3.02	30.4	3.46	29.6	
	1,400	3.28	32.8	3.40	32.0	Mid-volume = flood, 1,178 feet; ebb, 1,051 feet.
	1,500	3.42	34.5	3.38	33.7	Plane of reference for depths above mean low-water:
	1,600	3.48	39.3	3.31	38.5	Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,700	3.39	42.6	3.06	41.8	
	1,800	3.21	42.3	2.76	41.5	
	1,900	3.13	39.2	2.28	38.4	
	2,000	2.94	17.8	1.73	17.0	
	2,100	0.33	8.5	0.35	7.7	
	2,110	0.00	....	0.00	....	
	2,200	....	5.6	....	4.8	
	2,300	....	4.8	....	4.0	
	2,400	....	4.6	....	3.8	
	2,500	....	4.1	....	3.3	
	2,600	....	3.6	....	2.8	
	2,700	....	2.8	....	2.0	
	2,800	....	0.6	....	−0.2	[tude, 75° 07′ 25″.5.
	2,820	....	0.0	....	....	Position of end of section: Latitude, 39° 54′ 53″.4; longi-
Section 32........	0	....	1.7	....	0.9	Position of origin of section (marsh, Pennsylvania side):
	50	0.00	....	0.00	....	Latitude, 39° 54′ 36″.4; longitude, 75° 07′ 59″.4.
	100	1.84	6.0	1.74	5.2	
	200	3.11	20.2	3.69	19.4	
	300	3.31	25.2	4.01	24.4	
	400	3.34	28.4	4.12	27.6	
	500	3.22	27.9	4.12	27.1	
	600	2.93	28.1	3.84	27.3	
	700	2.48	24.6	3.72	23.8	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Stations.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 32—Cont'd.	800	2.40	33.3	3.73	22.5	
	900	2.45	24.6	3.76	23.8	
	1,000	2.56	25.7	3.75	24.9	
	1,100	2.66	28.5	3.73	27.7	
	1,200	2.76	31.5	3.58	30.7	
	1,300	2.84	32.7	2.81	31.9	Area of section: Flood, 66,757 square feet; ebb, 64,533 square feet.
	1,400	2.91	33.3	2.91	32.5	
	1,500	3.00	36.2	3.33	35.4	Mid-area=1,330 feet.
	1,600	3.13	40.1	3.68	39.3	
	1,700	3.28	43.1	3.07	42.3	Mid-volume=flood, 1,322 feet; ebb, 1,115 feet.
	1,800	3.30	43.4	3.02	42.6	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,900	3.08	40.7	2.97	39.9	
	2,000	2.80	33.6	0.35	32.8	
	2,030	....	....	0.00	....	
	2,100	2.53	18.1	....	17.3	
	2,200	2.24	8.6	....	7.8	
	2,300	1.80	6.7	....	5.9	
	2,400	1.52	5.2	....	4.4	
	2,500	1.11	4.3	....	3.5	
	2,600	0.42	5.1	....	4.3	
	2,630	0.00	....	....	....	
	2,700	....	4.8	....	4.0	[tude, 75° 07′ 23″.7.
	2,780	....	1.8	....	1.0	Position of end of section: Latitude, 39° 54′ 38″.1; longi-
Section 33........	0	0.00	21.3	0.00	20.5	Position of origin of section (entrance of slip, Pennsylvania side): Latitude, 39° 54′ 29″.2; longitude, 75° 07′ 57″.6.
	80	3.14	27.3	3.50	26.5	
	180	3.66	32.0	2.95	31.2	
	280	3.80	34.0	3.02	33.2	
	380	3.26	32.9	3.92	32.1	
	480	2.72	27.0	4.12	26.2	Area of section: Flood, 67,368 square feet; ebb, 65,584 square feet.
	580	2.55	23.9	3.92	23.1	
	680	2.46	23.2	3.79	22.4	Mid-area=1,215 feet.
	780	2.44	22.2	3.72	21.4	Mid-volume=flood, 1,171 feet; ebb, 1,073 feet.
	880	2.55	23.5	3.70	22.7	Plane of reference of depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	980	2.90	27.1	3.57	26.3	
	1,080	3.24	29.7	3.32	28.9	
	1,180	3.36	32.1	2.89	31.3	
	1,280	3.42	36.6	2.99	35.8	
	1,380	3.44	39.3	3.31	38.5	
	1,480	3.45	40.3	3.47	39.5	
	1,580	3.43	40.4	3.50	39.6	
	1,680	3.03	40.3	3.40	39.5	
	1,780	2.44	40.7	2.92	39.9	
	1,880	2.04	35.9	2.06	35.1	
	1,980	1.74	29.6	1.17	28.8	
	2,080	1.38	22.3	0.37	21.5	
	2,130	....	....	0.00	....	
	2,180	0.74	8.3	....	7.5	[tude, 75° 07′ 29″.1
	2,230	0.00	4.8	....	4.0	Position of end of section: Latitude, 39° 54′ 27″.6; longi-
Section 34........	0	....	....	....	....	Position of origin of section (levee, Pennsylvania side): Latitude, 39° 54′ 19″.2; longitude, 75° 08′ 05″.5.
	70	....	0.0	....	....	
	100	....	0.6	....	− 0.2	
	200	....	2.5	....	1.7	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 34—Cont'd.	300	....	4.4	....	3.6	
	330	0.00	....	....	....	
	400	2.85	13.1	....	12.3	
	410	....	....	0.00	....	
	500	3.62	26.6	2.63	25.8	
	600	3.60	29.9	3.07	29.1	
	700	3.53	30.9	3.21	30.1	
	800	3.39	31.0	3.47	30.2	
	900	2.97	32.0	3.90	31.2	Area of section: Flood, 69,120 square feet; ebb, 66,920 square feet.
	1,000	2.69	30.3	4.01	29.5	
	1,100	2.79	28.7	3.89	27.9	Mid-area = 1,563 feet.
	1,200	2.96	28.0	3.73	27.2	
	1,300	3.16	25.8	3.70	25.0	Mid-volume = flood, 1,462 feet; ebb, 1,480 feet.
	1,400	3.34	26.5	3.70	25.7	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,500	3.50	29.5	3.45	28.7	
	1,600	3.55	33.5	3.18	32.7	
	1,700	3.55	38.0	3.01	37.2	
	1,800	3.50	39.8	3.12	39.0	
	1,900	3.38	39.2	3.36	38.4	
	2,000	3.29	38.6	3.56	37.8	
	2,100	3.21	38.3	3.56	37.5	
	2,200	3.08	35.8	3.33	35.0	
	2,290	0.00	....	....	....	
	2,300	....	31.3	2.53	30.5	
	2,400	....	20.0	1.00	19.2	
	2,500	....	16.6	0.07	15.8	
	2,510	....	....	0.00	....	
	2,600	....	11.6	....	10.8	
	2,700	....	8.8	....	8.0	Position of end of section: Latitude, 39° 54′ 16″; longitude, 75° 07′ 30″.5.
	2,750	....	4.8	....	4.0	
Section 35	0	....	− 1.2	....	− 2.0	Position of origin of section (levee, Pennsylvania side): Latitude, 39° 54′ 09″.5; longitude, 75° 08′ 07″.4.
	100	....	+ 1.8	....	+ 1.0	
	170	0.00	....	0.00	....	
	200	2.00	6.3	1.74	5.5	
	300	2.69	23.7	2.94	22.9	
	400	3.04	25.0	3.39	24.2	Area of section: Flood, 62,460 square feet; ebb, 60,540 square feet.
	500	3.24	26.1	3.54	25.3	
	600	3.33	27.3	3.57	26.5	Mid-area = 1,298 feet.
	700	3.35	26.8	3.47	26.0	Mid-volume = flood, 1,261 feet; ebb, 1,201 feet.
	800	3.38	30.4	3.30	29.6	
	900	3.49	31.5	3.13	30.7	Plane of reference of depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,000	3.55	32.2	3.10	31.4	
	1,100	3.55	32.6	3.48	31.8	
	1,200	3.54	32.4	3.92	31.6	
	1,300	3.55	32.7	3.75	31.9	
	1,400	3.58	35.6	3.58	34.8	
	1,500	3.58	38.2	3.63	37.4	
	1,600	3.58	39.2	3.70	38.4	
	1,700	3.48	38.9	3.75	38.1	
	1,800	3.23	37.2	3.80	36.4	
	1,900	3.00	34.3	3.68	33.5	
	2,000	2.65	29.1	3.30	28.3	
	2,100	2.10	21.3	2.75	20.5	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood.		Ebb.		Remarks.
		Maximum velocity (reduced to mean tide).	Depth (at time of maximum velocity).	Maximum velocity (reduced to mean tide).	Depth (at time of maximum velocity).	
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 35—Cont'd.	2,200	0.46	14.0	1.60	13.2	
	2,270	0.00	...	0.00	...	
	2,300	...	7.5	...	6.7	
	2,400	...	7.2	...	6.4	Position of end of section: Latitude, 39° 54′ 03″.6; longitude, 75° 07′ 38″.
Section 36	0	...	0.6	...	−0.2	Position of origin of section (levee, Pennsylvania side): Latitude, 39° 53′ 58″.4; longitude, 75° 08′ 11″.8.
	100	...	2.0	...	+1.2	
	200	...	3.6	...	2.8	
	230	0.00	...	0.00	...	
	300	2.20	17.8	2.89	17.0	
	400	2.81	26.0	3.62	25.2	
	500	3.18	28.4	3.43	27.6	
	600	3.48	32.0	3.26	31.2	
	700	3.60	36.3	3.15	35.5	
	800	3.68	40.6	3.26	39.8	
	900	3.69	41.4	3.55	40.6	Area of section: Flood, 60,701 square feet; ebb, 58,765 square feet.
	1,000	3.70	39.7	3.86	38.9	
	1,100	3.74	36.1	4.09	35.3	Mid-area = 1,152 feet.
	1,200	3.76	34.2	4.12	33.4	Mid-volume = flood, 1,131 feet; ebb, 1,174 feet.
	1,300	3.76	33.8	4.06	33.0	Plane of reference of depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,400	3.72	32.9	4.01	32.1	
	1,500	3.70	32.0	3.99	31.2	
	1,600	3.67	31.2	3.93	30.4	
	1,700	3.65	30.5	3.82	29.7	
	1,800	3.57	29.4	3.77	28.6	
	1,900	3.14	27.7	3.67	26.9	
	2,000	2.30	21.5	3.43	20.7	
	2,100	0.82	13.8	2.56	13.0	
	2,130	0.00	...	...	...	
	2,200	...	9.2	0.66	8.4	
	2,230	...	...	0.00	...	
	2,300	...	5.2	...	4.4	
	2,400	...	3.3	...	2.5	
	2,420	...	3.3	...	2.5	Position of end of section: Latitude, 39° 53′ 49″.5; longitude, 75° 07′ 42″.9.
Section 37	0	...	2.7	...	1.9	Position of origin of section (marsh, Pennsylvania side): Latitude, 39° 53′ 44″.8; longitude, 75° 08′ 16″.5.
	10	0.00	...	0.00	...	
	100	1.19	4.8	1.40	4.0	
	200	2.06	16.8	2.47	16.0	
	300	2.57	23.0	2.81	22.2	
	400	2.84	28.1	3.01	27.3	Area of section: Flood, 64,860 square feet; ebb, 62,916 square feet.
	500	3.03	31.8	3.13	31.0	
	600	3.13	37.0	3.20	36.2	Mid-area = 1,040 feet.
	700	3.22	43.4	3.35	42.6	
	800	3.23	47.6	3.48	46.8	
	900	3.27	48.2	3.64	47.4	
	1,000	3.36	48.1	3.79	47.3	
	1,100	3.50	48.1	3.86	47.3	
	1,200	3.59	46.7	3.81	45.9	
	1,300	3.67	44.0	3.73	43.2	
	1,400	3.61	38.9	3.64	38.1	
	1,500	3.37	31.0	3.63	30.2	
	1,600	3.19	22.3	3.50	21.5	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 37—Cont'd.	1,700	3.13	18.2	3.52	17.4	
	1,800	3.08	15.2	2.96	14.4	Mid-volume = flood, 1,049 feet; ebb, 1,045 feet.
	1,900	2.84	13.6	1.13	12.8	
	2,000	2.21	17.3	0.04	16.5	Plane of reference of depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	2,010	....	....	0.00	....	
	2,100	1.06	9.6	....	8.8	
	2,200	0.03	6.8	....	6.0	
	2,210	0.00	....	....	....	
	2,300	....	5.8	....	5.0	
	2,400	....	4.8	....	4.0	
	2,430	....	4.5	....	3.7	Position of end of section: Latitude, 39° 53′ 37″.7; longitude, 75° 07′ 46″.8.
Section 38.........	0	0.00	0.0	....	−0.8	Position of origin of section (marsh, Pennsylvania side): Latitude, 39° 53′ 31″.1; longitude, 75° 08′ 39″.4.
	100	1.06	3.8	....	3.0	
	150	....	....	0.00	....	
	200	1.48	6.5	0.61	5.7	
	300	1.67	7.9	1.65	7.1	
	400	1.82	8.7	2.20	7.9	
	500	1.84	9.0	2.46	8.2	
	600	1.77	8.3	2.60	7.5	
	700	1.72	7.4	2.45	6.6	
	800	1.69	10.0	2.24	9.2	
	900	1.67	13.5	2.40	12.7	
	1,000	1.73	17.1	2.77	16.3	
	1,100	1.91	19.8	3.24	19.0	
	1,200	2.05	21.8	3.41	21.0	
	1,300	2.09	23.1	3.38	22.3	
	1,400	2.08	24.8	3.30	24.0	
	1,500	2.05	26.7	3.28	25.9	Area of section: Flood, 85,370 square feet; ebb, 81,850 square feet.
	1,600	2.01	28.2	3.30	27.4	
	1,700	2.17	31.0	3.37	30.2	Mid-area = 2,144 feet.
	1,800	2.49	34.6	3.50	33.8	
	1,900	2.77	38.4	3.62	37.6	Mid-volume = flood, 2,178 feet; ebb, 1,984 feet.
	2,000	2.96	43.0	3.72	42.2	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	2,100	3.03	45.8	3.80	45.0	
	2,200	3.04	46.6	3.81	45.8	
	2,300	3.14	46.6	3.62	45.8	
	2,400	3.22	45.1	3.23	44.3	
	2,500	2.95	39.8	2.71	39.0	
	2,600	2.87	32.6	2.19	31.8	
	2,700	2.84	24.3	1.73	23.5	
	2,800	2.75	21.0	1.35	20.2	
	2,900	2.53	20.1	1.06	19.3	
	3,000	2.31	19.0	0.84	18.2	
	3,100	2.20	18.3	0.61	17.5	
	3,200	2.16	15.8	0.41	15.0	
	3,300	2.12	14.7	0.18	13.9	
	3,390	....	....	0.00	....	
	3,400	2.02	13.1	....	12.3	
	3,500	1.91	12.0	....	11.2	
	3,600	1.75	11.0	....	10.2	
	3,700	1.43	9.4	....	8.6	
	3,800	0.99	8.5	....	7.7	
	3,900	0.00	6.8	....	6.0	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 38 – Cont'd.	4,000	....	6.0	....	5.2	
	4,100	....	5.5	....	4.7	
	4,200	....	5.1	....	4.3	
	4,300	....	4.2	....	3.4	
	4,400	....	0.8	....	0.0	
	4,410	....	0.8	....	....	Position of end of section: Latitude, 39° 53′ 15″.4; longitude, 75° 07′ 40″.6.
Section 39	0	....	0.0	....	− 0.8	Position of origin of section (marsh, Pennsylvania side) Latitude, 39° 53′ 25″.4; longitude 75° 08′ 53″.
	100	....	5.3	....	4.5	
	110	0.00	....	0.00	....	
	200	1.35	9.2	0.94	8.4	
	300	1.89	12.0	1.60	11.2	
	400	2.04	13.4	1.73	12.6	
	500	2.04	13.3	1.74	12.5	
	600	2.00	11.7	1.65	10.9	
	700	1.97	8.8	1.60	8.0	
	800	1.98	7.4	1.54	6.6	
	900	1.97	8.1	1.53	7.3	
	1,000	1.96	9.8	1.69	9.0	
	1,100	1.92	12.0	2.01	11.2	
	1,200	1.87	15.1	2.51	14.3	
	1,300	1.83	17.4	3.03	16.6	
	1,400	1.76	18.9	3.32	18.1	
	1,500	1.74	19.3	3.31	18.5	
	1,600	1.74	19.7	3.23	18.9	
	1,700	1.89	20.3	3.21	19.5	
	1,800	2.31	21.0	3.18	20.2	Area of section: Flood, 96,093 square feet; ebb, 91,693 square feet.
	1,900	2.55	22.0	3.20	21.2	
	2,000	2.64	22.8	3.21	22.0	
	2,100	2.69	23.7	3.24	22.9	Mid-area = 2,652 feet.
	2,200	2.69	24.8	3.30	24.0	Mid-volume = flood, 2,611 feet; ebb, 2,481 feet.
	2,300	2.69	27.2	3.33	26.4	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	2,400	2.70	31.8	3.35	31.0	
	2,500	2.76	38.8	3.37	38.0	
	2,600	2.80	43.8	3.42	43.0	
	2,700	2.76	45.2	3.47	44.4	
	2,800	2.65	43.1	3.46	42.3	
	2,900	2.50	40.6	3.31	39.8	
	3,000	2.47	37.6	3.03	36.8	
	3,100	2.61	34.2	2.60	33.4	
	3,200	2.73	30.8	2.16	30.0	
	3,300	2.66	26.5	1.74	25.7	
	3,400	2.40	21.2	1.33	20.4	
	3,500	2.12	19.2	1.07	18.4	
	3,600	1.99	17.8	0.96	17.0	
	3,700	1.88	16.5	0.87	15.7	
	3,800	1.84	15.7	0.77	14.9	
	3,900	1.76	15.2	0.68	14.4	
	4,000	1.68	14.5	0.57	13.7	
	4,100	1.63	12.0	0.41	11.2	
	4,200	1.57	10.4	0.21	9.6	
	4,300	1.49	9.5	0.00	8.7	
	4,400	1.36	8.1	....	7.3	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 39—Cont'd.	4,500	1.17	7.3	....	6.5	
	4,600	0.96	6.5	....	5.7	
	4,700	0.73	6.3	....	5.5	
	4,800	0.43	6.1	....	5.3	
	4,900	0.13	6.1	....	5.3	
	5,000	0.00	5.6	....	4.8	
	5,100	....	5.5	....	4.7	
	5,200	....	5.5	....	4.7	
	5,300	....	5.4	....	4.6	
	5,400	....	5.3	....	4.5	
	5,500	....	5.1	....	4.3	[tude, 75° 07′ 57″.8.
	5,560	....	4.8	....	4.0	Position of end of section: Latitude, 39° 52′ 50″.6; longi-
Section 40.........	0	0.00	3.3	0.00	2.5	Position of origin of section (marsh, Pennsylvania side):
	100	1.46	5.7	0.84	4.9	Latitude, 39° 53′ 21″.2; longitude, 75° 09′ 08″.5.
	200	1.97	10.1	1.41	9.3	
	300	2.31	13.2	1.70	12.4	
	400	2.51	16.8	1.85	16.0	
	500	2.52	17.6	1.92	16.8	
	600	2.52	17.1	1.92	16.3	
	700	2.51	15.6	1.86	14.8	
	800	2.49	14.0	1.81	13.2	
	900	2.51	13.5	1.76	12.7	
	1,000	2.52	12.3	1.72	11.5	
	1,100	2.36	8.0	1.71	7.2	
	1,200	2.06	7.5	1.97	6.7	
	1,300	2.05	9.6	2.69	8.8	
	1,400	2.20	11.6	3.20	10.8	
	1,500	2.33	14.6	3.43	13.8	
	1,600	2.44	15.7	3.52	14.9	Area of section: Flood, 100,520 square feet; ebb, 95,820
	1,700	2.56	17.3	3.53	16.5	square feet.
	1,800	2.63	17.8	3.51	17.0	Mid-area = 3,010 feet.
	1,900	2.65	18.0	3.47	17.2	Mid-volume = flood, 2,810 feet; ebb, 2,770 feet.
	2,000	2.64	18.5	3.43	17.7	
	2,100	2.60	18.5	3.36	17.7	Plane of reference for depths above mean low-water:
	2,200	2.56	18.8	3.22	18.0	Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	2,300	2.61	19.2	3.12	18.4	
	2,400	2.64	20.2	3.19	19.4	
	2,500	2.78	20.8	3.33	20.0	
	2,600	2.65	21.8	3.42	21.0	
	2,700	2.51	22.9	3.43	22.1	
	2,800	2.28	27.1	3.43	26.3	
	2,900	2.16	33.1	3.45	32.3	
	3,000	2.42	42.4	3.52	41.6	
	3,100	2.52	44.9	3.56	44.1	
	3,200	2.56	45.0	3.47	44.2	
	3,300	2.55	43.4	3.14	42.6	
	3,400	2.42	39.1	2.74	38.3	
	3,500	2.25	34.6	2.29	33.8	
	3,600	2.12	27.3	1.91	26.5	
	3,700	2.01	23.9	1.58	23.1	
	3,800	1.93	22.3	1.30	21.5	
	3,900	1.83	21.5	1.10	20.7	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood.		Ebb.		Remarks.
		Maximum velocity (reduced to mean tide).	Depth (at time of maximum velocity).	Maximum velocity (reduced to mean tide).	Depth (at time of maximum velocity).	
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 40—Cont'd.	4,000	1.70	20.3	0.95	19.5	
	4,100	1.58	19.0	0.80	18.2	
	4,200	1.50	17.6	0.57	16.8	
	4,300	1.38	15.5	0.38	14.7	
	4,400	1.28	13.6	0.20	12.8	
	4,470	....	....	0.00	....	
	4,500	1.16	11.1	....	10.3	
	4,600	1.01	8.6	....	7.8	
	4,700	0.86	7.8	....	7.0	
	4,800	0.72	7.2	....	6.4	
	4,900	0.58	7.0	....	6.2	
	5,000	0.43	6.6	....	5.8	
	5,100	0.30	6.7	....	5.9	
	5,200	0.15	6.8	....	6.0	
	5,300	0.00	6.5	....	5.7	
	5,400	....	6.3	....	5.5	
	5,500	....	5.7	....	4.9	
	5,600	....	5.5	....	4.7	
	5,700	....	5.4	....	4.6	
	5,800	....	4.8	....	4.0	
	5,900	....	0.0	....	0.0	Position of end of section: Latitude, 39° 52′ 31″.9; longitude, 75° 08′ 28″.
Section 41........	0	0.00	4.2	....	3.4	Position of origin of section (marsh, League Island): Latitude 39° 53′ 17″.3; longitude, 75° 09′ 22″.5.
	70	....	....	0.00	....	
	100	1.68	9.3	0.72	8.5	
	200	2.21	18.4	1.81	17.6	
	300	2.52	19.5	1.98	18.7	
	400	2.69	20.2	1.99	19.4	
	500	2.68	19.8	1.99	19.0	
	600	2.54	19.4	1.97	18.6	
	700	2.35	18.5	1.84	17.7	
	800	2.31	15.9	1.34	15.1	
	900	2.33	9.5	0.73	8.7	
	1,000	2.39	9.1	0.62	8.3	Area of section: Flood, 99,780 square feet; ebb, 95,100 square feet.
	1,100	2.51	8.8	1.26	8.0	
	1,200	2.62	8.9	2.22	8.1	Mid-area = 2,765 feet.
	1,300	2.71	11.1	2.65	10.3	Mid-volume = flood, 2,389 feet; ebb, 2,700 feet.
	1,400	2.81	13.1	2.80	12.3	
	1,500	2.96	14.7	2.87	13.9	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,600	3.07	17.8	2.84	17.0	
	1,700	3.14	18.5	2.78	17.7	
	1,800	3.17	20.2	2.69	19.4	
	1,900	3.18	21.0	2.66	20.2	
	2,000	3.20	22.1	2.64	21.3	
	2,100	3.13	22.6	2.63	21.8	
	2,200	3.03	23.5	2.63	22.7	
	2,300	2.98	23.7	2.81	22.9	
	2,400	2.92	24.1	3.05	23.3	
	2,500	2.84	24.6	3.18	23.8	
	2,600	2.76	26.8	3.16	26.0	
	2,700	2.71	29.8	3.06	29.0	
	2,800	2.69	33.6	3.01	32.8	
	2,900	2.66	36.4	3.08	35.6	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood.		Ebb.		Remarks.
		Maximum velocity (reduced to mean tide).	Depth (at time of maximum velocity).	Maximum velocity (reduced to mean tide).	Depth (at time of maximum velocity).	
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 41—Cont'd.	3,000	2.62	37.5	3.19	36.7	
	3,100	2.49	38.6	3.30	37.8	
	3,200	2.29	39.8	3.34	39.0	
	3,300	2.13	39.7	3.32	38.9	
	3,400	1.99	35.5	3.20	34.7	
	3,500	1.87	28.3	2.96	27.5	
	3,600	1.76	24.3	2.56	23.5	
	3,700	1.65	23.1	2.18	22.3	
	3,800	1.47	21.1	1.79	20.3	
	3,900	1.18	17.8	1.42	17.0	
	4,000	0.73	14.3	1.04	13.5	
	4,100	0.42	9.8	0.65	9.0	
	4,200	0.17	7.6	0.26	6.8	
	4,270	....	....	0.00	....	
	4,300	0.03	7.0	....	6.2	
	4,350	0.00	....	....	....	
	4,400	....	6.8	....	6.0	
	4,500	....	6.8	....	6.0	
	4,600	....	6.2	....	5.4	
	4,700	....	6.3	....	5.5	
	4,800	....	6.2	....	5.4	
	4,900	....	6.1	....	5.3	
	5,000	....	6.0	....	5.2	
	5,100	....	6.0	....	5.2	
	5,200	....	6.0	....	5.2	
	5,300	....	6.0	....	5.2	
	5,400	....	6.0	....	5.2	
	5,500	....	6.0	....	5.2	
	5,600	....	5.6	....	4.8	
	5,700	....	5.3	....	4.5	
	5,800	....	3.8	....	3.0	
	5,870	....	0.0	....	−0.8	Position of end of section: Latitude, 39° 52′ 20″.7; longitude, 75° 09′ 06″.
Section 42........	0	....	2.9	....	2.1	Position of origin of section (marsh, League Island): Latitude, 39° 53′ 15″.7; longitude, 75° 09′ 33″.5.
	85	0.00	....	0.00	....	
	100	0.19	6.8	0.19	6.0	
	200	0.98	15.1	1.12	14.3	
	300	1.96	18.5	2.00	17.7	
	400	2.67	20.9	2.59	20.1	
	500	2.78	21.1	2.72	20.3	
	600	2.81	21.6	2.56	20.8	Area of section: Flood, 92,495 square feet; ebb, 88,175 square feet.
	700	2.77	21.4	2.15	20.6	
	800	2.65	20.7	1.71	19.9	Mid-area = 2,406 feet.
	900	2.54	15.1	1.71	14.3	Mid-volume = flood, 2,123 feet; ebb, 2,450 feet.
	1,000	2.52	11.9	1.87	11.1	
	1,100	2.61	12.2	2.00	11.4	
	1,200	2.85	13.8	2.19	13.0	
	1,300	3.03	15.1	2.40	14.3	
	1,400	3.15	17.4	2.59	16.6	
	1,500	3.22	19.8	2.67	19.0	
	1,600	3.31	22.1	2.69	21.3	
	1,700	3.40	23.3	2.69	22.5	
	1,800	3.47	24.3	2.68	23.5	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 42—Cont'd.	1, 900	3. 53	24. 8	2. 74	24. 0	
	2, 000	3. 44	24. 9	2. 88	24. 1	
	2, 100	3. 28	24. 5	2. 98	23. 7	
	2, 200	3. 11	24. 9	3. 09	24. 1	
	2, 300	3. 04	25. 5	3. 23	24. 7	
	2, 400	3. 01	28. 1	3. 38	27. 3	
	2, 500	2. 97	31. 0	3. 52	31. 1	
	2, 600	2. 92	34. 8	3. 56	34. 0	
	2, 700	2. 86	36. 5	3. 61	35. 7	
	2, 800	2. 76	36. 7	3. 62	35. 9	
	2, 900	2. 68	36. 0	3. 60	35. 2	
	3, 000	2. 57	35. 1	3. 49	34. 3	
	3, 100	2. 44	34. 5	3. 32	33. 7	
	3, 200	2. 20	33. 1	3. 06	32. 3	
	3, 300	1. 81	30. 4	2. 69	29. 6	
	3, 400	1. 31	27. 0	2. 27	26. 2	
	3, 500	0. 70	21. 9	1. 88	21. 1	
	3, 600	0. 27	13. 2	1. 45	12. 4	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	3, 690	0. 00	....	....	....	
	3, 700	....	7. 8	1. 06	7. 0	
	3, 800	....	6. 7	0. 69	5. 9	
	3, 900	....	6. 3	0. 31	5. 5	
	3, 965	....	....	0. 00	....	
	4, 000	....	6. 0	....	5. 2	
	4, 100	....	5. 5	....	4. 7	
	4, 200	....	4. 8	....	4. 0	
	4, 300	....	4. 1	....	3. 3	
	4, 400	....	3. 9	....	3. 1	
	4, 500	....	3. 6	....	2. 8	
	4, 600	....	3. 5	....	2. 7	
	4, 700	....	3. 5	....	2. 7	
	4, 800	....	3. 6	....	2. 8	
	4, 900	....	3. 6	....	2. 8	
	5, 000	....	3. 6	....	2. 8	
	5, 100	....	3. 6	....	2. 8	
	5, 200	....	3. 6	....	2. 8	
	5, 300	....	3. 6	....	2. 8	[tude, 75° 09′ 41″.8.
	5, 400	....	3. 6	....	2. 8	Position of end of section: Latitude, 39° 52′ 27″.7; longi-
Section 43	0	0. 00	1. 8	....	1. 0	Position of origin of section (levee, League Island): Latitude, 39° 53′ 12″.5; longitude 75° 09′ 58″.2.
	100	1. 13	7. 7	....	6. 9	
	150	....	....	0. 00	....	
	200	1. 91	17. 1	0. 52	16. 3	
	300	2. 37	23. 3	1. 52	22. 5	
	400	2. 64	25. 1	2. 23	24. 3	
	500	2. 77	27. 3	2. 40	26. 5	
	600	2. 79	28. 2	2. 35	27. 4	Area of section: Flood, 83,435 square feet; ebb, 80,779 square feet.
	700	2. 80	27. 1	2. 21	26. 3	
	800	2. 83	25. 8	2. 01	25. 0	Mid-area = 1,742 feet.
	900	2. 92	25. 5	2. 00	24. 7	
	1, 000	3. 00	24. 0	2. 44	23. 2	
	1, 100	3. 06	23. 9	2. 72	23. 1	
	1, 200	3. 13	25. 0	2. 77	24. 2	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 43—Cont'd.	1,300	3.17	26.6	2.83	25.8	
	1,400	3.26	27.0	2.89	26.2	
	1,500	3.35	28.2	2.95	27.4	
	1,600	3.40	29.0	3.01	28.2	
	1,700	3.30	29.2	3.05	28.4	
	1,800	3.22	30.3	3.05	29.5	
	1,900	3.14	32.0	3.11	31.2	
	2,000	3.03	33.9	3.49	33.1	Mid-volume = flood, 1,662 feet; ebb, 1,966 feet.
	2,100	2.93	35.6	3.94	34.8	Plane of reference for depths above mean low-water:
	2,200	2.81	36.3	4.22	35.5	Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	2,300	2.72	35.8	4.12	35.0	
	2,400	2.62	35.0	3.94	34.2	
	2,500	2.54	33.9	3.73	33.1	
	2,600	2.45	32.3	3.56	31.5	
	2,700	2.34	31.0	3.35	30.2	
	2,800	2.10	27.5	3.11	26.7	
	2,900	1.77	21.3	2.56	20.5	
	3,000	1.36	15.0	1.87	14.2	
	3,100	0.49	6.6	1.16	5.8	
	3,110	0.00	....	....	....	
	3,200	....	6.2	0.55	5.4	
	3,300	....	4.4	0.06	3.6	Position of end of section: Latitude, 39° 52′ 40″; longitude, 75° 10′ 10″.9.
	3,320	....	3.8	0.00	3.0	
Section 44.........	0	....	2.3	....	1.5	Position of origin of section (levee, League Island): Latitude, 39° 53′ 11″.1; longitude, 75° 10′ 20″.9.
	70	0.00	....	0.00	....	
	100	0.62	9.6	0.21	8.8	
	200	1.66	20.5	1.00	19.7	
	300	2.06	27.6	2.12	26.8	
	400	2.36	29.3	2.49	28.5	
	500	2.63	31.3	1.92	30.5	
	600	2.83	32.0	1.65	31.2	
	700	2.91	32.4	1.86	31.6	
	800	2.91	32.9	2.40	32.1	Area of section: Flood, 84,375 square feet; ebb, 81,599 square feet.
	900	2.95	33.2	2.81	32.4	
	1,000	3.11	32.3	2.90	31.5	Mid-area = 1,480 feet.
	1,100	3.29	31.5	2.96	30.7	Mid-volume = flood, 1,422 feet; ebb, 1,661 feet.
	1,200	3.42	31.5	2.99	30.7	
	1,300	3.51	32.3	3.04	31.5	Plane of reference for depths above mean low-water:
	1,400	3.39	33.3	3.09	32.5	Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	1,500	3.20	33.8	3.12	33.0	
	1,600	3.08	34.7	3.25	33.9	
	1,700	2.95	35.3	3.50	34.5	
	1,800	2.87	35.3	3.95	34.5	
	1,900	2.82	35.4	4.21	34.6	
	2,000	2.78	35.4	4.19	34.6	
	2,100	2.69	35.3	4.07	34.5	
	2,200	2.54	33.8	3.92	33.0	
	2,300	2.25	32.3	3.75	31.5	
	2,400	2.26	30.1	3.52	29.3	
	2,500	2.34	23.4	3.20	22.6	
	2,600	2.36	14.8	2.72	14.0	
	2,700	2.26	10.5	2.25	9.7	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 44—Cont'd.	2,800	2.07	9.0	1.74	8.2	
	2,900	1.80	8.0	1.25	7.2	
	3,000	1.49	6.1	0.76	5.3	
	3,100	1.12	5.2	0.31	4.4	
	3,170	....	....	0.00	....	
	3,200	0.78	5.1	....	4.3	
	3,300	0.36	4.7	....	3.9	
	3,370	0.00	....	....	....	
	3,400	....	3.9	....	3.1	[tude, 75° 10′ 18″.3.
	3,470	....	0.8	....	0.0	Position of end of section: Latitude, 39° 52′ 36″.8; longi-
Section 45.........	0	....	3.1	....	2.3	Position of origin of section (levee, League Island): Latitude, 39° 53′ 11″.8; longitude, 75° 10′ 45″.6.
	100	....	4.1	....	3.3	
	200	....	5.1	....	4.3	
	300	....	15.8	....	15.0	
	315	0.00	....	0.00	....	
	400	2.10	22.1	1.39	21.3	
	500	2.29	25.6	1.82	24.8	
	600	2.42	30.4	1.96	29.6	
	700	2.50	32.9	2.09	32.1	
	800	2.54	33.0	2.31	32.2	
	900	2.66	34.8	2.57	34.0	
	1,000	3.14	36.4	2.75	35.6	
	1,100	3.42	36.3	2.82	35.5	
	1,200	3.42	36.4	2.86	35.6	
	1,300	3.37	36.2	2.82	35.4	
	1,400	3.25	35.8	2.82	35.0	Area of section: Flood, 90,055 square feet; ebb, 87,099 square feet.
	1,500	3.14	34.8	2.82	34.0	
	1,600	3.05	35.0	2.92	34.2	
	1,700	2.93	35.1	3.22	34.3	Mid-area=1,636 feet.
	1,800	2.78	35.1	3.51	34.3	Mid-volume=flood, 1,557 feet; ebb, 1,799 feet.
	1,900	2.58	35.6	3.72	34.8	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	2,000	2.35	35.3	3.87	34.5	
	2,100	2.30	34.6	3.95	33.8	
	2,200	2.28	34.4	3.94	33.6	
	2,300	2.19	32.8	3.82	32.0	
	2,400	2.18	29.9	3.61	29.1	
	2,500	2.22	24.4	3.33	23.6	
	2,600	2.28	20.6	2.82	19.8	
	2,700	2.35	13.9	2.28	13.1	
	2,800	2.41	9.6	1.87	8.8	
	2,900	2.47	13.3	1.83	12.5	
	3,000	2.52	15.0	2.23	14.2	
	3,100	2.57	16.3	2.45	15.5	
	3,200	2.58	17.3	2.30	16.5	
	3,300	2.54	17.1	1.68	16.3	
	3,400	2.10	10.1	0.30	9.3	
	3,415	....	....	0.00	....	
	3,500	1.01	6.4	....	5.6	
	3,600	0.06	5.7	....	4.9	
	3,615	0.00	....	....	....	[tude 75° 10′ 41″.6.
	3,695	....	2.8	....	2.0	Position of end of section: Latitude, 39° 52′ 35″.4; longi-

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—**Continued.**

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 46.........	0	....	3.4	....	2.6	Position of origin of section (levee, League Island): Latitude, 39° 53′ 08″.2; longitude, 75° 11′ 13″.1.
	100	....	5.0	....	4.2	
	140	0.00	....	....	....	
	200	0.16	5.4	0.00	4.6	
	300	0.82	7.1	0.41	6.3	
	400	2.38	23.3	1.42	22.5	
	500	2.91	30.0	2.23	29.2	
	600	2.91	31.6	2.38	30.8	
	700	2.80	31.7	2.38	30.9	
	800	2.71	30.8	2.37	30.0	
	900	2.71	30.3	2.44	29.5	
	1,000	2.77	29.1	2.59	28.3	
	1,100	2.93	29.0	2.79	28.2	
	1,200	3.12	30.7	2.96	29.9	
	1,300	3.15	33.0	2.96	32.2	
	1,400	3.06	34.4	2.88	33.6	
	1,500	2.97	33.5	2.77	32.7	
	1,600	2.87	32.3	2.72	31.5	
	1,700	2.73	32.6	2.88	31.8	Area of section: Flood, 92,495 square feet; ebb, 89,055 square feet.
	1,800	2.48	32.2	3.10	31.4	
	1,900	2.29	31.9	3.34	31.1	Mid-area = 1,787 feet.
	2,000	2.20	31.6	3.50	30.8	
	2,100	2.13	31.1	3.61	30.3	Mid-volumes = flood, 1,580 feet; ebb, 1,775 feet.
	2,200	2.07	29.8	3.68	29.0	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	2,300	1.94	28.5	3.72	27.7	
	2,400	1.84	25.0	3.74	24.2	
	2,500	1.76	22.0	3.56	21.2	
	2,600	1.77	20.4	3.22	19.6	
	2,700	1.77	16.9	2.82	16.1	
	2,800	1.79	10.6	2.46	9.8	
	2,900	1.92	6.6	2.16	5.8	
	3,000	2.18	11.3	1.92	10.5	
	3,100	2.48	14.8	1.87	14.0	
	3,200	2.73	18.0	1.86	17.2	
	3,300	2.84	20.6	1.83	19.8	
	3,400	2.87	22.7	1.82	21.9	
	3,500	2.88	23.5	1.88	22.7	
	3,600	2.84	22.8	2.00	22.0	
	3,700	2.42	20.1	1.89	19.3	
	3,800	1.30	14.8	1.46	14.0	
	3,900	0.20	6.9	0.76	6.1	
	3,940	0.00	....	....	....	
	4,000	....	5.1	0.16	4.3	
	4,040	....	....	0.00	....	
	4,100	....	4.6	....	3.8	
	4,200	....	3.8	....	3.0	
	4,300	....	1.8	....	1.0	Position of end of section: Latitude, 39° 52′ 26″.7; longitude, 75° 11′ 00″.8.
Section 47.........	0	0.00	0.8	....	0.0	Position of origin of section (levee, League Island): Latitude, 39° 53′ 06″.6; longitude, 75° 11′ 28″.3.
	100	1.01	6.7	....	5.9	
	200	1.12	6.6	....	5.8	
	300	0.95	6.1	....	5.3	
	360	....	....	0.00	....	

DELAWARE RIVER—Continued.

Transverse curves of velocity, and perimeters—Continued.

Sections.	Distance from origin.	Flood. Maximum velocity (reduced to mean tide).	Flood. Depth (at time of maximum velocity).	Ebb. Maximum velocity (reduced to mean tide).	Ebb. Depth (at time of maximum velocity).	Remarks.
		Feet per sec.	*Feet.*	*Feet per sec.*	*Feet.*	
Section 47—Cont'd.	400	1.01	11.1	0.43	10.3	
	500	1.73	19.0	1.24	18.2	
	600	2.63	26.0	1.82	25.2	
	700	2.76	29.2	2.06	28.4	
	800	2.56	30.5	2.19	29.7	
	900	2.39	31.2	2.29	30.4	
	1,000	2.39	31.1	2.38	30.3	
	1,100	2.49	30.9	2.47	30.1	
	1,200	2.66	33.2	2.55	32.4	
	1,300	2.84	34.8	2.64	34.0	
	1,400	3.00	34.4	2.73	33.6	
	1,500	3.15	33.6	2.80	32.8	
	1,600	3.24	32.5	2.84	31.7	
	1,700	3.28	31.9	2.92	31.1	
	1,800	3.18	32.5	3.09	31.7	Area of section: Flood, 96,480 square feet; ebb, 92,960 square feet.
	1,900	2.91	31.1	3.27	30.3	
	2,000	2.61	29.8	3.43	29.0	
	2,100	2.37	28.8	3.57	28.0	Mid-area = 1,915 feet.
	2,200	2.23	27.2	3.69	26.4	Mid-volume = flood, 1,801 feet; ebb, 2,035 feet.
	2,300	2.11	26.1	3.82	25.3	
	2,400	1.98	23.3	3.87	22.5	Plane of reference for depths above mean low-water: Maximum flood, 4.75 feet; maximum ebb, 4 feet.
	2,500	1.91	22.2	3.78	21.4	
	2,600	1.89	21.3	3.68	20.5	
	2,700	1.89	20.0	3.61	19.2	
	2,800	1.91	17.8	3.64	17.0	
	2,900	1.92	16.6	3.66	15.8	
	3,000	1.89	15.6	3.43	14.8	
	3,100	1.85	13.1	3.03	12.3	
	3,200	1.80	11.1	2.53	10.3	
	3,300	1.80	8.9	1.74	8.1	
	3,400	1.92	10.2	1.31	9.4	
	3,500	2.16	13.8	1.49	13.0	
	3,600	2.47	18.4	1.88	17.6	
	3,700	2.75	24.1	2.14	23.3	
	3,800	2.95	28.1	2.12	27.3	
	3,900	2.92	29.5	2.14	28.7	
	4,000	2.66	29.3	2.00	28.5	
	4,100	2.11	22.8	1.72	22.0	
	4,200	1.10	7.8	1.10	7.0	
	4,300	0.00	3.8	0.00	3.0	[tude, 75° 11′ 13″.1.
	4,400	....	0.7	....	0.0	Position of end of section: Latitude, 39° 52′ 24″.2; longi-
	4,460	....	−0.2	....	....	

TABLE No. 48.

Locus of the center of the cross-section in Delaware River.

[The origin of the ordinates in this table is at intersection of meridian 75° 08′ 00″ with parallel 39° 56′ 00″.]

No. of cross-section.	Distance, in feet, on parallel x.	Distance, in feet, on meridian y.	Radius of curvature, in feet.	Direction of concavity.
47	−15, 730	−19, 386	∞	....
46	−14, 659	−19, 126	198, 000	N.
45	−12, 773	−18, 651	9, 700	S.
44	−10, 898	−15, 556	130, 000	S.
43	− 9, 365	−18, 681	152, 000	S.
42	− 7, 606	−19, 021	156, 000	N.
41	− 5, 816	−19, 161	5, 800	N.
40	− 3, 720	−18, 606	5, 100	N. W.
39	− 2, 072	−17, 311	6, 300	N. W.
38	− 1, 065	−15, 839	9, 500	N. W.
37	− 285	−14, 000	29, 000	N. W.
36	+ 160	−12, 748	24, 400	S. E.
35	+ 680	−11, 505	43, 400	N. W.
34	+ 1, 115	−10, 380	9, 200	N. W.
33	+ 1, 390	− 9, 260	4, 000	N. W.
32	+ 1, 375	− 8, 375	5, 300	S. W.
31	+ 1, 110	− 7, 180	16, 600	S. W.
30	+ 610	− 5, 600	19, 000	S. W.
29	+ 200	− 4, 580	17, 000	S. W.
28	− 375	− 3, 400	14, 300	S. W.
27	− 760	− 2, 745	3, 000	N. E.
26	− 945	− 2, 150	34, 700	W.
25	− 1, 270	− 985	8, 300	N. E.
24	− 1, 415	+ 80	103, 400	....
23	− 1, 585	+ 1, 235	....	....

	EAST CHANNEL.				WEST CHANNEL.			
No. of cross-section.	Distance, in feet, on parallel x.	Distance, in feet, on meridian y.	Radius of curvature, in feet.	Direction of concavity.	Distance, in feet, on parallel x.	Distance, in feet, on meridian y.	Radius of curvature, in feet.	Direction of concavity.
22	−510	+2, 235	....	....	−2, 135	+2, 240	....	....
21	−400	+2, 860	8, 191	N. W.	−2, 175	+3, 035	6, 058	N. E.
20	−330	+3, 820	....	....	−2, 075	+4, 035	....	....

TABLE No. 48—Continued.

No. of cross-section.	Distance, in feet, on parallel x.	Distance, in feet, on meridian y.	Radius of curvature, in feet.	Direction of concavity.
19	− 1,650	+ 5,040	....	....
18	− 1,245	+ 5,945	3,300	N. W.
17	− 1,120	+ 6,950	9,000	S. E.
16	− 860	+ 8,010	3,200	S. E.
15	− 505	+ 8,615	3,800	S. E.
14	+ 155	+ 9,320	5,600	S. E.
13	+ 830	+ 9,840	8,800	S. E.
12	+ 1,620	+10,330	3,800	S. E.
11	+ 2,380	+10,590	....	....

	NORTH CHANNEL.				SOUTH CHANNEL.			
No. of cross-section.	Distance, in feet, on parallel x.	Distance, in feet, on meridian y.	Radius of curvature, in feet.	Direction of concavity.	Distance, in feet, on parallel x.	Distance, in feet, on meridian y.	Radius of curvature, in feet.	Direction of concavity.
10	....	....	....	....	+ 3,145	+ 9,580	....	....
9	+ 3,180	+11,980	....	....	+ 4,846	+ 9,270	....	....
8	+ 4,505	+12,752	18,000	S. E.	+ 6,741	+ 8,965	2,600	N. W.
7	+ 5,736	+13,342	13,200	S. E.	+ 8,016	+ 9,585	6,100	N. W.
6	+ 7,081	+13,820	19,800	N. W.	+ 9,342	+10,730	8,100	N. W.
5	+ 8,401	+14,400	11,300	S. E.	+10,052	+11,620	3,400	S. E.
4	+10,572	+14,940	....	....	+11,216	+12,315	5,000	....

No. of cross-section.	Distance, in feet, on parallel x.	Distance, in feet, on meridian y.	Radius of curvature, in feet.	Direction of concavity.
3	+13,072	+14,300	....	....
2	+15,958	+15,400	5,800	N. W.
1	+17,953	+17,751	....	....

REPORT OF THE SUPERINTENDENT OF THE UNITED STATES COAST AND GEODETIC SURVEY.

APPENDIX No. 10.

METEOROLOGICAL RESEARCHES.

By WILLIAM FERREL.

PART II.

ON CYCLONES, TORNADOES, AND WATERSPOUTS.

CHAPTER I.

THE THEORY OF CYCLONES.

1. In the general motions of the atmosphere, treated of in the first part of these Researches, the disturbing cause is the difference of density, arising mostly from the difference of temperature between the equatorial and polar regions of the globe, and each one of the two similar systems of atmospheric circulation arising from this cause, and having one of the poles of the earth for its center, embraces a whole hemisphere. In the ordinary cyclonic disturbances of the atmosphere the causes are similar, but more local, embracing generally a comparatively small part of a hemisphere, and consist in a difference of density arising mostly from a difference of temperature between some central area and the external surrounding parts of the atmosphere. In the latter case the conditions to be satisfied are those expressed by equations (22), Part I. Where, however, the part of the earth's surface to which these equations are to be applied is of such extent only that the whole area may be regarded as a plane, and the curvature of the surface may be neglected without material error, we have the approximate and more simple equations of (23), Part I. In these latter equations $\cos\psi$ (sine of the latitude), belonging to the center, is used instead of $\cos\delta$, which differs in value between the equatorial and polar sides of the cyclone. This makes the equations symmetrical on all sides of the cyclone, when the disturbing cause is symmetrical, and consequently simplifies them very much, but it should be understood that these equations cannot be applied to very large cyclones without considerable error.

2. The expressions of some of the terms of these equations may be simplified by adopting the following notation, putting

$r=$ the linear distance from the center;

$u=$ the velocity in the direction of r;

$v=$ the velocity in a direction perpendicular to r on the left;

and putting also F_u and F_v for the components of F in the directions, respectively, of the velocities u and v, we shall then have in equations (23), Part I, $D_u \log P' = D_r \log P'$, $D_u \log a' = D_r \log a'$, $D_t^2 u = D_t u$, $D_t^2 v = D_t v$, $D_t u = u$, $F_u = F_u$, and $F_v = F_v$. If we also assume that a' is a function of r simply, or, in other words, that the disturbing cause is symmetrical on all sides, we shall have $D_v \log a' = 0$, and consequently $D_v \log P' = 0$. With these changes, by omitting the accent on a', since we for the present assume a to be the same for all altitudes, the equations become

$$(1) \quad \begin{cases} D_r \log P' = a\left((2n\cos\psi + \nu)\,v - F_u - D_t u\right) + gh\,D_r\,a. \\ 0 = 2\,(n\cos\psi + \nu)\,u + F_v + D_t v. \end{cases}$$

in which

$$(2) \quad \nu = \frac{v}{r}$$

is the angular gyratory velocity around the center, relative to the earth's surface, $n\cos\psi$ being the angular gyratory velocity due to the earth's rotation on its axis.

These are the equations of a regular cyclone, symmetrical on all sides, but it must be borne in mind, that in addition to these equations the condition of continuity must also be satisfied, or, in other words, that the motions of the air satisfying these equations must also be such as will not cause a greater or less volume of air to flow into any given space than flows out of it at the same time. This condition is satisfied by gyratory motions with uniform velocities on all sides at the same distances from the center, and is also satisfied by interchanging motions between the internal and external parts of the atmosphere of the disturbed area, provided these are such as to cause no

greater or less volume of air to move toward than from the central part, at any given distance from the center, unless there is a corresponding change in the amount of air and of barometric pressure in the interior part.

From (11) and (20), Part I, we get for the expression of the symbol a, adopted for the convenience of expression,

$$(3) \qquad a = \frac{1}{gl(1.00154 + .004t)} = \frac{1}{78455\,(1 + .004t)} \text{ very nearly,}$$

in which $g = 9.8053^{\text{m}}$ is the acceleration per second of the velocity of a falling body near the earth's surface on the parallel of 45°, and t is the temperature of the air in centigrade degrees, and $l = 7989^{\text{m}}$, is the height of a homogeneous atmosphere. For the expression of a', which is the value of a at the earth's surface, t becomes t', the temperature at the surface.

Where a cannot be assumed to have the same value at all altitudes, without its leading to too much error, but must be regarded as a function of h, then, instead of a or a' in (23), Part I, we must put $\frac{\int_h a}{h}$, which will include the small neglected term $f(h)$ in (14), Part I, from which the preceding equations have been deduced. Since a is a function which varies with the variable t, where the rate of decrease of temperature with the altitude varies much at different distances from the center of the cyclone, the use of the expression $\frac{\int_h a}{h}$ instead of a, may have considerable effect upon the conditions of the cyclone, expressed by equations (1), especially when h is great, while the effect from the absolute change of the value of a would be very little.

3. In the case of no friction we have $F_v = 0$, and from the last of (1) and from (2) we get in this case, by putting for u its equivalent $D_t r$,

$$0 = 2\,(n \cos \psi + \nu)\,D_t r + r\,D_t \nu.$$

This gives by integration

$$(4) \qquad . \quad . \quad . \quad . \quad r^2\,(n \cos \psi + \nu) = c,$$

in which c is a constant depending upon the initial state or amount of gyratory velocity of any given particle of air considered independently of the others, each particle being supposed to be free and not acted upon through friction by the contiguous particles, as implied by putting $F_v = 0$.

Since $(n \cos \psi + \nu)$ is the angular gyratory velocity due both to the earth's rotation on its axis, and the gyratory motion around the center relative to the earth's surface, the first member of (4) expresses the area swept over by the radius r in a unit of time in consequence of both parts of the gyration, and hence this equation is simply the expression of the well-known principle of the preservation of areas where there are only centripetal or centrifugal forces.

If we put

r_0 = the initial distance or value of r,
ν_0 = the initial angular gyratory velocity;

we shall have

$$c = r_0^{\,2}\,(n \cos \psi + \nu_0).$$

For the different particles of air, therefore, there must in general be different values of the constant c, for any given initial state whether of motion or of rest relative to the earth's surface.

If we now suppose that the particles of air have a mutual action upon one another through friction, but that there is no friction between the air and the earth's surface, all the particles then, whatever may have been their original state with regard to motion or rest, will be brought to have the same gyratory velocity at all altitudes at the same distances from the center, and we must in this case have the sum of the areas for all the particles equal to the sum of the initial areas, since the equality of areas cannot be affected by the mutual actions of the particles upon one another. Equation (4) then will have the same constant for each one of the particles of air, and we shall have

$$(5) \qquad . \quad . \quad . \quad . \quad r^2\,(n \cos \psi + \nu) = C$$

in which, putting m for the mass of the air,

$$(6) \quad . \quad . \quad . \quad C = \frac{\int_m c}{m} = \frac{\int_m r_0^2 (n \cos \psi + \nu_0)}{m}$$

This is upon the hypothesis of an interchanging motion simply between the interior and exterior parts, due to any cause whatever, and of no friction between the air and the earth's surface, and does not hold strictly when we take into account this friction and other conditions of the problem, as will be shown.

If the air has no initial gyratory motion relative to the earth's surface, we have $\nu_0 = 0$, and if the part of the atmosphere under consideration is circular, having the radius of the limit equal to R, we then get, upon the hypothesis that the height of the air is not materially changed by the gyrations,

$$(7) \quad . \quad . \quad . \quad C = \frac{\int r_0^2 n \cos \psi}{m} = \tfrac{1}{2} R^2 n \cos \psi$$

From (5) and (7) we get for the case in which the air is supposed to have no initial gyratory motion,

$$(8) \quad . \quad . \quad . \quad . \quad \nu = \left(\frac{R^2}{2 r^2} - 1 \right) \cos \psi$$

In the northern hemisphere, where $\cos \psi$ is positive, this expression gives a positive value of ν in the interior part, which approximates to infinity at the center, where $r = 0$, and in the external part a negative value; and hence, according to the definition of v in § 2, upon which ν depends, it gives a gyratory motion from right to left in the interior part, and from left to right in the exterior part. In the southern hemisphere where $\cos \psi$ is negative the gyrations are all reversed.

If in (8) we put $\nu = 0$, we get

$$r = \frac{R}{\sqrt{2}}$$

for the distance from the center in this case, at which the gyratory motion relative to the earth's surface vanishes and changes sign.

4. If we should substitute the value of ν in (8) in the first of (1), this equation, then, could be treated like the similar equation in the general motions of the atmosphere in § 35, Part I, and we should get an expression for the value of log P′ similar to that of (41). We should consequently have the barometric pressure at the earth's surface equal 0 at the center, and any infinitely thin stratum of equal density and pressure, of any altitude whatever in the exterior part, would be brought down to the earth's surface near the center at a greater or less distance, according to the height of the stratum in the exterior part, and the rapidity of the gyrations. In the case of a homogeneous fluid without friction, since in this case there would be a definite outline or surface, the surface of the earth would be left entirely bare for some distance from the center of gyrations. It is readily seen from the expressions that in the case of no friction the force which tends to drive the air from the center is mostly the centrifugal force arising from the great rapidity of the gyrations near the center.

At the equator, where $\cos \psi = 0$, (8) gives $\nu = 0$; hence at the equator no gyrations can arise from the influence of the earth's rotation on its axis, and if there are any gyrations there they must arise from initial gyrations whose angular gyratory velocity is represented by ν_0 in (6).

5. The preceding results have been obtained upon the hypothesis of no friction between the air and the earth's surface, and are independent of any form of the disturbing function contained in the term $g\, h\, D_r\, a$ in the first of (1), but are deduced from the last of these two equations upon the assumption simply that there is an interchanging motion between the interior and the exterior parts of the disturbed area, due to any centripetal or centrifugal forces whatever, and giving a value to u, or its equivalent $D_t\, r$, in the last of (1). But these results are very much modified in the real case of nature in which there is friction between the air and the earth's surface. We must, however, have in some degree the same kind of gyrations, since friction between the air and the

earth's surface being a function of the velocity which vanishes when the latter vanishes, there can be no friction to resist this tendency of the air to run into these gyrations under the circumstances until there is some gyratory motion produced. But the amount of the gyratory velocity is not now determined by the principle of the preservation of areas, as in the case of no friction between the air and earth's surface, but upon the amount of this friction and the force there is to overcome it. From the last of (1) we have in the case of friction

$$(9) \quad . \quad . \quad . \quad F_v + D_t v = -2(n \cos \psi + \nu) u$$

The first member of this equation expresses the friction and the inertia of the air belonging to the gyratory motion, and it is seen that the force arising from the earth's rotation, expressed by the second member of this equation, and which overcomes this friction and inertia, requires that u shall have a value; that is, that there shall be a motion of the air either toward or from the center, according to the sign of the first member under the circumstances. But this value of u, considered with regard to all the particles of air, must be such as to satisfy the equation of continuity, as well as the other conditions of the problem, and as its value must depend, therefore, upon an interchanging motion of the air between the central and exterior parts of the area of the gyrations, its value must vanish both at the center and the exterior limit, and consequently must become small as it approximates these limits. The gyrations, therefore, must vanish at both these limits and become small near these limits, especially where there is much friction. Hence, instead of the almost infinitely great gyratory velocities near the center, as in the case of no friction, we have in the case of friction between the air and the earth's surface only gyrations of small velocity, since there is little force to overcome the frictional resistances to these gyrations on account of the small value of u near the center. In large cyclones these gyrations near the center become insensible to observation over a considerable area, so that there is apparently a perfect calm.

The gyrations in this case being so much modified by the effect of friction, the amount of depression of the infinitely thin strata of equal density and pressure and diminution of barometric pressure in the central region must be also very much modified, so that instead of a very great diminution of pressure in the central part, and even a vacuum at the center, we have now in general only a comparatively small decrease of barometric pressure there.

6. The amount of gyratory velocity, in any given case, cannot be determined from the conditions, both on account of the uncertainty in the friction constant, and also on account of the complexity of the problem. There is one important principle, however, from which we may make some important deductions with regard to these gyrations, which is, that the sum of the moments arising from the actions of the gyratory motions of the air through friction upon the earth's surface, taken for the whole area of the gyrations, must equal 0, else these actions would have a tendency to turn the whole earth around the axis of which the center of the cyclone is the pole, which we know can only arise from the action of external forces.

Since F_v is the force required to overcome the friction of the gyrations, if we put F_v' for the value of F_v at the earth's surface, and let σ represent the surface, we shall have from this principle

$$(10) \quad . \quad . \quad . \quad . \quad . \quad \int \sigma r F_v' = 0$$

in which the integration must extend over the whole area of the gyrations. The values of v', taken for all values of r, must be such as to satisfy the principle expressed by the preceding equation, and for any particular value of r, such as to satisfy (9), in which ν by (2) is a function of v. From the preceding equation it follows, first, that where there are gyrations in either direction in the interior there must at the same time be counter-gyrations on the exterior part; and, secondly, since F_v' is a function which must increase with the gyratory velocity, this velocity in the interior gyrations, where r is smaller, must be much greater than in the exterior counter-gyrations, where r is much greater, unless the area of the interior gyrations, which is not in any case probable, is much greater than that of the exterior gyrations. We should therefore conclude *a priori*, as being at least most probable, that the interior gyrations are much more rapid than the exterior ones.

7. The frictional resistance to which any stratum of air is subject does not in general depend upon the absolute velocity relative to the earth, but upon the relative velocities between it and the two contiguous strata above and below, and upon the differences of density and pressure. If the actions of the contiguous strata above and below upon the intermediate one are exactly equal and

in opposite directions, the stratum is not affected by friction, since the two counter actions exactly neutralize each other. If the strata all have the same velocity in the same direction there is no friction, however great the velocity, except between the lower stratum and the earth's surface, and if the relative velocities between any stratum and the two contiguous ones are the same, the counter-actions of the two contiguous strata exactly neutralize each other, except so far as those actions differ on account of a difference of density and pressure on the upper and under side of the stratum acted upon. These relative velocities, however, may be such that the intermediate stratum is not affected by friction, whatever the absolute velocities may be.

The velocity of the stratum next the earth's surface must be of the same order as the relative velocities of the strata immediately above it, and these relative velocities must decrease with the altitude where the forces acting upon the different strata are in the same direction, since the whole action of the earth's surface through friction upon the lower stratum is distributed through the strata above by means of friction arising from the differences of the relative velocities. The velocities, therefore, must in general increase with the altitude most rapidly near the earth's surface, while the relative velocities decrease most rapidly. The friction, therefore, to which any stratum or particle of air is subject is generally greater at and near the earth's surface, where the relative velocities decrease most rapidly with the altitude, and where the pressure is greatest, than in the upper regions, where the relative velocities of the strata are generally small and the pressure less.

8. At and near the earth's surface, where the relative velocities between the strata are greatest we may assume that the force required to overcome the friction, although it depends upon the differences in the relative velocities, and not upon the absolute velocities, is sensibly in the same direction as the velocity relative to the earth's surface. If we, therefore, put

$s =$ the resultant of the component velocities u and v,

$F_s =$ the force in the direction of s necessary to overcome the friction,

$i =$ the angle between u and v, reckoned from v toward the left, called the *inclination*,

we shall have $u = -s \sin i$, $v = s \cos i$, $F_u = -F_s \sin i$, $F_v = F_s \cos i$; and supposing i to be constant we shall likewise have

$$D_t u = -D_t s \sin i, \text{ and } D_t v = D_t s \cos i.$$

We shall, therefore, have

$$F_u + D_t u = -(F_s + D_t s) \sin i,$$
$$F_v + D_t v = (F_s + D_t s) \cos i.$$

in which F_s is always positive, that is, in the direction of s, but $D_t s$ may be either positive or negative, according as the velocity is increasing or decreasing. From these and from (9) we get, by putting for u its equivalent $- s \sin i$,

$$(11) \quad . \quad . \quad \tan i = \frac{F_u + D_t u}{2(n \cos \psi + \nu) u} = \frac{F_s + D_t s}{2(n \cos \psi + \nu) s}.$$

It must be remembered that this expression is only applicable at and near the earth's surface, where it can be assumed that the force required to overcome the friction is in the direction of the velocity s.

It is not claimed that the preceding assumptions are strictly correct, and therefore this expression must only be regarded as being approximate, but as it depends mostly upon friction, the effect of which may generally be regarded as being a quantity of the second order, the error will generally be small, and at least the principal part of the effect of friction will be taken in.

Since ν is a function of i (11) cannot be regarded as an expression of $\tan i$, except approximately where ν is small in comparison with $n \cos \psi$.

If in the preceding equation we suppose that F_s is a function which increases in proportion to the velocity we can put $F_s = fs$, and neglecting $D_t s$, which is usually very small in comparison, we get

$$\tan i = \frac{f}{2\left(n \cos \psi + \frac{s \cos i}{r}\right)}.$$

If we put $f = .455$, on the latitude of 45° in which $\psi = 45°$, and with the velocity $s = 30^{km}$ per hour, we get, with $r = 1000^{km}$, $i = 48°$; with $r = 500^{km}$, $i = 45°$; with $r = 200^{km}$, $i = 37°$; and

with $r = 100^{km}$, $i = 27°$. Hence we see that, upon the above hypothesis, the inclination diminishes considerably with the decrease of distance. If we put $s = 60^{km}$ and $r = 100^{km}$, we get $i = 18°$ instead 27°, as above, with $s = 30^{km}$. Hence the inclination likewise decreases with increase of velocity, but this does not amount to much except near the center. Again, if we put $r = 500^{km}$ and $s = 30^{km}$, the value of f remaining the same as above, we get, for lat. 60°, $i = 39°$; for lat. 30°, $i = 55°$; and for lat. 20°, $i = 62°$. Hence we see that the inclination, all other things being the same, increases with the decrease of latitude.

9. With regard to the term $D_t s$ in this expression, expressing the inertia of the air, it has been shown in Part I, § 42, that where the change of hourly velocity amounts to 10 kilometers (6.2 miles) per hour, at the earth's surface the force required to overcome the inertia corresponds to that of a barometric gradient of 0.035^{mm} in the distance of one degree of the meridian. A gradient, therefore, of only 1^{mm} would produce an acceleration of about 300 kilometers per hour, in the hourly velocity, and hence it is seen how small a force is necessary to overcome the inertia of the air in all ordinary rates of acceleration, and how little the barometric gradients are affected by the term $D_t s$, or its components $D_t u$ and $D_t v$.

In the case of no friction we get from (5) by differentiation, since $r\nu = v$,

$$D_r v = -\left(2 n \cos \psi + \frac{v}{r}.\right)$$

The second member of this equation expresses the rate with which the gyratory velocity v changes with a change of r. Where the unit of velocity is one hour we have, for the parallel of 45°, $2 n \cos \psi = .3714$. With a gyratory velocity v of 50^{km} per hour at the distance from the center of 500^{km}, we get $\frac{v}{r} = 0.1$, and hence $D_r v = -0.4714$, or $\delta v = -0.4714\ \delta r$. Hence the increase in the hourly gyratory velocity is 0.2857 times the amount of approximation of a particle of air toward the center. If we assume $r = 100^{km}$ and $v = 50^{km}$ we get $D_r v = -0.8714$, or $\delta v = -0.8714\ \delta r$ and hence nearer the center we have a still greater rate of change in the gyratory velocity corresponding to a given change of r. In the first case we should have for each kilometer of approximation toward the center, an increase of 0.4714^{km}, and in the last, of 0.8714^{km} in the hourly gyratory velocities. These rates of change in the gyratory velocities are much greater than those which usually occur in any case of nature, in which the gyrations are retarded by friction. Hence the forces which, in the case of no friction, would produce so great an acceleration of the gyratory velocities, in the case of friction are mostly spent in overcoming the friction. In such cases, therefore, the term F_r must be much greater than the term $D_t v$, and consequently F_s must be much greater than $D_t s$.

10. Having shown that the absolute value of $D_t s$ is usually very small, and also that it is generally very small in comparison with F_s at the earth's surface, we can now make from (11) the following important deductions:

(*a*) At and near the earth's surface, where $D_t s$ is small in comparison with F_s and especially where it is positive, as in increasing velocities, we have in the northern hemisphere, where $\cos \psi$ is positive, $\tan i$ also positive, and hence the velocity s is either in the direction of the first or third quadrant, according as the sign of u is negative or positive, that is, according as the air in its gyrations around the center, is approaching to or receding from the center. Hence in ordinary cyclones there is an inclining of the direction at the earth's surface from the tangent toward the center on the left, and the amount of inclination is nearly in proportion to the amount of friction. In the southern hemisphere the inclination is to the right of the direction of the tangent.

(*b*) Since, according to § 7, F_s is greatest at the surface of the earth for the same value of s, and diminishes with the altitude near the surface, so the value of i is greatest at the surface and diminishes with the altitude, and therefore the gyrations are more nearly circular at some distance from the earth's surface than near the surface.

(*c*) Toward the center of a cyclone where the gyratory angular velocity is great on account of the smallness of r, and where consequently the value of ν in the denominator in the expression of $\tan i$ is great, the value of i, everything else remaining the same, is in general smaller than at greater

distances from the center, where r is larger, and consequently ν smaller. The gyrations, therefore, near the center are more nearly circular than at a great distance from the center. The greater the velocity also, everything else remaining the same, the smaller the inclination.

(d) Since $n \cos \psi$ becomes small near, and vanishes at, the equator, so the value of i, all other circumstances remaining the same, increases with the decrease of latitude, and at the equator where $\cos \psi = 0$, and where also ν vanishes, it becomes either 90° or 270°, and the motion of the air is either exactly toward or from the center, and there are no gyrations.

(e) In the upper regions of the atmosphere where there is not much friction, the inclination or deviation from the tangent is small wherever the gyratory velocity is considerable, and as the motion of the air in ordinary cyclones is *toward* the center below it must be *from* it above. Hence the gyrations at high altitudes must be nearly circular but incline a little from the tangent. This must be especially the case at medium altitudes, and at all altitudes near the center, since the radial motion either toward or from the center is small.

(f) In a cyclone increasing in violence the term $D_t s$, (11) taken for any given place, and so regarded as a function of the time simply, is positive, but in a cyclone decreasing in violence, it is negative; hence the effect of this term in the former case is to increase i, and in the latter to decrease it, but on account of the smallness of this term, as shown in the preceding section, its effect must generally be small.

11. From (11) we get

$$F_u + D_t u = 2\,(n \cos \psi + \nu)\, u \tan i.$$

With this expression of $F_u + D_t u$ substituted in the first of (1) we get, by putting for u and v their equivalents $-s \sin i$ and $s \cos i$,

$$(12) \qquad D_r \log P' - gh\, D_r\, a = D_r \log P = as \cos i\,(2\,n \cos \psi + \nu)\,(1 + e \tan^2 i)$$

in which

$$e = \frac{2\,(n \cos \psi + \nu)}{2\,n \cos \psi + \nu} = 1 + \frac{\nu}{2\,n \cos \psi + \nu}.$$

Where the gyratory angular velocity ν is so small that it may be neglected in comparison with $2\,n \cos \psi$, as in large cyclones and at a great distance from the center, we have $e = 1$, but where the angular gyratory velocity is so great, as in small cyclones very near the center, that $2\,n \cos \psi$ may be neglected in comparison with ν, we have $e = 2$. Hence the value of e must in all cases fall between these extremes.

At the distance of 1000 kilometers from the center with a gyratory velocity of 50 kilometers per hour, we should have, on the parallel of 45°, $2\,n \cos \psi = 0.371$ and $\nu = .050$. With these values we should have $e = 1.119$, for which unity could be used in the small term in which e occurs, without sensible error. At the distance of 100 kilometers, however, with the same gyratory velocity, we get $e = 1.574$, and for greater gyratory velocities and smaller distances from the center, the value of e approximates still nearer to the maximum limit 2. But we have seen § 8, (c) that where the conditions are such as to make the value of e differ much from unity, the value of $\tan^2 i$ becomes very small on account of the diminished value of i, and hence we may in all cases put $e = 1$ without material error.

12. Putting $e = 1$, (12) becomes

$$(13) \qquad D_r \log P = a \frac{2\,n \cos \psi + \nu}{\cos i}\, s = a \frac{2\,n \cos \psi + \nu}{\cos^2 i}\, v$$

in which v is the gyratory component of s.

We have seen (§ 6) that if v is positive in the interior gyrations it is negative in the exterior ones, and *vice versa*. But v, and consequently ν, have the same sign as $\cos \psi$ in the interior gyrations, positive in the northern and negative in the southern hemisphere. Hence from (13) $D_r \log P$ is always positive in the interior and negative in the exterior gyrations in both hemispheres, and the barometric pressure, P, is a minimum at the center, and a maximum at the distance from the center at which v changes sign and becomes reversed.

At the earth's surface, according to § 10 (a), $\tan i$ is positive in the northern hemisphere and negative in the southern, and hence in all cases it has the same sign in the interior and exte-

rior gyrations, either both positive or negative. But from the definitions of u, v and i, we have $\tan i = -\frac{u}{v}$. Hence where v changes and the gyrations are reversed, the sign of u must also change. If, therefore, v is positive, as in the northern hemisphere, the sign of u must be negative, that is, the radial component of motion must be *toward* the center, but in the exterior gyrations where v becomes negative the sign of u must be positive, that is, the radial component of motion must be *from* the center.

We see, therefore, that in the northern hemisphere the interior gyrations at the earth's surface are positive, that is, from right to left, and the air at the same time approaches the center, so that the gyrations are in a spiral around and *toward* the center, but the exterior gyrations are in the contrary direction and the air flows from the center, so that the gyrations are around and *from* the center. Since the maximum pressure is where the gyrations are reversed, the air flows out from beneath the area of high pressure, on the one hand toward the center, and on the other hand from it, and at the same time gyrates around the center. The directions of motion of the air at the earth's surface, as indicated by the solid arrows, and the circular isobars are represented in Fig. 1. In the southern hemisphere, the gyrations are all reversed.

For reasons already given in § 6, the interior gyrations are generally by far the most rapid, and from a mere inspection of (13) it is readily seen that these produce by far the greatest effect upon $D_r \log P$, or, in other words, upon the barometric gradient, since ν there becomes much greater for the same value of v. These interior gyrations, therefore, and their effects upon barometric pressure, are often alone observed, while the comparatively moderate gyrations of the exterior part and their small effect upon the barometric pressure often pass unnoticed, especially on land where there are a great many disturbing causes to interfere with the development of a regular and complete cyclone. The former gyrations form the cyclone proper, and the latter the anti-cyclone. Hence, *every cyclone is accompanied by a corresponding anti-cyclone, and the former cannot exist without the latter.*

13. The preceding results with regard to cyclones, deduced from the mathematical expressions, may be made more intelligible to many by means of a more popular demonstration or explanation of them, based upon certain well-known principles. The instantaneous motion due to the earth's rotation on its axis, of any part of its surface except the equator, and so small that it may be regarded as a plane surface, may be resolved into two parts, one of translation, and one of gyration around the the center of the area under consideration. At the poles of the earth it is readily seen that this gyratory motion is the same as that of the earth's rotation on its axis, and that at the equator it must vanish. For any intermediate latitude it is expressed by $n \cos \psi$, or n multiplied into the sine of the latitude. If now, for any reason whatever, there is kept up a continued interchange of the air between the central and exterior part of this area continually gyrating around its center in consequence of the earth's rotation, it will be understood by most persons that, by the principle of the preservation of areas, the air of the interior part of this area will receive a gyratory motion around the center, relative to the earth's surface, in the same direction in which the area comprising the atmosphere under consideration is turning by virtue of the earth's rotation, that is, from right to left in the northern hemisphere, and the contrary in the southern, and that the gyrations of the exterior part of this area must be contrary to those of the interior part. These gyrations, especially near the center and the exterior limit, would be very rapid if it were not for the friction between the air and the earth's surface. This frictional resistance prevents in a great measure the rapidity which the gyrations would otherwise have, but there must necessarily be some gyratory motion, as in the case of no friction, since without some such motion frictional resistance would not be brought into action.

If we now bear in mind the well established principle that wherever any body or particle of air moves in any direction over the earth's surface, there is a force arising from the earth's rotation on its axis which tends to deflect it to the right in the northern hemisphere, and the contrary in the southern, it is seen that, in either hemisphere, there is a force arising from these interior gyrations and the earth's rotation which tends to drive the air from the center and cause a minimum pressure there. The exterior gyrations being in the contrary direction, of course this force is, in this case, in the contrary direction, that is, toward the center. These two forces, then, depending upon the interior and exterior gyrations, both press the air toward the place where the gyratory

velocity vanishes and changes sign, and cause a maximum barometric pressure there. Of course only the gyratory component of motion is here considered. When there is also a radial component, the deflecting force arising from the earth's rotation is required to overcome the frictional resistance to the gyrations, and so, when there is no friction, no radial component is necessary and the gyrations may be entirely circular.

In addition to this force depending upon the earth's rotation there is, likewise, the centrifugal force due to the gyrations. This force is of course *from* the center in both the cyclone and anti-cyclone, and hence in the former it is combined with that depending upon the earth's rotation, but in the latter in the contrary direction, and in some measure counteracts the force arising from the earth's rotation. The centrifugal force, however, in the anti-cyclone is very small, both on account of the small velocity of gyration and the distance from the center, but in the cyclone, very near the center, it may be much the greater of the two forces.

Since the amount of gyratory motion arising from the interchanging motions of the air between the interior and exterior parts depends upon friction, and the less the amount of friction the more nearly these gyrations approximate to what they would be in the case of no friction, these gyrations, especially the initial ones, must be much more rapid in the upper regions of the atmosphere, where there is comparatively little friction, than near the earth's surface where the friction is very much greater. Hence the accumulation of air with its maximum at the dividing line between the cyclone and anti-cyclone, as already explained, is due at the start mostly to the gyrations in the upper part of the atmosphere, and the pressure from this accumulation tends to force the air out from beneath on the one side toward the center of the cyclone and on the other toward the outer limit of the anti-cyclone. The deflective force, now, arising from these radial motions, as explained above, aids in overcoming the greater friction of the gyrations near the earth's surface, but these latter gyrations must not be so great that the force pressing from both sides toward the place of maximum pressure would prevent the outflow on both sides below, else the deflecting force then which overcomes the resistances to the gyrations would be cut off.

According to the preceding reasoning and explanations, therefore, wherever there are disturbing causes to produce an interchanging motion between the interior and the exterior parts of the disturbed atmosphere, from the principle of the preservation of areas, and the forces depending upon the earth's rotation, there are necessarily interior gyrations of the air in one direction and counter-gyrations in the exterior part and a region of high barometric pressure with its maximum at the dividing line between the two kinds of gyrations, and a flow of air at the surface from the region of high pressure, on the one side toward the center, and the other toward the outer limit of the anti-cyclone, which, combining with the gyratory motions, gives rise to a spiral motion in the interior around and toward the center, and in the exterior part around the contrary way and from the center, just as has been deduced from the preceding mathematical expressions.

14. From (13) and (3) we get, by putting $P' = 0.76^{m}$,

$$(14) \quad . \quad . \quad D_r P = \frac{2 n \cos \psi + \nu}{\cos i \, (1.00154 + .004 t)} \cdot \frac{s}{103070} \cdot \frac{P}{P'}$$

in which s is the velocity in meters per second. At the earth's surface, or rather at the altitude at which the barometric pressure is 0.76^{m}, the last factor in the expression becomes unity. If we now put

$$(15) \quad . \quad . \quad . \quad D_r P = \frac{\delta P}{\delta r} = \frac{G}{111111111}$$

G will express the barometric gradient in millimeters per degree of a great circle, or 60 geographic miles. With this value of $D_r P$ (14) gives sensibly,

$$(16) \quad . \quad . \quad . \quad G = \frac{1076.4 \, (2 n \cos \psi + \nu) \, s}{\cos i \, (1 + .004 \, t)} \frac{P}{P'}$$

in which by (3), putting for v its equivalent $s \cos i$,

$$(17) \quad . \quad . \quad . \quad . \quad . \quad . \quad \nu = \frac{s \cos i}{r}$$

Equation (16) shows the relation between the barometric gradient G and the velocity s of the wind, and is substantially the same as that which I had published in Silliman's Journal for 1874,

except that the temperature correction, here given in the expression of G, was there simply referred to in the text, but the gradients and units of measure here used are different.

Two years before this, Mr. Peslin, mining engineer at Tarbes, France, had obtained relations substantially the same as the preceding, except the effect of friction contained in the term $\cos i$, which in the case of no friction becomes unity.* According to the theory this effect becomes especially large towards the equator, and it will be shown from observation that in all latitudes the effect is very considerable, and that where the angle i is not observed, the theoretical relation above is not confirmed by observation, except upon the hypothesis of a large value of i, and consequently of a large effect due to friction.

The fundamental equations from which the relation of (16) has been obtained, except so far as friction is concerned, were first given by the author in a paper in Runkle's Mathematical Monthly in 1859, equations (38). At that time the term "gradient" was not in use. By expressing the altitude h in these equations in terms of the barometric pressure P of the preceding equations, and applying the friction terms, as above, and introducing the barometric gradient instead of the differential coefficient, the relations above are readily obtained.

The value of $\cos i$ in (16) depends upon the amount of friction, and hence must be obtained from observation, and when friction vanishes we have $\cos i = 1$. Since (11) is strictly applicable only near the earth's surface, and this equation has been used in obtaining (13), the expression of G is strictly applicable only at and near the earth's surface; but the error at a distance above the surface is only in the effect of friction, and since friction is small there, and the gyrations more nearly circular than at the earth's surface, for considerable altitudes we can put $\cos i = 1$, and with this value of $\cos i$ this expression of G is applicable to these altitudes without material error.

The coefficient of t in the expression G is such as to include in the temperature correction that also for the average amount of aqueous vapor, taken over the earth generally, and the balance may be neglected. The value of $2n$, where one second is the unit of time, is equal to .00014585, and the value of ν is given by (17).

From (16) and (17) we get

$$s^2 + a\,s = b\,G \tag{18}$$

in which

$$\left\{\begin{aligned} a &= \frac{.00014585\, r \cos \psi}{\cos i} \\ b &= \frac{r\,(1 + .004\,t)}{1076.4}\,\frac{P}{P'} \end{aligned}\right. \tag{19}$$

From (18) we have

$$s = -\frac{1}{2}\,a \pm \sqrt{\frac{1}{4}a^2 + b\,G} \tag{20}$$

The expression of (16) is convenient for computing the gradient corresponding to a given velocity s, given in meters per second, but the latter can be most conveniently used in computing the velocity s, corresponding to a given gradient G.

Where the unit of distance is one kilometer and the unit of time one hour, we have

$$\left\{\begin{aligned} a &= \frac{0.52505\, r \cos \psi}{\cos i} \\ b &= \frac{r\,(1 + .004\,t)}{.083055}\,\frac{P}{P'} \end{aligned}\right. \tag{21}$$

When the unit of distance is one mile and that of time one hour, and the gradient is expresed in inches per 60 geographic miles, we have

$$\left\{\begin{aligned} a &= \frac{0.52505\, r \cos \psi}{\cos i} \\ b &= \frac{r\,(1 + .004\,t)}{.005262}\,\frac{P}{P'} \end{aligned}\right. \tag{22}$$

In all the preceding expressions, as in the whole theory of cyclones, the effect of friction comes

* Bulletin International de l'Obs. de Paris et de l'Obs. Phys. Cent. de Montsouris, 1872. Translation by Prof. Cleveland Abbe, Smithsonian Report for 1877.

in in the value of i, which depends mostly upon friction, and i is therefore an unknown quantity which must be determined either directly in each case from observation, or if we assume some function to represent the law of friction, as in § 8, then the unknown constants in this expression depending upon friction must be determined from observation.

15. The preceding results so far are all independent of any particular form of the expression of t, regarded as a function of r, since the term $gh\,D_r\,a$ in the first of the fundamental equations (1), which alone contains t, does not enter into any of the preceding expressions. It has merely been assumed that there is a disturbing force of some form to keep up an interchanging motion between the interior and exterior portions of the air over a given area. It is readily seen from an inspection of equations (1) that if $D_r\,a = 0$, that is, if t in the expression of a (2) is independent of r, the equations in the case of friction are satisfied with $u = 0$ and $v = 0$, that is, with a state of rest; for if there were initial motions they would soon be destroyed by friction. The interchanging motions, therefore, necessary to keep up the gyrations of the cyclone depend upon a variation of t at different distances from the center. It becomes now important to ascertain the effects produced by different functions of t entering into a in the term of $gh\,D_r\,a$. This can be done only approximately on account of the uncertainty of the friction terms. It has been shown that both the friction terms, and those expressing the inertia of the air, are generally small in comparison with those depending upon the earth's rotation and with the centrifugal force arising from the gyrations where r is small, so that these terms may be neglected without much error, in obtaining approximate results.

If in the first of (1), therefore, we neglect F_u and $D_t\,u$, putting as heretofore ν' and v' for the values of ν and v at the earth's surface, we get

$$(2\,n\cos\varphi + \nu')\,v' - (2\,n\cos\varphi + \nu)\,v = gh\,D_r\log a.$$

Except near the center where ν may be great in comparison with $2\,n\cos\varphi$, we can put $\nu = \nu'$ especially when h is not large, and we then get approximately

$$(23) \quad . \quad . \quad . \quad . \quad (v' - v =) \frac{gh\,D_r\log a}{2\,n\cos\varphi + \nu'}$$

Equation (5) deduced from the hypothesis simply of an interchanging motion between the exterior and interior parts, gives the same gyratory velocity at all altitudes for the same distance from the center, but now upon taking into account another condition of the problem, namely, the difference of density or temperature between the exterior and interior part, which gives a value to $D_r\,a$, it is seen that these velocities must vary some at different altitudes, depending upon the amount of difference temperature.

16. Let us suppose that t in (3) is such a function of r that we shall have

$$(24) \quad . \quad . \quad . \quad . \quad . \quad t = t_o + C + C\cos\varphi.$$

in which

$$\varphi = \frac{r}{R}\,\pi.$$

R being the value of r at the outer limits of the supposed circular part of the atmosphere affected by a difference of temperature, and hence the value of φ ranges from 0 to 180°. This expression makes the temperature a maximum or a minimum at the center, according as the constant C is plus or minus, t_o being the undisturbed temperature at and beyond the limit of the supposed circular portion of the atmosphere.

From (24) we get

$$(25) \quad . \quad . \quad . \quad D_r\,t = \frac{\delta\,t}{\delta\,r} = -\,C\,\frac{\pi}{R}\sin\varphi$$

which is the expression of the temperature gradient, and is as the constant C and inversely as the radius R. It has its maximum where r is equal to $\frac{1}{2}$ R, and vanishes at the center and the outer limit.

If t_c is temperature at the center we have

$$(26) \quad . \quad . \quad . \quad . \quad . \quad . \quad C = \tfrac{1}{2}\,(t_c - t_o).$$

If t_c and t_o increase or decrease with the altitude h, in the same ratio, then C is a constant for all altitudes, and depends upon the surface temperature.

In the original equations, for the sake of simplicity, the temperatures were supposed to be the

same at all altitudes, in which case C is independent of h. But if t_o and t_c vary with the altitude at the same rate, C is still independent of h, and in this case the expression of (23) can be used by using a at half the altitude of h.

From (3) and (25), where C is constant for all altitudes, we get

$$(27) \quad . \quad . \quad . \quad gh\,\mathrm{D}_r \log a = \frac{.004\,\mathrm{C}\,\pi}{1 + .004\,t_o}\frac{h}{\mathrm{R}}\,g \sin \varphi.$$

With this expression (23) gives, putting $g = 9.805^{\mathrm{m}}$,

$$(28) \quad . \quad . \quad (v' - v) = \frac{0.1226^{\mathrm{m}}}{1 + .004\,t_o}\,\frac{h}{\mathrm{R}}\,\frac{\mathrm{C}}{2\,n \cos \varphi + \nu} \sin \varphi.$$

in which t_o may be taken at the earth's surface, but more accurately at half the altitude h.

17. From the first of (1) we get for the initial state, before gyrations are caused by the interchanging motions between the interior and exterior parts, and when we have yet $v = o$.

$$(29) \quad . \quad . \quad . \quad a\,(\mathrm{F}_u + \mathrm{D}_t\,u) = gh\,\mathrm{D}_r\,a - \mathrm{D}_r \log \mathrm{P}'$$

In the case in which C is positive, and the temperature gradient by (25) is negative, that is, in which the temperature decreases from the center out, $gh\,\mathrm{D}_r\,a$ by (27) is positive, and $\mathrm{D}_r \log \mathrm{P}'$, which is not a function of the altitude, must have such a value that $gh\,\mathrm{D}_r\,a - \mathrm{D}_r \log \mathrm{P}' = 0$, at some intermediate altitude, or value of h, below which the last member of (29) is negative and tends to produce motion toward the center, and above, negative and tends to produce motion from the center. This value of h and the whole distribution of the motions must be such as to satisfy the condition of continuity. In order to do this there must be an ascending motion in the interior part, and a descending one in the exterior part, both generally very small in comparison with the horizontal motions toward and from the center. When C is negative, and consequently the temperature is a minimum in the center and increases from the center out, of course these initial interchanging motions are reversed, being from the center below, and toward it above, with a comparatively small descending motion in the central part and ascending motion in the exterior part. When the temperature gradient, or value of C, is not the same at all altitudes, these interchanging motions are more complex.

We have seen that gyratory motions are produced from any kind of interchanging motions, giving rise to a cyclone and its corresponding anti-cyclone. After these have once set in from either of these interchanging motions toward the center below, and from it above, or the reverse, and an accumulation of air has been caused thereby, with its maximum where the gyrations are reversed, the current flowing out from beneath this accumulation on both sides, as has been explained, combines with the general current in the lower part of the air, toward the center in the case of a cyclone with a warm center, and increases it in the cyclone, but counteracts and completely reverses it near the earth's surface in the anti-cyclone, so that the air there, regarding only its interchanging motions, flows from the center, although at a little elevation above the surface of the earth it flows toward the center. In the case of a cyclone with a cold center the reverse takes place, the general interchanging current of the lower part of the air being generally diminished or even reversed near the earth's surface in the cyclone but increased in the anti-cyclone.

Equation (28) has been obtained upon the hypothesis that F_u and $\mathrm{D}_t\,u$ in the first of (1) may be neglected in comparison with the terms arising from the effect of the earth's rotation and the centrifugal force of the gyrations. It has been shown that the absolute value of $\mathrm{D}_t\,u$ is very small, §9, and in the general circulation in vertical section toward and from the center, it has in general the same sign above as below, so that it would have little effect upon the value of $(v'-v)$, even if the term were large. The term F_u vanishes in the case of no friction, so that the less the friction the smaller is this term. The approximate expression, therefore, of $v'-v$ in (28) must be very nearly correct for the upper regions of the atmosphere, and even very near the earth's surface, where the friction is much greater, the effect of the neglected terms is generally small.

18. According to (28), $(v'-v)$ is positive where C is positive, and is in proportion to the altitude h. Hence in a cyclonic system, with a warm center, the gyratory velocity v decreases algebraically with the altitude, or, in other words, decreases in the cyclone and increases in the anti-cyclone; but in the case of a cold center the reverse takes place, and the gyratory velocity of the cyclone

increases and that of the anti-cyclone decreases with the altitude h. In the system, therefore, with a warm center, the distance from the center at which the gyrations are reversed from the cyclonic to the anti-cyclonic decreases with the increase of altitude, and the more so the greater the value of C, so that at a considerable altitude, and with a large temperature gradient, or valve of C, the gyratory velocity v may become negative at all distances from the center, however small, although it may have a large positive value at the earth's surface, and thus above that altitude the gyrations at all distances from the center are anti-cyclonic. But in the case of a cold center the distance from the center at which the gyrations change from the cyclonic to the anti-cyclonic increases with the altitude, so that at a certain altitude less or greater, according to the negative value of C in this case, the gyrations may all become cyclonic, however great the negative value of v', the value of v at the earth's surface may be in the anti-cyclone.

In the case of an ordinary cyclonic system the directions of the currents in the upper regions are represented by the dotted arrows in Fig. 1, in which the anti-cyclonic gyrations above are supposed to overlap a considerable part of the cyclonic gyrations at the surface.

19. From (16) it is seen, remembering that $s = \frac{v}{\cos i}$, that the barometric gradient at any altitude, that is, for any value of P, has the same sign as the gyratory velocity v, positive where the gyrations are cyclonic and negative where they are anti cyclonic. Hence at all altitudes, as well as at the earth's surface, the maximum barometric pressure is where the gyrations are reversed, so that in a cyclonic system with a warm center the greatest pressure, at a considerable altitude, may be very near or even at the center, and the pressure decrease thence to the outer limit of the anti-cyclone, although at the earth's surface there may be a low minimum barometric pressure at the center. This result, deduced from (16), is also evident from other considerations, for cyclonic gyrations tend to drive the air *from* the center, but anti-cyclonic ones *toward* the center. If all the gyrations, therefore, above a given altitude become anti-cyclonic, as they may according to the preceding section, they must cause a heaping up of the air in the central part in such a manner that the maximum pressure is at the center, and the thin strata of equal density and pressure become convex in the central part instead of concave as at the earth's surface, and the greater the altitude the more so. The thicknesses of the strata bounded by surfaces of equal pressure are therefore greater in the rarefied central part than in the more dense exterior part, but so far as weight or pressure at the earth's surface are concerned, the rarefaction at the center is exactly compensated by the greater thicknesses of the strata, so that the pressure P′ at the earth's surface is entirely independent of the temperature gradient, and depends only upon the amount of gyratory velocity v' at the earth's surface, as shown by (13), and upon friction as affecting the value of cos i.

In a cyclonic system with a cold center we have the reverse. At the earth's surface the anti-cyclonic gyrations give a negative gradient, according to (16), but at a certain altitude, where all the gyrations may become cyclonic, the gradient is positive from the center to the outer limit of the anti-cyclone, so that the pressure at that altitude is a minimum at the center and a maximum at the exterior limit. The cyclonic gyrations drive the air from the center and cause a depression of the strata bounded by surfaces of equal pressure, and the greater the altitude the greater is this depression. The thicknesses of these strata, therefore, are less in the central than in the exterior part; but this diminution of thickness, so far as pressure on the earth's surface is concerned, is exactly compensated by the greater density in the cold central part, so that the pressure then depends only upon the gyrations there, and upon friction in affecting the value of cos i in (13), and not upon the temperature gradient.

At the equator, where the gyrations vanish and the motions all become radial, § 10 (d), the difference in barometric pressure depends upon the friction and inertia terms alone, but these must be such that the force arising from the difference of pressure due to the temperature gradient is just sufficient to overcome them, and hence the velocity in the case of friction must be such that the amount of friction is equal to the force, and where there is no friction the whole force is spent in overcoming the inertia of the air, and there is then a continued acceleration of radial interchanging motion between the central and exterior parts of the air, disturbed by a difference of temperature, so long as this difference is maintained.

FIG. 1

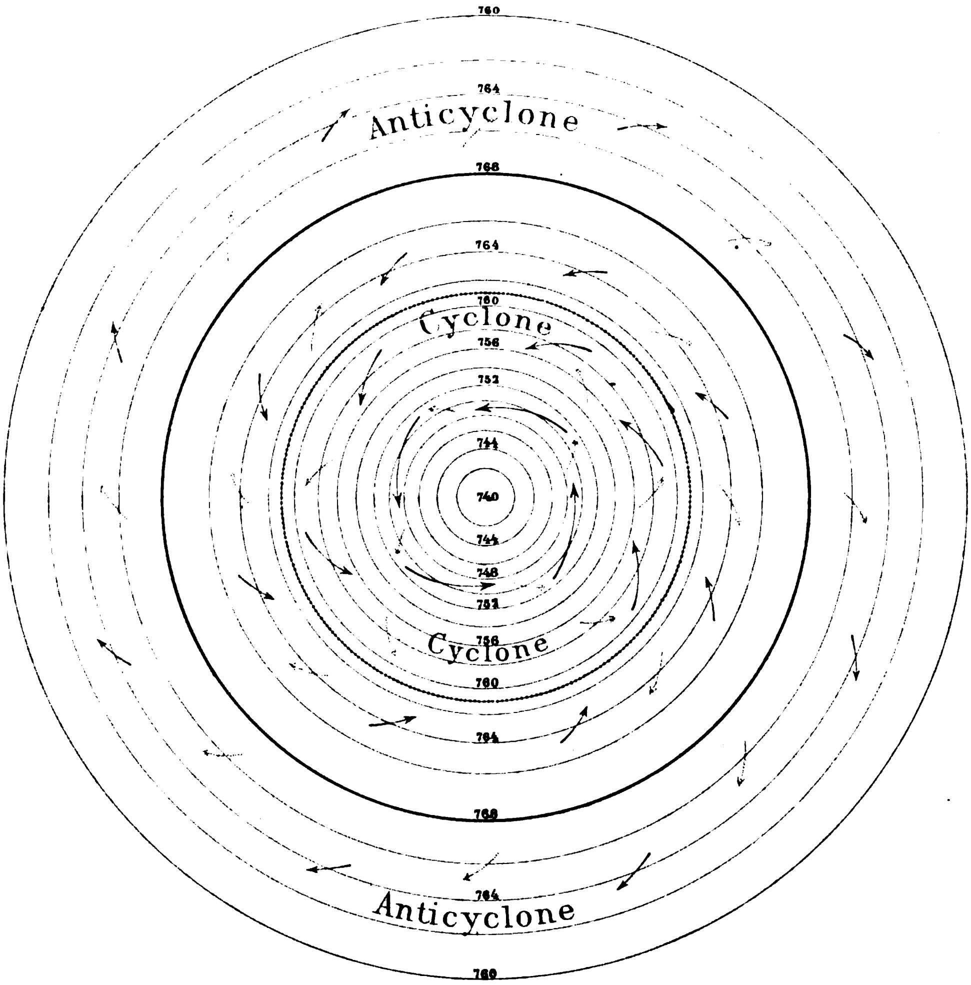


FIG. 2

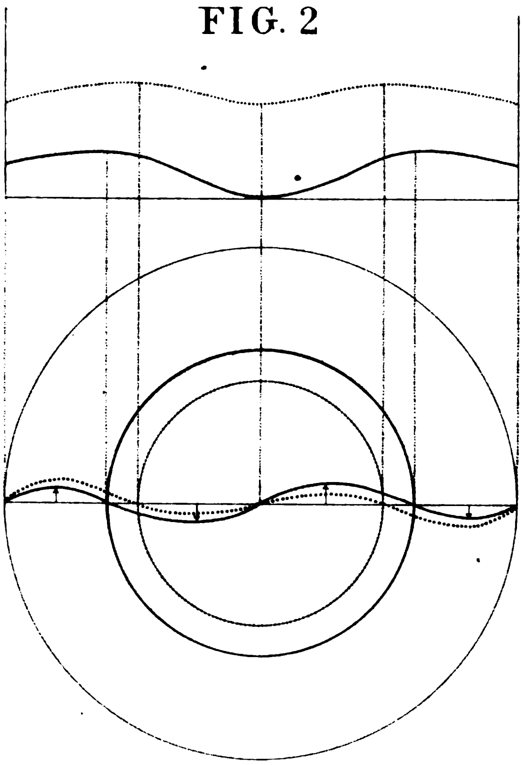
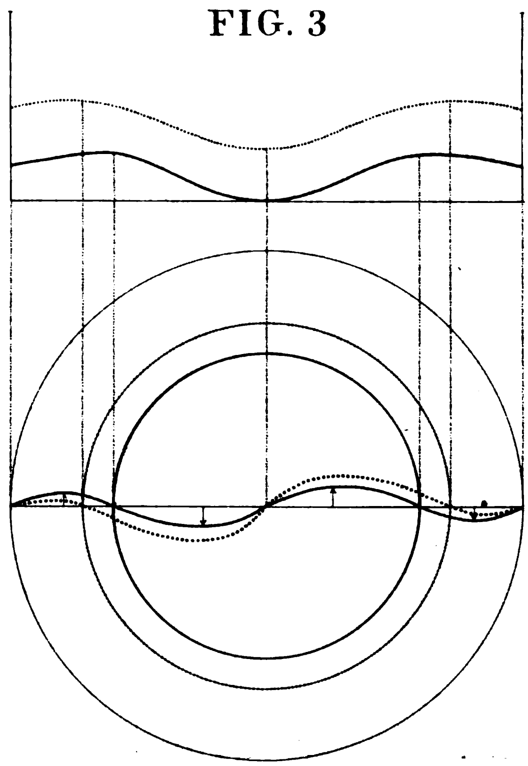


FIG. 3



Julius Bien Lith.

In Fig. 2 and Fig. 3 is given a graphic representation of the gyratory velocities of cyclones at the earth's surface, and of the relations between these and the gyratory velocities at any altitude above the surface, together with the forms of the corresponding barometric curves of a section passing through the center of the cyclone. The former belongs to a cyclone with a warm center, in which the cyclonic gyratory velocities are diminished above, and the latter to a cyclone with a cold center in which the reverse takes place. The co-ordinates of the unbroken curved lines perpendicular to the radius, as indicated by the arrows, represent the gyratory velocities at the earth's surface, represented by v', and the corresponding curved line of the same kind in the upper part of the figure represents the barometric curve of pressure at the earth's surface. The maximum of pressure is at the distance at which the gyratory velocities vanish and change signs, as already shown. In like manner the perpendicular co-ordinates of the dotted curved lines show the gyratory velocities at some altitude above the earth's surface, greater or less, according to the temperature gradient, and the corresponding dotted curved line in the upper part of the figure represents the corresponding barometric curve. In this case, also, the maximum of barometric pressure is at the distance at which the gyrations vanish and change signs, which, in the cyclone with a warm center, is nearer the center, and in the one with a cold center further from the center than in the case of the gyrations at the earth's surface.

Either the altitude or the temperature gradient, or both, might be so great as to make the gyratory velocities and the co-ordinates representing them all negative in the cyclonic system with a warm center and positive in the one with a cold center. The maximum of the curves then representing the barometric pressure at a considerable altitude would be at the center in the former case and at the outer limit of the anti-cyclone in the latter.

20. According to (28) the gyratory velocities of a cyclone with a warm center decrease as the altitude h where C, and consequently the temperature gradient, are the same at all altitudes. But since the friction terms were neglected in obtaining (28) it is only approximate, and near the earth's surface where friction is great but rapidly diminishes with the altitude, the gyratory velocities increase with the altitude much more rapidly than at greater altitudes where the amount of friction is small, and where, consequently, the velocities must have nearly the relations given by (28). If instead of using the first of (1), neglecting the friction and inertia terms, we had used (13), putting $v = s \cos i$, we should have obtained for the first member of (28) $\left(\frac{v'}{\cos^2 i'} - \frac{v}{\cos^2 i}\right)$, and since at and near the earth's surface $\cos^2 i'$ is much less than $\cos^2 i$ at some altitude above, since i increases with the amount of friction, v', in order to satisfy the conditions where friction is considered, must increase with the altitude more rapidly near the earth's surface than at considerable altitudes. But for the reason that these gyratory velocities in comparison with those above are less near the surface than is required to satisfy (28), the force which arises from these gyrations and tends to drive the air from the center is much less there, and consequently the air flows in toward the center with much greater facility than at higher altitudes where the gyratory velocities are much greater. The velocity, therefore, of inflow toward the center is much the greatest near the earth's surface, while the maximum velocity of the gyrations is at a greater altitude. The directions, therefore, of the currents in the lower strata of a cyclone must be more inclined toward the center than at a very moderate altitude above the surface, as is likewise shown by the greater value of i near the earth's surface, as deduced from (11) in § 10 (b). If an observer at the earth's surface, therefore, stands with his face toward the wind of a cyclone, the currents, and likewise the directions of the clouds, at some altitude above him, must be from some point of the compass at his right hand in the northern hemisphere, and the contrary in the southern, and at very great altitudes, as in the region of the cirrus clouds, where the gyratory velocities are much less in the case of ordinary cyclones, and may even be reversed, and where the radial motion is outward from the center instead of toward it, these directions may vary so much that they may seem to cross one another at right angles, or even to be somewhat counter to one another. The directions of motion in the upper regions are indicated by the dotted arrows in Fig. 1. The area of cyclonic gyrations and also their velocities here, according to what precedes, is much diminished, and those of the anti-cyclone much

increased, especially at very high altitudes, and the dividing line between the cyclonic and anti-cyclonic gyrations much nearer the center. The interior part of the upper anti-cyclone consequently overlaps the exterior part of the cyclone below, and over this region the directions of motion of the air above and below are very nearly or quite in opposite directions.

21. So far we have regarded the area of cyclonic disturbance, and likewise the gyrations, as having a definite circular limit with the radius R. In the case of nature, however, this is not the case, but the gyrations of the anti-cyclone, by means of friction, are continually bringing into circulation, both gyratory and vertical, new portions of the surrounding air, by which the area of gyrations is enlarged, and, in consequence of the corresponding enlarged vertical circulation, the area of the temperature gradient is also enlarged in somewhat the same manner. As the area of the gyrations is enlarged the condition expressed by (10) must still be satisfied, which will in general require that the cyclone and anti-cyclone both enlarge simultaneously. There is, however, evidently a limit in all cases beyond which the power of the cyclone cannot extend, or at least make itself sensibly felt.

22. If we regard C, and consequently the temperature gradient, as a function increasing or decreasing with the time, as it generally is, we somewhat change the conditions of the problem as treated so far, but the effects depending upon this change are so small that it is not worth while to enter into the consideration of them here. While C is increasing of course the whole activity of the cyclone must increase, and this introduces a change in the amount of inertia to be overcome by the forces, so that the term $D_t s$, and the components $D_t u$, and $D_t v$ entering into the fundamental equations, taken either positively or negatively, must increase as C increases, and inversely, but as C generally changes very slowly with the time, and the amount of force necessary to overcome the inertia of the air in any ordinary rate of increase or decrease of velocities is very small, (§ 9), the conditions of the problem, and all the results, may be regarded at any instant as being sensibly the same as if C were constant.

While C is increasing the whole activity and the area of the cyclones are increasing, but the latter for reasons given above, and not on account of the increase of C, so that the area continues to enlarge while C decreases, and although while C decreases the velocities are being gradually diminished by friction, and as C vanishes these velocities also vanish, yet the area of the cyclones is not thereby diminished, but rather enlarged to the last.

23. By comparing the fundamental equations (1) of a cyclone and the results deduced from them, in the preceding pages, with the equations of the general motions of the winds, and the results deduced from them, in Part I of these Researches, it is seen that they are similar in the two cases, those of the general motions of the atmosphere differing a little from those of a cyclone, and its accompanying anti-cyclone, on account of the greater extent of the earth's surface included, and the necessity, therefore, of taking into account the curvature of the earth's surface instead of regarding it as a plane surface, as in ordinary cyclones. The two systems of winds, therefore, belonging to the northern and southern hemispheres of the globe are simply two great cyclonic systems with a cold center, having the cold poles of the earth for their centers. The motions of the air eastward around and toward the poles, in the middle latitudes, giving rise in those latitudes to the normal southwest winds in the northern, and northwest winds in the southern hemisphere, form the cyclones, and the trade-wind region the corresponding anti-cyclones, with the equatorial calm-belt for the common limit of the two systems. The tropical calm-belt and corresponding maxima of barometric pressure near the parallels of 30° correspond to the similar calm and dividing line between the cyclone and anti-cyclone in the ordinary and smaller cyclonic systems. The mean-temperature gradients of the northern and southern systems are about the same, but since there is much more land in the northern than in the southern hemisphere, and consequently the resistance to the gyrations of the atmosphere around the globe from mountain ranges and the inequalities of land surface generally is much less in the latter than in the former, the southern cyclonic system is the more violent of the two, so that the barometric depression below the mean is much greater at the south than at the north pole, and the mean position of the equatorial calm belt is a little north of the equator.

During the winter of the northern hemisphere, the temperature gradient of the northern cyclonic system is much greater and that of the southern some less than the mean of winter and summer; hence at the former time the violence of the northern system is much greater and that of the southern some less than the mean, and the equatorial and tropical calm-belts are driven south of their mean positions. In the summer of the northern hemisphere the reverse takes place, and the positions of these calm-belts is a little north of their mean positions. Hence there is a regular oscillation of these calm-belts and of the whole system, with the changes of the seasons.

In § 16 it has been assumed that the function expressing the temperature upon which the disturbance of static equilibrium and the maintenance and activity of the cyclones depend is of the form of (24). Of course the real forms of this function in nature are generally only rough approximations to this, which has been assumed in order to simplify the problem: The results, therefore, which have been obtained upon this assumption are not strictly such as are usually observed in nature, and they can only be used in a general explanation of the phenomena observed, or in the comparisons of theory with observation where we have the averages of great numbers of observations, from which the effects of abnormal disturbances and irregularities are eliminated.

24. There are several causes which produce variations of temperature in the atmosphere, which may be divided into primary and secondary. The first give rise to the initial cyclonic motions of the air, and the latter can only come into play after these initial motions take place. If, in a quiescent atmosphere, any portion receives more caloric, from the sun's rays, from radiation from the earth's surface, from conduction, &c., than it loses by radiation, conduction, &c., its temperature is increased; and, on the contrary, if it loses more caloric than it receives, from the several causes mentioned, its temperature is diminished. Inequalities of temperature are thus produced in different parts of the air which give rise to temperature gradients. If the isotherms be somewhat circular, as they frequently may be, we have approximately the conditions of a cyclone, with a warm or a cold center, according as the temperature gradients are negative or positive. The several causes mentioned above, upon which these conditions depend, are primary causes, which alone, unaided by the secondary causes, may give rise to and keep up a cyclone of more or less violence for a considerable time.

After the cyclonic disturbance due to the primary causes has once set in, and ascending and descending currents are produced, the temperature of the air is decreased in some parts by expansion in ascending, and increased in others by compression in descending, and when the ascending currents are saturated with aqueous vapor the temperature is increased by the latent caloric of the vapor set free in its condensation by the cold of expansion in the ascent of the air. These latter are the secondary causes which only take place after the initial disturbances arising from the primary causes.

25. From (29) we get, since the second member is equal to $-\,D_r \log P$,

$$a\,(F_u + D_t u) = -\,D_r \log P = -\,\frac{D_r P}{P}$$

The condition of initial motion toward the center at or near the earth's surface, and consequently of ascending currents in the interior of the cyclone in the lower strata, is that the first member of this equation taken at or near the earth's surface shall be negative, or, in other words, that $D_r P'$ shall be positive, P' representing the barometric pressure there. After the initial motions the term in the first of (1) depending upon v, which was neglected in obtaining (29), begins to have a value in the origination of the cyclone; but this is only a modifying force depending upon the earth's rotation and the centrifugal force of the gyrations, and cannot become so great as to destroy and change the sign of the first member of (29), else the radial motions would become reversed without any change in the temperature conditions upon which they depend. The condition of ascending currents, therefore, in the interior of a cyclone in the lower strata is that $D_r P'$, regarded as a function varying with the temperature only, or rather with the density, for this depends also slightly upon the amount of aqueous vapor in the air, shall be positive.

Now we have as an expression of P′—

$$P' = \int_h gk$$

$$= \int_h \frac{P}{l\,(1 + .004t)} \text{ by (9) Part I, and by (3),}$$

$$= \int_h \frac{P}{l}\,(1 - .004t) \text{ very nearly,}$$

in which $l = 7989^m$ is the height of a homogeneous atmosphere. In this expression, it will be remembered, the average principal part generally of the effect of the aqueous vapor in a quiescent atmosphere has been taken into account by changing a little the coefficient of t.

From this expression we get—

$$(30) \qquad D_r\,P' = -\ .004 \int_h \frac{P}{l}\,D_r\,t$$

$$= \ .002\,\frac{\pi}{R}\,\sin\varphi \int_h \frac{P}{l}\,(t_c - t_o) \text{ by (25) and (26),}$$

$$= \frac{c}{R}\sin\varphi \int_h (t_c - t_o)\,\frac{P}{P'},$$

in which the value of c is of no consequence for our purpose. The integration must extend from $h = 0$ to the top of the atmosphere, or at least so high that the balance may be neglected without material error.

In the preceding expression of P′, P is the pressure independent of temperature, and consequently of r, and P′ in the expression of $D_r\,P'$, especially at the time of incipient disturbance before gyrations have set in, to which time the expression refers, may be regarded as a constant. $D_r\,P'$ is the incipient force which overcomes the inertia and friction of radial motions to or from the center, upon the hypothesis that there has been yet no change in the height of the atmosphere or of the strata of equal density in the upper part, but as the motions set in the height of the atmosphere and of the strata of equal density are elevated or depressed, as the case may be, enough to give rise to a gradient either from or toward the center in the upper part of the atmosphere, contrary to the radial motion below, else the equation of continuity could not be satisfied. But this must be only sufficient to divide the force $D_r\,P'$ between the upper and lower strata, and consequently only such as to diminish $D_r\,P'$ and not enough to entirely destroy it or reverse the sign of it; so that according as the sign of $D_r\,P'$ is positive or negative will the motion of the air below be toward the center and rise up in the interior part of the cyclone, or the reverse.

The preceding expression of $D_r\,P'$ is applicable to any other stratum of the atmosphere of which the pressure is P″, provided that h in that case is reckoned from that stratum instead of from the earth's surface. If $D_r\,P'$ is positive at and near the surface there must be motion toward the center and a rising up of the air in the central part, but this ascent of the air may not extend to the top of the atmosphere, where the temperature gradients are not the same at all altitudes, but these may be such a function of the altitude that the reverse radial currents will take place at some intermediate part of the atmosphere. If $D_r\,P$ is positive for the lower and upper parts of the atmosphere and negative for the intermediate part, the radial motion will be toward the center below and above, and from it in the intermediate parts, and the vertical currents be ascending ones below and descending ones above, with little or no change in the height of the atmosphere. Of course the reverse would take place if $D_r\,P$ were negative below and above, and positive for the intermediate altitudes. When $D_r\,P$ is positive for all altitudes, and generally where it is so for only the lower half of the atmosphere, the currents in the interior will be ascending ones to the top of the atmosphere, and the flow of the air will be out from the center above, since if $D_r\,P$ is negative above in the initial state it becomes reversed in the upper part by the increased height of the atmosphere, and the reverse of this if $D_r\,P$ is negative for all altitudes, and generally so if for only the lower half of the atmosphere.

If $D_r P = 0$ for the lower strata of the atmosphere, then these strata remain undisturbed, except so far as they are acted upon, through friction, by the radial and gyratory motions of the disturbed strata above them.

If we put

$\Delta_c, \Delta_o =$ the decrease in temperature for each 100 meters of increase of altitude in the center and exterior limit of the cyclone respectively,

$t'_c, t'_o =$ the corresponding values of t_c and t_o at the earth's surface, we have

$$t_c = t'_c - \frac{1}{100}\int_h \Delta_c$$

$$t_o = t'_o - \frac{1}{100}\int_h \Delta_o$$

With these values of t_c and t_o in (30), we get

$$(31) \quad D_r P' = \frac{c}{R} \sin\varphi \int_h \left\{ (t'_c - t'_o) - \frac{1}{100}\int_h (\Delta_c - \Delta_o) \right\} \frac{P}{P'}$$

$$= \frac{c}{R} \sin\varphi \left\{ (t'_c - t'_o)\, l - \frac{1}{100}\int_h \left(\frac{P}{P'} \int_h (\Delta_c - \Delta_o)\right) \right\}$$

in which the integrations must commence at the earth's surface or at any assumed stratum of which the barometric pressure is P′.

In the case of descending currents if we wish $(t'_c - t'_o)$ to represent the temperature disturbance from the primary causes, we must commence the integration at the top of the atmosphere, or wherever the current commences and where there is no disturbance of temperature from the primary causes, and hence the sign in this case must be changed. If the integration commence at the earth's surface, then $(t'_c - t'_o)$ must include the whole temperature disturbance arising from both the primary and secondary causes.

In order to have motion toward the center at the earth's surface and ascending currents, we must have $D_r P'$ in (31) positive, regarded as a function of temperature, and unaffected by gyratory motion, and hence

$$(32) \quad . \quad . \quad . \quad (t'_c - t'_o)\, l > \frac{1}{100}\int_h \left(\frac{P}{P'} \int_h (\Delta_c - \Delta_o)\right)$$

For the condition of motion from the center and descending currents near the earth's surface in the interior we must consequently have

$$(t'_c - t'_o)\, l < \frac{1}{100}\int_h \left(\frac{P}{P'} \int_h (\Delta_c - \Delta_o)\right)$$

For any other stratum with pressure P″ l must be the height of the homogeneous atmosphere above that stratum and the integrations commence from that stratum. If (32) is satisfied for the lower half of the atmosphere or thereabout, the currents will ascend to the top of the atmosphere, but otherwise they may not.

26. In the initial state, before motion has commenced, Δ_c may have any value, differing generally, however, not very much from that of Δ_o; but as ascending or descending currents are generated and kept up, its value approximates and finally reaches the value of 1° very nearly, in the case of air unsaturated with aqueous vapor, but in the case of saturated air and ascending currents the value to which it approximates, and finally reaches, is different for different altitudes and temperatures. This matter has been thoroughly worked up by various investigators, and the most important of their propositions have been deduced and presented in the most simple manner by Dr. Hann, so that it is not necessary to enter into this subject here, but merely to copy a few of the results from his paper,* which it may be necessary to use in the following pages. These are contained in the following table:

*See Zeit. Oest. Gesell. Met. 1874, ix, p. 321 *et seq.* Also a translation by Prof. Cleveland Abbe, Smithsonian Report for 1877.

TABLE I.—*Diminution of the temperature of ascending saturated air for each 100 meters of ascent.*

Initial pressure.	Initial temperature.									Altitude for 0°.
	−10°	−5°	0°	5°	10°	15°	20°	25°	30°	
Millimeters.	°	°	°	°	°	°	°	°	°	*Meters.*
760	0.76	0.69	0.63	0.60	0.54	0.49	0.45	0.41	0.38	20
700	.74	.68	.62	.59	.53	.48	.44	.40	.37	680
600	.71	.65	.58	.55	.49	.44	.40	.37	....	1,910
500	.68	.62	.55	.52	.46	.41	.38	....	....	3,360
400	.63	.57	.50	.47	.42	.38	....	....	....	5,150
300	.57	.51	.44	.42	....	....	....	....	....	7,430
200	.40	.43	.38	....	....	....	....	....	....	10,670
Weight of aqueous vapor in a kilogram of saturated air.										
Millimeters.	*Grams.*	*Grams.*	*Grams.*	*Grams.*	*Grams.*	*Grams.*	*Grams.*	*Grams.*	*Grams.*	*Meters.*
760	1.7	2.6	3.8	5.4	7.6	10.5	14.4	19.5	26.3	20
600	2.2	3.2	4.6	6.8	9.6	13.3	18.3	24.8	....	1,910
400	3.3	4.8	7.2	10.2	14.4	20.0	....	....	....	5,150
200	6.5	9.7	....	....	....	....	....	....	....	10,670

It is seen from this table that Δ_c, the rate of diminution of temperature in ascending saturated air, is much less than in the case of unsaturated air, which is 1° for each 100 meters of ascent. This difference is due to the latent caloric set free in the condensation of the vapor, and is especially large in warm summer temperatures, when, as seen in the last part of the table, the amount of aqueous vapor is comparatively very great. Hence the free caloric arising from the condensation of the vapor forms a very important part in the preceding conditions of (32) after the ascending currents have once set in, and likewise in the power of the cyclone, as seen from the expression of $D_r P'$, upon which this power depends. In descending currents saturated air at once becomes unsaturated, and hence we have in this case $\Delta_c = 1°$, as in ascending currents of unsaturated air.

If Δ_o at all altitudes is greater than Δ_c in the case of an ascending current, then $(\Delta_c - \Delta_o)$ is negative for all altitudes, and the conditions of (32) and (33) are satisfied in the case of $(t'_c - t'_o) = 0$; that is, in the case of no primary disturbance of temperature, provided that ascending or descending currents, however slight, are once generated by any small instantaneous impulse or temporary cause of disturbance; for the initial value of Δ_c being the same as Δ_o, as soon as the vertical current, either ascending or descending, sets in, the value of $\Delta_c = \Delta_o$ at the start approximates to the value of Δ_c in an ascending or descending current, and consequently becomes less than Δ_o at all altitudes, and the conditions of both (32) and (33) are satisfied with $t'_c - t'_o = 0$ for all altitudes, and this continues so until by a partial or complete inversion of the atmosphere, or from the action, meanwhile, of the primary causes of disturbance of temperature, the value of Δ_o in the surrounding parts comes to have a value equal to or less than that of Δ_c in the case of ascending currents, after which the conditions are no longer satisfied with $(t'_c - t'_o) = 0$, and all motion ceases. If the first impulse from the instantaneous or temporary cause is upward in the central part of a given area, then the currents will continue to ascend in this central part to the top of the atmosphere, and consequently to descend gently in the surrounding parts, and the currents will be toward the center below and from it above; but if the first impulse is downward in the central part the vertical and horizontal motions will all be reversed. The state of the atmosphere in which (32) and (33) are satisfied with $(t'_c - t'_o) = 0$, after the temperature disturbances are called into play by any instantaneous impulse, is called the *state of unstable equilibrium.*

If Δ_o is greater than 1° in the case of unsaturated air, or than the values given in the preceding table in the case of saturated air, we have this state of unstable equilibrium, but in the latter case the primary instantaneous impulse must be upward. Hence the state of unstable equilibrium is produced by a much less rapid rate of diminution of temperature with increase of altitudes in the

case of saturated air than in that of unsaturated air, and this is especially the case for warm temperatures and at great altitudes, as is seen from the preceding table. For a temperature of 30° it is only necessary that Δ_o should be greater than 0°.38.

In order to have the state of unstable equilibrium it is not necessary to have Δ_o greater than Δ_c for all altitudes, but merely that there should be a preponderance of such values, and mostly in the lower parts of the atmosphere, so that $\int_h (\Delta_c - \Delta_o)$ shall be negative. If this be the case for the lower strata only of the atmosphere, and for all those above $\int_h (\Delta_c - \Delta_o)$ should be positive, then the ascending current would not ascend to the top of the atmosphere, and the conditions may be such that they will ascend only to a moderate height when the air will move out from the center to supply the inflowing currents below.

27. It is seen from (30) that a state of unstable equilibrium in the lower strata only of the atmosphere has little effect in producing a cyclone of large extent with ascending or descending currents extending to the upper regions of the atmosphere, on account of the short extent of the integrations with regard to h in the expression of D_rP', upon which the power of the cyclone depends, and also on account of the large value of R, in a large cyclone, for this shows that the temperature gradient and power of the cyclone are inversely as the extent of the temperature disturbance, the difference of temperature between the center and the external part remaining the same. If Δ_o were so much less than Δ_c that the average of $(\Delta_c - \Delta_o)$ for a stratum of 1,000 meters in depth were 9°, the effect would only be equal to about a difference of 1° in $(t'_c - t'_o)$; that is, to a difference of 1° between the central and exterior part of the area of temperature disturbance arising from the primary causes, and extending to the top of the atmosphere. But this is an extreme assumption; for the atmosphere most probably never becomes in this state to the height of 1,000 meters, even when the difference between Δ_c and Δ_o is very small; and where there is a very rapid diminution of the temperature with the altitude, and consequently Δ_o is large, the height to which such a state extends must generally be small, and this only in cases where there have been no disturbances of the atmosphere from the primary causes for a considerable time.

The state of unstable equilibrium occurs mostly at some distance above the earth's surface, and in the case of saturated air; since in this case the diminution of temperature with altitude, or value of Δ_o, required is much less, especially in the summer season, as is seen from Table I; but in order to have the air saturated, it is necessary that there shall have been first an ascending current long enough to cool the air down to the dew-point, and this can only arise from the disturbances of temperature due to the primary causes. But after this has once taken place, if the value of Δ_o is less than the value of Δ_c in Table I, we have a state of unstable equilibrium, since the condition of (32) would be satisfied if t'_c were equal to t'_o, taken in the lowest saturated stratum; and if the upward currents were to cease at the time of reaching saturation, any instantaneous upward impulse would bring Δ_c to have a less value than Δ_o, and this would continue as long as the air could be kept saturated at any given altitude by means of the ascending currents.

Where $(t'_c - t'_o)$ has a finite value, either positive or negative, the primary causes combine with the secondary causes of disturbance in the state of unstable equilibrium; and where the state of the atmosphere only approximates to this unstable state the conditions of (32) and (33) can still be satisfied if the amount of primary disturbance of temperature $(t'_c - t'_o)$ is sufficiently great; but it may be such only that it is entirely counteracted by the disturbance of temperature from the secondary causes after the motions upon which these depend have once set in, so that in such a case there can be but little atmospheric disturbance.

28. If we knew the value of Δ_c and Δ_o for all the strata of the atmosphere, and also the amount of primary temperature disturbance $(t'_c - t'_o)$, in any case, we could determine by computation which of the conditions of (32) or (33) are satisfied. Unfortunately we have no means of knowing the values of Δ_o at any special time, and we have few observations from which we can deduce general average values of it. Glaisher's observations, during numerous balloon voyages, gave, for the free atmosphere, the following rates for the diminution of temperature with the increase of altitude, taken from Dr. Hann's paper, already referred to, deduced from results given originally in the Report of the British Association, 1864.

TABLE II.—*Diminution of temperature per* 100 *meters of altitude.*

Weather.	Altitude in thousands of English feet.							
	0 – 1	1 – 2	2 – 3	3 – 4	4 – 5	5 – 10	10 – 15	15 – 20
	°	°	°	°	°	°	°	°
Clear	0.98	0.71	0.55	0.55	0.55	0.46	0.39	0.30
Cloudy	0.86	0.73	0.73	0.56	0.55	0.45	0.40	0.25

For the first 1,000 feet, in clear weather, it is seen that the average state of the atmosphere at the time of these balloon ascensions differed but little from a state of unstable equilibrium for unsaturated air. In cloudy weather the earth's surface, and the lower strata of the atmosphere, are not so much heated by the direct rays of the sun, and the rate of diminution of temperature, then, for the first 1,000 feet, is less than in clear weather. Above the altitude of 3,000 feet it is seen that the rate is nearly the same in clear and cloudy weather, and very much less than near the earth's surface and than the rate required for unstable equilibrium of unsaturated air. Different balloon ascents often gave very different results, and in some cases the rates became negative in some parts of the ascent, and the temperature increased with altitude. These rates were also found, in the lower part of the atmosphere, to be different at different seasons of the year and different times of the day. At different times, therefore, the rates of decrease, no doubt, differ very much from the averages given above, and even from the averages which would be given by observations made at the same season of the year and the same time of day. They probably differ also, under the same circumstances, in different latitudes.

The rates given in the preceding table are deduced from observations made mostly in the summer half year, especially those of high altitudes, and the latter are based upon but few observations, so that the results from many observations, made at all seasons of the year and all times of the day, might differ considerably from these, though the rates of decrease of temperature in high altitudes are perhaps not affected very much by the seasons or the times of day.

From observations made by the United States Signal Service on the top of Mount Washington and at Burlington and Portland, and on the top of Pike's Peak and at surrounding stations on the plateau of Colorado, and also from observations made in Switzerland, we have the following rates given by Dr. Hann: *

TABLE III.

Stations.	Winter.	Spring.	Summer.	Autumn.	Year.
	°	°	°	°	°
Mount Washington—Burlington	0.40	0.59	0.67	0.52	0.56
Mount Washington—Portland	0.46	0.60	0.57	0.55	0.55
Switzerland by Weilenman	0.45	0.67	0.73	0.52	0.58
Plateau of Colorado					0.61

It is seen that these rates differ considerably in different seasons of the year, but the averages of summer and autumn do not differ much from the average for corresponding altitudes given in Table II.

* Zeit. Oest. Gesell. Met. xi, 1876, p. 84.

The following table of results for the lower strata of the atmosphere, taken from a paper* by Glaisher, is very important in this connection:

TABLE IV, *showing the decrease of temperature with increasing elevation at every* 100 *feet up to* 1,000 *feet.*

Height above the ground.	Clear sky.						Cloudy sky.				
	10 to 11	3 to 4	4 to 5	5 to 6	6 to 7	7 to 7.30	3 to 4	4 to 5	5 to 6	6 to 7	7 to 7.30
Feet.	°	°	°	°	°	°	°	°	°	°	°
0 to 100	1.0	1.5	1.1	0.9	0.5	0.0	1.2	1.2	0.6	0.5	0.5
100 to 200	0.9	0.8	0.7	.6	5	.1	0.9	0.6	.6	.6	.5
200 to 300	.9	.8	.7	.6	5	.3	.9	.5	.6	.5	.5
300 to 400	.9	.7	.6	.6	5	.4	.6	.6	.6	.5	.4
400 to 500	.8	.6	.6	.6	5	.3	.4	.4	.5	.4	.5
500 to 600	.8	.5	.5	.5	4	.3	.4	.4	.5	.5	.5
600 to 700	.7	.5	.5	.4	4	.4	.4	.4	.5	.4	.4
700 to 800	.7	.5	.4	.4	4	.4	.5	.4	.5	.5	.5
800 to 900	.6	.5	.4	.4	4	.3	.4	.4	.5	.5	.5
900 to 1,000	.6	.4	.4	.3	4	.2	.5	.4	.4	.4	.5

To reduce these results in degrees F. per 100 feet to degrees C. per 100 meters multiply by 1.8. The observations were made May 5, July 12, 17, 23, 24, 28, August 4, 7, 1869, and hence mostly in the summer season. It is seen that during the day until 6 o'clock in the afternoon, with a clear sky, the air on the average was in a state of unstable equilibrium up to the height of 500 feet, and nearly up to the same height with a cloudy sky. But near sunset, with a clear sky, there was no decrease of temperature with altitude near the earth's surface, and the results show evidently that during the night the temperature increased with altitude, owing, no doubt, to the rapid radiation into the clear sky. But the observations indicate that this was not the case with a cloudy sky, as we would expect.

We have seen from the preceding tables that the decrease of temperature for considerable altitudes is less in winter than in summer, so that near the surface the difference must also be very great, and hence this table is not applicable in the winter season.

29. The value of Δ_c is known very nearly in some cases, but not even approximately in many others. Where an ascending current of air has been kept up for some time, so that the air at any given altitude has ascended from the earth's surface, and during the same time has not received or lost caloric from the primary causes mentioned in § 24, we know that Δ_c must equal 1° nearly, in the case of unsaturated air; and in the case of saturated air it must have the value given in Table I, which we may denote by Δ'_c. The same is true if the air has ascended from any given altitude and no air has ascended from the strata beneath that altitude. The same is also true in the case of descending currents under the same limitations. The more rapid the ascending or descending current the more rapidly Δ_c approximates to the value Δ'_c at any altitude, but reaches it first in the lower strata.

To satisfy the conditions of a cyclone it may be necessary in some cases to have ascending currents at some altitudes and descending ones at others, but generally they may be satisfied with either ascending or descending currents throughout the whole height of the atmosphere. In this case Δ_c comes to have the value of Δ'_c first at the earth's surface in the case of ascending currents, and gradually for higher altitudes Δ_c becomes equal to Δ'_c, but in the extreme upper parts of the atmosphere Δ_c may be but little changed from Δ_o unless the vertical circulation, which is generally slow, should be kept up for a very long time.

Let us assume that the atmosphere is completely saturated with vapor, and that there is a temperature disturbance due to the primary causes of $(t'_c - t'_o)$ which is the same at all altitudes. In the initial state of motion we should therefore have $(\Delta_c - \Delta_o) = 0$, and hence the condition of (32) satisfied at all altitudes if $(t'_c - t'_o)$ is positive, but that of (33), if $(t'_c - t'_o)$ is negative. The initial motions, therefore, in the former case are toward the center below and upward to the top of the atmosphere in the interior part of the disturbed area, and the reverse in the latter case. Let us take first the case of ascending currents in the interior, and suppose that these have continued so long

* On the Changes of Temperature and Humidity of the Air up to 1,000 feet, from observations made in the car of M. Gifford's captive balloon; by James Glaisher, F. S., F. R. A. S., &c. Report of British Association, 1869. Part ii, page 27.

that Δ_c has come to have the value of Δ_c', or nearly, up to the top of the atmosphere. Let us also suppose that the state of the undisturbed atmosphere, so far as it regards the rate of decrease of temperature with the altitude, is the same as the average in Glaisher's balloon ascents in clear weather and that $t'_o = 15°$. We therefore get from Tables I and II the values of Δ_c and Δ_o, and of P: P′ in the following table, corresponding to the altitudes of the first column:

TABLE V.

1	2	3	4	5	6	7	8	9	10	11	12	13
Altitude.	Δ_o	$\frac{P}{P'}$	$(\Delta_c-\Delta'_c)$	$(\Delta_c-\Delta_o)$	$10\int_h(\Delta_c-\Delta_o)$	$10\frac{P}{P'}\int_h(\Delta_c-\Delta_o)$	Δ_c	$10\int_h(\Delta_c-\Delta_o)$	$10\frac{P}{P'}\int_h(\Delta_c-\Delta_o)$	Δ_c	$10\int_h(\Delta_c-\Delta_o)$	$10\frac{P}{P'}\int_h(\Delta_c-\Delta_o)$
Meters.			°	°	°	°	°	°	°	°	°	°
500	0.68	0.94	0.48	−0.20	− 2.00	−1.87	0.50	−1.80	− 1.69	0.80	+ 1.20	+ 1.13
1,500	.55	.83	.53	−0.02	2.20	1.83	.54	1.90	1.58	.70	2.70	2.24
2,500	.45	.74	.57	+0.12	− 1.00	−0.74	.55	−0.90	− 0.51	.60	4.20	3.20
3,500	.40	.65	.62	0.22	+ 1.20	+0.78	.53	+0.40	+ 0.26	.50	5.20	3.38
4,500	.35	.57	.68	0.33	4.50	2.56	.48	1.70	0.97	.40	5.70	3.25
5,500	.30	.50	.73	0.43	8.80	4.40	.42	2.90	1.45	.30	5.70	2.85
6,500	.25	.44	.77	0.52	14.00	6.16	.34	3.80	1.67	.25	5.70	2.51
7,500	.20	.39	.84	+0.64	+20.40	+7.96	.24	+4.20	1.64	0.20	5.70	2.22
....				....	....	....	....	....	+12.58	....	....	+17.05

With these data the computation of the last number of (32) may be carried out as in the succeeding columns by means of finite integrations, regarding the thickness of each stratum of 1,000 meters, for the middle of which the data are given, as a unit, which requires the numerical coefficient in (32) to be 10 instead of $\frac{1}{100}$. The summation of the seventh column to the top of the atmosphere would give the value of the last member of (32). If we suppose a temperature disturbance amounting to a difference of 10° between the central part and exterior limit of the disturbed area, the value of the first member of (32) would be 80 very nearly, since with our new unit of 1,000 meters the value of l is very nearly 8 instead of 7989. We do not have the data for extending the computation of Table V above the altitude of 8,000 meters, but with any probable values of Δ_c and Δ_o for the higher altitudes, it is seen from a mere inspection of the results in the seventh column that the integral of this column, extended to the top of the atmosphere, would be much more than 80; in fact, that (32) would not be satisfied with any reasonable assumption of the value of (t'_c-t_o').

We have seen that any primary disturbance of the atmosphere, represented by $(t_c'-t_o')$, in which t'_c is greater than t'_o, and the temperature gradients are the same at all altitudes, must give rise to cyclonic disturbances, in which the currents of the interior central part are ascending ones to the top of the atmosphere, but the preceding result shows that such cyclonic action cannot be kept up indefinitely with any reasonable amount of primary disturbance, although the ascending currents should be continually supplied below with air saturated with aqueous vapor. The reason of this is readily seen from a general consideration of the subject without conditions expressed by mathematical equations; for if the ascending currents have been kept up long enough to bring Δ_c to the value of Δ_c' in Table I, and there has been no accession of temperature meanwhile from the primary causes, the temperature of the air above becomes so reduced, and the density consequently so great, that the atmospheric pressure, even of the lower strata, so far as it depends upon temperature, becomes as great in the interior as in the exterior part of the disturbed area, and then the power of the cyclone and all motion must cease.

The air is likewise cooled from expansion in horizontal currents where it passes from under a greater to a less barometric pressure, as from the exterior to the interior of a cyclone. The change of temperature from this cause is a little more than 1° for every 10^{mm} of difference of pressure.

It does not follow, however, from the preceding result, that the cyclonic action may not be very violent and kept up for a considerable time, for the values of Δ_c may very slowly acquire such values, especially in the upper part of the atmosphere, as will make the last member of (32) equal to the first one. If, after the cyclone has been in action for some time, we suppose that the values

of $\varDelta_o$ have changed from their initial values $\varDelta_o$ to those in the eighth column of Table V, and carry out the computation upon this supposition as before, we get the results in the ninth column instead of those of the seventh. In this example we get $10\int_h(\varDelta_c - \varDelta_o)$, which is the expression of the amount of diminution of the temperature gradient, equal to 4°.2 for the upper or eighth stratum in Table V. Supposing $(\varDelta_c - \varDelta_o)$ to be equal to 0 for the strata above, the preceding value of 4°.2 remains constant for all the strata above. We shall, therefore, have for the integral of column 9, above the altitude of 8,000 meters,

$$10\int_h \frac{P}{P'} \int_h (\varDelta_c - \varDelta_o) = 4.2\int_h \frac{P}{P'} = 4.2\ l\,\frac{P''}{P'} = 12.58$$

in which P″ is the barometric pressure at the altitude of 8,000 meters. With this value in column 9 the summation, commencing at the earth's surface or at any altitude above it, gives a value for the last member of (32) which satisfies it for more than the lower half of the atmosphere if $(t'_c - t'_o)$ is 7°.5 or more, and hence we have the conditions which will produce or keep up ascending currents to the top of the atmosphere. The summation from the earth's surface of the tenth column gives 14.63 for the value of the last member of (32). This is less than the first member when $(t'_c - t'_o)$ is 1°.8 or more, and hence the conditions of (32) are satisfied at the surface. For the altitude of 6,000 meters we get from the summation of the tenth column above that altitude 15.89. But for the altitude of 6,000 meters we must have P″ the pressure at that altitude instead of P′. Hence we must divide this by P′ : P″ = 0.47 in order to get the true integral in this case. We thus get 34, which satisfies (32) when $t'_c - t'_o = 4°.5$. With this value for all the strata above 8,000 meters we have $D_r\,P = 0$, and hence we must have the condition of (30) satisfied with a value less than 4°.3 and for any value greater, and consequently ascending currents to the top of the atmosphere. But with a temperature disturbance due to the primary causes corresponding to $(t'_c - t'_o) = 5°$, or a little more, the two members of (32) are so nearly equal that the cyclone has very little power left, as may be seen from (31), for the power of the cyclone depends upon the values of D_rP for the lower strata. If, however, $(t'_c - t'_o)$ is equal say to 10° or more, the cyclone still has great power, and must continue until $\varDelta_c$ acquires such values for the different strata as will make the last member of (32) equal to the first, when the power of the cyclone is gone.

The condition that a cyclone must be perpetual with any given amount of primary disturbance is that the last member of (32) is less than the first when $\varDelta_c$ has acquired its extreme value $\varDelta'_c$, which is that of Table I in the case of saturated air, but which in the case of dry air is 1°. We have seen in the results of the first part of Table IV that with the values of $\varDelta_o$ given by Glaisher's balloon ascents, the cyclone could not be perpetual without an unreasonably large value of $(t'_c - t'_o)$, the exact value being unknown on account of the uncertainty of the integral of the seventh column for the upper strata. It is readily seen from an inspection of the formula and its computation in Table V that the greater the values of $\varDelta_o$ the greater the power of the cyclone and the longer it must continue in action before the values of $\varDelta_c$ become so great as to destroy the condition of (32), and also the less the amount of primary disturbance required to make it perpetual.

But since the balloon ascents gave very different values of $\varDelta_o$ at different times the atmosphere is no doubt in a state frequently when the values of $\varDelta_o$ are much greater than the average, which are the values used in the preceding example, and then the power of the cyclone would be much greater with the same amount of disturbance due to the primary causes, and would be kept longer in action, and perhaps would become perpetual for ordinary amounts of primary temperature disturbance if this could continue unchanged.

It should be borne in mind here that the theory assumes that the gradient of primary disturbance of temperature remains the same above as below, and consequently that the very cold air of the upper part of the ascending currents does not receive any caloric, meanwhile, from any of the primary causes of disturbance of temperature. If they do, of course the power of the cyclone is greater and is kept up a longer time, and the cyclone would become perpetual with a less value of $(t'_c - t'_o)$, which is the amount of temperature disturbance at the earth's surface.

It is seen from Table III that the values of $\varDelta_o$, at least for the lower strata of the atmosphere, are greater in summer than in winter, and from Table I that the values of $\varDelta_c$ are less in summer than in winter. Hence the conditions for cyclonic disturbances, for both reasons, are more favor-

able in the former than in the latter season. If Δ_o were equal to Δ'_c, the limiting value of Δ_c, the condition of (32) would be satisfied for any value of $(t'_c - t'_o)$ however small; for then the last member of (32) could not acquire any value, but must remain equal to 0. When Δ_o is greater than Δ'_c, we have the state of unstable equilibrium in which $(t'_c - t'_o)$ may be almost infinitely small or even equal to 0, provided there is merely an instantaneous impulse to give origin to an upward current. For in this case the initial value of $\Delta_c = \Delta_o$ gradually approximates to its extreme and final value Δ'_c, and in doing so gives to the last member of (32) a negative and increasing value, so that after once receiving the first impulse, the condition of (32) is satisfied and continues so with $(t'_c - t'_o) = 0$, and the cyclonic disturbance is perpetual as long as Δ_o is greater than Δ'_c. Of course, after a long continuance, these conditions would be changed, and Δ_o would come to have the value of Δ'_c from the effect of the very gradual descent of the air in the surrounding and comparatively undisturbed portions of the atmosphere, in order to supply the upper currents in the interior, and then a state of static equilibrium would take place.

Where the air of the ascending currents is not saturated with aqueous vapor, other things remaining the same, the conditions for cyclonic disturbances are less favorable, for then the value of Δ'_c, to which the values of Δ_c approximates and finally reaches is 1° in the lower strata, instead of the values in Table I. If the temperature of the dew-point were 10° below that of the air, then the air would have to ascend 1,000 meters before condensation would take place, and before we should have the values in Table I for Δ'_c. It is readily seen from an inspection of the mathematical expression in (31) that the power of the cyclone is weakened with increased values of Δ_c, and from (32) and the computation of its last member in Table IV that the conditions of the cyclone are destroyed in a shorter time, since Δ_c increases with the continuance of the ascending currents. In this case we should have $\Delta_c = 1°$ for the lower stratum in the computations of Table IV, instead of 0°.48 in the first example and 0°.50 in the second, and the same for the second stratum if the temperature of the dew-point were 20° below that of the air.

In this case since the diminution of temperature or value of Δ_c in the ascending current up to where the vapor is condensed into cloud approximates to that of 1°, it consequently soon becomes larger than Δ_o under all usual circumstances, especially immediately under the cloud where the ascending currents are more rapid than nearer the earth's surface, and hence since cloud is mostly formed by ascending currents, the value of Δ_c under a cloud must generally be greater than in the surrounding parts where there are very gentle descending currents, and consequently clear weather. Hence in Glaisher's balloon ascents the rate of diminution of temperature at an altitude from two to three thousand feet, which may be supposed to have been, on the average, under the base of the cloud, was for cloudy weather 0°.73, while for clear weather it was only 0°.55, as seen in Table II, § 28.

In the case of a perfectly dry air we should have the final values of $\Delta_c = 1°$ for all the strata to the top of the atmosphere, and hence, after a short continuance of the cyclone, unless the primary disturbance was very great, the cyclone would have little power left, or a state of complete static equilibrium would take place. It is seen from Table IV that if we were to use $\Delta_c = 1°$ for the greater part or all of the strata instead of the values in column second, the integral of column seventh to the top of the atmosphere would be very great, and hence the value of the last member of (32) in this case would be very great, and the condition of the cyclone would not be satisfied except with a value of $(t'_c - t'_o)$ enormously greater than in the case of saturated air.

If $(t'_c - t'_o)$ is negative, we have the condition which gives rise to a cyclone with a cold center and descending currents in the interior. The condition which must be satisfied in this case is that of (33), which is the same as that of (32) if we take $(t'_c - t'_o)$ without regard to sign. In this case, however, we have $\Delta'_c = 1°$ for all the strata, whether the air be saturated or unsaturated at the start; for, if saturated, on account of the descent of the currents in the interior, it at once becomes unsaturated. Hence all that has been stated with regard to cyclones with warm centers and ascending currents in the interior, in the case of perfectly dry air, is applicable in this case. Cyclones, therefore, of this sort have comparatively little power and require a large amount of temperature disturbance due to the primary causes to keep them long in action, and especially to satisfy the condition of a perpetual cyclone, supposing the amount of primary disturbance to remain unchanged.

So far we have considered the temperature disturbance due to the primary causes as being the

same at all altitudes, and hence, in the initial state of cyclonic disturbance, that $\Delta_c = \Delta_o$. This, as is readily seen from (32) and (33), gives initial ascending currents from the bottom to the top of the atmosphere where $(t'_c - t'_o)$ is positive, and the reverse where it is negative. We shall now assume a case in which the rate of diminution of temperature due to the primary causes is not the same in the central and exterior part of the disturbed area, and in which, consequently, the initial values of Δ_c are not equal to those of Δ_o. If we assume the values in column 11, Table V, for those of the initial values of Δ_c in this case, still using the values of Δ_o in column two, deduced from Glaisher's balloon ascents, and carry out the computation as in the other examples, we get column 12 for the amount of diminution of $(t_c - t_o)$ for the middle of the several strata, which in this case in the initial state is not constant and the same as $(t'_c - t'_o)$ at the earth's surface. In column 13 are the numbers of which the summation to the top of the atmosphere is the value of the last member of (32). The number 17.05 is the summation of the strata above 8,000 meters of altitude obtained as in the last example, since with the assumed values of Δ_c and Δ_o, $10\int_h(\Delta_c - \Delta_o)$ is a constant above that altitude, and even above the fifth stratum. If we assume that the temperature gradients remain undisturbed above the fifth stratum, and that the primary disturbances of temperature are confined to the strata below, we must put $(t'_c - t'_o) = 5.70$, which with the value of $l = 8$ gives the first member of (32) equal to 45.60. The summation of column 13 gives 37.83. Hence the condition of (32) is satisfied at the earth's surface. It is also readily seen that (30) is satisfied in this case, since above the fifth stratum we have $D_r P = 0$, or the last member of (32) equal to the first, but below that less than the first. Hence the currents are ascending ones in the interior to the top of the atmosphere.

If in this example the atmospheric disturbance extended to a very small altitude, it is readily seen that the value of the last member of (32) would be so nearly equal to the first that we should have the value of $D_r P$ for all the strata so small that the cyclone would have very little power, and there would be only very feebly ascending currents, as already pointed out in § 27, although $(t'_c - t'_o)$ might be large.

It is seen from what precedes that all cyclonic disturbances of the atmosphere depend upon the temperature disturbances due to the primary causes, and that without these there is no disturbance of static equilibrium. After the initial cyclonic motions due to the primary causes have once set in, the secondary causes of temperature disturbance are called into play, which, as a whole, are antagonistic to the primary disturbances, except in the case of unstable equilibrium, and, unless the latter are very great, may entirely counteract them, after which static equilibrium again takes place. The more nearly the atmosphere is in a state of unstable equilibrium, that is, the more nearly Δ_o equals Δ_c for all the strata, the less is the counteracting influence of the temperature disturbances due to the secondary causes. The caloric arising from the condensation of the aqueous vapor diminishes Δ_c, by reducing it from the values of $\Delta_c = 1°$ to the value in table I, and hence puts the atmosphere in a state of unstable equilibrium with a smaller value of Δ_o, and, for any given value of Δ_o less than those of Δ_c in Table I, gives the cyclone more power. The effect of the cold of expansion, or loss of sensible caloric, in the ascending currents is opposed to that arising from the primary disturbances, but the latent caloric set free in the condensation of the aqueous vapor in part replaces the loss of sensible caloric by expansion, more than one-half generally, as is seen from Table I, and hence diminishes the counteracting effect of the loss of sensible caloric from expansion of the air. The condensation of aqueous vapor, therefore, plays an important part in cyclonic disturbances, but is by no means either a primary or a principal cause of cyclones.

30. In a cyclonic system with a warm center, we have seen that the air ascends in the central part, or cyclone, and descends in the anti-cyclone. Since, therefore, in ascending currents of only partially saturated air the vapor must be condensed to cloud at a height greater or less according to the dew-point of the air, and in descending currents there is no condensation of vapor but the vapor already condensed into cloud is evaporated, it is evident that cyclonic areas, in general, are those of cloud and rain, and anti-cyclonic those of clearing and clear weather. Of course it is not to be supposed that these areas correspond with each other with much accuracy either in form or extent. The cloud of the ascending part of the atmosphere may be carried by the outward upper currents over into the anti cyclone area before it falls as rain or is evaporated again in the descending currents of the anti-cyclone.

Cloud is generally formed in ascending air only, so that the initial cloud particles, or extremely fine drops of rain, are at first still carried upward by the ascending current, the smaller ones faster than the larger ones, until by meeting in being carried by one another and agglomerating they become too heavy to be sustained by the upward current, for the larger the drop the stronger the current required to keep it up. It then commences to descend, but still grows from meeting with smaller ascending particles in its descent. If the upward current is very strong the drops are carried very high before they begin to descend, and then, having a long distance of dense cloud to fall through, they may become very large before they reach the base of the cloud and the earth.

Where the ascending current is very gentle, the small particles or drops may be so small when they have descended to the base of the cloud that they have evaporated before reaching the earth, and in such cases no rain falls to the earth.

In a cyclonic system with a cold center no rain or cloud can be formed in the cyclone, since the air is gradually descending, but if there is cloud or rain it must be in the anti-cyclone. In the two great examples of this kind, embracing the northern and southern hemispheres of the globe, § 23, the cloudy and rainy parts are the outer limits of the anti-cyclones, which, falling together, form the equatorial rain-belt of the globe. Of course this is to be understood only of the general motions of the systems, and not of those depending upon minor local disturbances. There is much rain in the higher latitudes of both hemispheres, but this arises from the local cyclonic disturbances, which are cyclones within the great hemispheric systems of cyclones, and their conditions form no part of those of the general system in which local disturbances of temperature are not taken into account.

31. So far we have regarded the forces depending upon the earth's rotation as being equal on all sides of the cyclone, and so have used the approximate equations (1) in which ψ, the polar distance of the center of the cyclone, is put for δ, which is the polar distance of each part of the cyclone. The expression of the accelerating force, in the direction of r, depending upon the earth's rotation is $2\,nv \cos \delta$, in which v is the gyratory linear velocity around the center. Now, on the polar side of the cyclone, where $\cos \delta$ is larger, this force is greater than on the equatorial side where $\cos \delta$ is smaller. The polar components, then, of these radial forces on the polar side of the cyclone are greater than the equatorial components, in the contrary direction, of the equatorial side, and hence there is a greater force on the polar side of the cyclone, arising from the earth's rotation, causing pressure toward the poles, than on the other, causing pressure toward the equator. The resultant, therefore, of all the forces, of every part of the cyclone, tends to drive it toward the pole.

The residual, or neglected part, of this force, for any part of the cyclone, in the preceding theory, is $2\,nv\,(\cos \delta - \cos \psi)$. This being resolved into components in the direction of the meridian and perpendicular to it, we get for the component of force toward the pole $2\,nv\,(\cos \delta - \cos \psi) \cos \varphi$, in which φ is the angle expressing the bearing of each particle from the direction of the pole. On the polar side of the cyclone δ is less than ψ and hence $(\cos \delta - \cos \psi)$ is positive, and likewise $\cos \varphi$. On the equatorial side both are negative. Hence for all parts of the cyclone, $2\,nv\,(\cos \delta - \cos \psi) \cos \varphi$, which expresses the polar components of the neglected forces depending upon the neglected terms, is positive, and hence the integration of this force for all parts of the cyclone gives a force which tends to drive it toward the pole, as stated above.

Since v in the above term is negative in the anti-cyclone, of course the effect of the neglected terms, in the preceding treatment of cyclones, is to drive it toward the equator, so that there is a tendency of the cyclone to cut its way through the anti-cyclone, but on account of the comparatively small gyratory velocities in the anti-cyclone the forces in this case for any given particle of air are much less, in general, than in the cyclone.

As the cyclone has a tendency to cut its way through the anti-cyclone, and the atmosphere generally, and move toward the pole, and the condensation of aqueous vapor and the whole power of the system are contained mostly within it, the tendency is for the whole cyclonic system, of both cyclone and anti-cyclone, to be drawn toward the pole, in case it is not confined by definite limits but is free to assume a progressive motion through the surrounding atmosphere. Of course it is not to be supposed that the cyclone, in its progressive motion, contains at all times the same air, but rather that new portions of the air are being continually drawn into circulation on the polar side, where the forces depending upon the earth's rotation are strongest, and that on the equatorial

FIG. 4

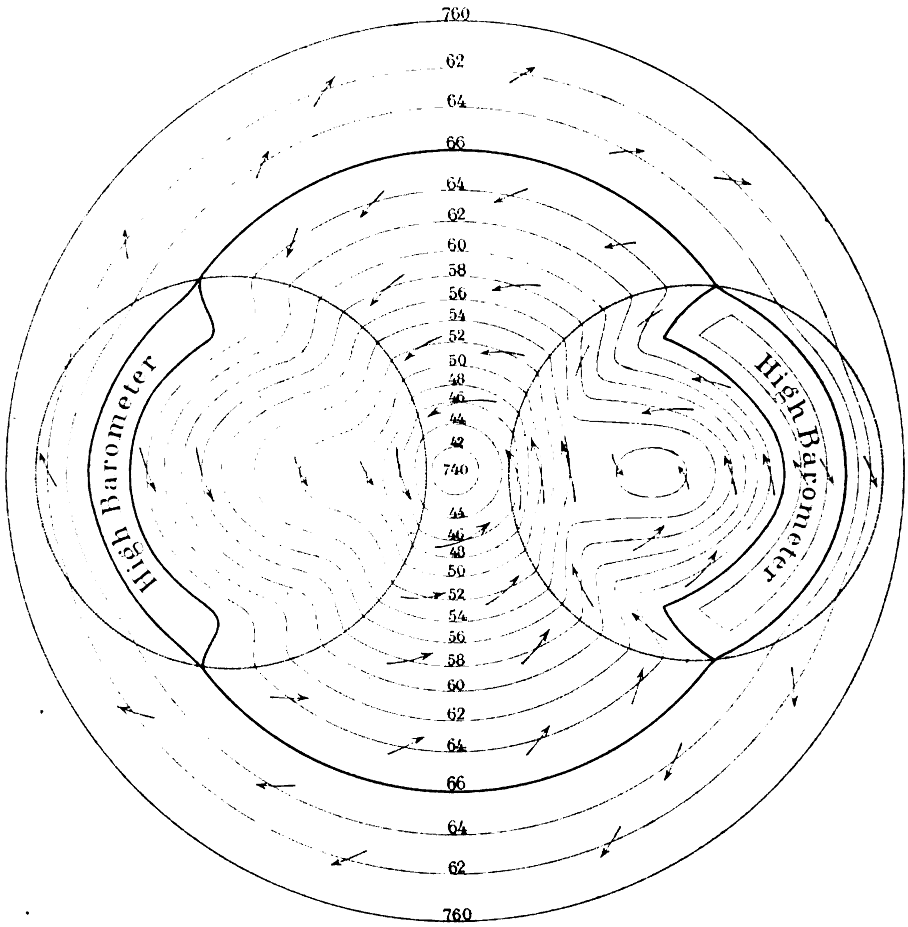
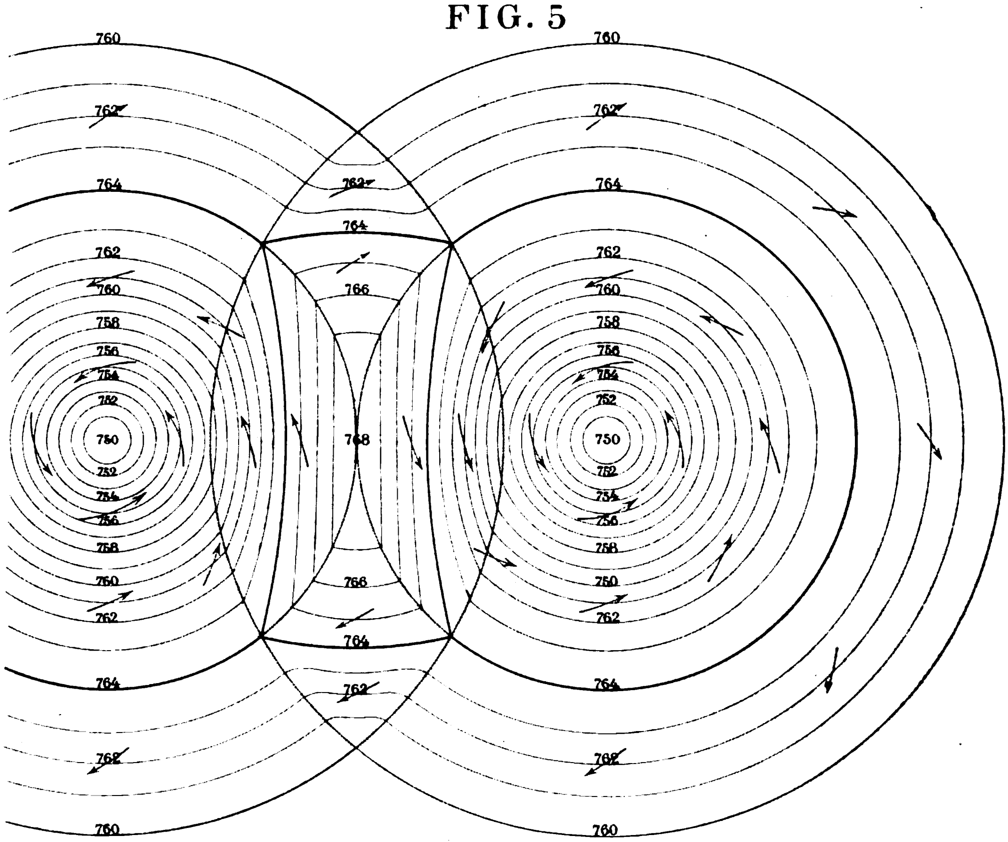

760
62
64
66
64
62
60
58
56
54
52
50
48
46
44
42
740
44
46
48
50
52
54
56
58
60
62
64
66
64
62
760
High Barometer
High Barometer

FIG. 5


760
762
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762

side, where these forces are weakest, other portions are being continually brought to rest by means of friction, and cease to be a part of the system, as the cyclone moves on toward the pole.

Since the force which tends to drive the cyclone toward the pole, as seen above, is as $(\cos \delta - \cos \psi)$, it is evident that this force, everything else being the same, is greater in low than in high latitudes, since $(\cos \delta - \cos \psi)$ is a maximum at the equator and vanishes at the poles.

32. It often happens that the conditions of a smaller cyclone, or even several of them, are contained within the limits of a larger one. The isobars then, which in a perfectly regular cyclone are circular, become very irregular. The contained cyclone may be such as to give two points of minimum barometric pressure, or it may simply produce derangements in the regularity of the isobars. When the isobars become more crowded, and consequently the barometric gradients steeper, the velocities are likewise greater, for in such places the velocities of the larger and smaller cyclone are in the same direction. Where the isobars are further apart, and gradients consequently smaller, the velocities are also smaller, since then the velocities are in contrary directions and partially neutralize each other.

Fig. 4 represents the effect upon the isobars and directions of the wind of smaller cyclones contained in larger ones. In the one on the left the depression was not sufficient to give rise to a secondary center of low barometer, but merely to cause considerable derangement in the isobars, crowding them together on the one side and widening them on the other, and changing somewhat the directions of the wind. The one on the right produces a secondary center with the wind blowing around it, but with much greater velocity on the right side, where the directions of the currents of both cyclones coincide, than on the other side where the gyratory velocity of the smaller cyclone is but a very little greater than the velocity there of the principal one in the contrary direction. The annulus of high barometer of the larger cyclone is increased in height in an irregular manner by those of the smaller ones falling somewhat upon it. Of course there are generally many other disturbances combined with these to disturb the regularity of the isobars.

Since the equations expressing the conditions of a cyclone are not strictly linear equations, especially so far as the centrifugal force of the gyrations is concerned, each set of conditions taken separately does not give results which, when combined, are precisely the same as the combined conditions would give, if the equations expressing them could be solved. The formula (16), therefore, showing the relation between the barometric gradient and the velocity of the wind, in some measure fails when there is an interference of cyclones, especially when the centrifugal forces are great and the radii of curvature of the isobars small. In such cases, however, the formula is applicable without material error if we use the radius of curvature of the distorted isobar for the radius of the cyclone, r, in the formula.

Generally two or more cyclones encroach upon or overlap one another. Fig. 5 represents the forms of the isobars where two cyclones, or rather cyclonic systems, of equal area and depression, such as represented in Fig. 1, overlap in such a manner that the cyclones touch, and each one is partly overlapped by the anti-cyclone of the other. The effect is an area of higher barometer than that of any part of the cyclones taken separately, with the winds blowing somewhat around the place of the maximum, but it is seen that in such cases the isobars are by no means circular, but, rather, that such areas of high barometer, especially when caused by the overlapping of more than two cyclones, often require lines with somewhat acute angles, or at least with very short curves to represent the isobars. Of course, in the real cases of nature, any sharp angles given by theory upon assumed regular conditions, are very much rounded off by various abnormal disturbances.

If we combine the conditions of the two cyclones in such cases in which the temperature gradients are such as given by the assumed form of (24), we do not get an area of maximum cold corresponding to the maximum barometric height, but one in which the temperature decreases from the point of highest barometer each way toward the centers of the cyclones, and increases in each direction from the same point perpendicular to the line joining the centers. An area of high barometer is not, therefore, necessarily one of maximum cold.

Such areas of high barometer are usually called anti-cyclones, and the air does in some manner move around them in a direction contrary to that of a cyclone, but this does not arise from a central area of greater cold, for it has been shown that such a condition would give rise to a cyclone and not to an anti-cyclone, and that the latter would be entirely at variance with fundamental and

well-established principles of mechanics. The areas of high barometer, theoretically, are not circular, and neither the gradients nor the motions around them on the different sides are symmetrical, and they are not anti-cyclones in any proper sense of the term.

33. So far we have regarded the cyclones as existing in an atmosphere having no motions except those belonging to the cyclones. But on account of the general motions of the atmosphere, treated of in the first part of these researches, cyclones usually occur in an atmosphere having a general motion independent of those of the cyclone itself. In order, therefore, to have the true motions relative to the earth's surface, we must take the resultant of the two motions. In the middle and higher latitudes of the northern hemisphere the general or normal motion of the air is from the west or southwest. In a cyclone in these latitudes this motion of the air coincides with that of the southern side of the cyclone, and increases the motion relative to the earth's surface, but on the northern side it is in the contrary direction, and counteracts in some measure that of the cyclone, or it may even entirely destroy it. On the east or northeast side of the cyclone and its opposite, this general motion from the west or southwest would have little effect upon the velocity relative to the earth's surface, but would affect mostly the *inclination*, that is, the angle between the tangent to the isobar and the direction of the wind, decreasing it on the east or northeast side of the cyclone and increasing it on the west or southwest side.

In the latitude of the trade-winds, where the general or normal direction is from the east or northeast, the reverse takes place, so that the velocities of the air belonging to the cyclone are diminished or, it may be, entirely destroyed by the general or normal motion on the south side of the cyclone, but increased on the north side. The effect upon the *inclination* is in this case likewise reversed, being increased on the east or rear side and decreased on the opposite side.

Since, as we have seen in § 23, the general motions of the atmosphere form two great systems of cyclone and its corresponding anti-cyclone, all ordinary cyclones are simply cyclones within a cyclone, so that all that has been stated in § 32 with regard to smaller cyclones within ordinary cyclones is applicable here to ordinary cyclones with reference to the general motions of the atmosphere. Hence the isobars of cyclones, as laid down upon the charts, are the resultants generally of two or more systems of cyclones contained within or encroaching upon one another, and to have the isobars belonging to any one system it would be necessary to eliminate the effects of the others. The isobars of all ordinary cyclones are therefore affected generally by the isobars belonging to the general motions of the atmosphere, as well as by the smaller class of cyclones which may be contained within the ordinary ones.

34. The progressive motion of a cyclone depends mostly upon the general motions of the atmosphere, but also upon the tendency of the cyclone to press toward the pole, as explained in § 32. In the trade-wind latitudes the wind at the earth's surface is westward, or at least has a large westerly component, and hence the cyclones in these latitudes are carried westward by this westward motion of the air, especially at certain seasons, and having likewise a tendency toward the pole the resultant of the two is a westward motion, inclined a little toward the poles, or in the northern hemisphere a motion about WNW. After having arrived at the parallel of 30° or 35° in the tropical calm-belt, where there is no westward motion, the progressive motion is a polar one mostly, but after progressing still nearer the pole, into the middle and higher latitudes, the general eastward motion of the atmosphere here, which is great in the upper regions, carries now the cyclone toward the east, and the direction of the progressive motion, which is usually about ENE., is the resultant of this eastward motion and the motion toward the pole. All well-developed cyclones, therefore, having their origin near the equator, have mostly a progressive motion represented by a curve somewhat in the form of a parabola, having its vertex in the tropical calm-belt at the parallel of 30° or 35°.

Both the progressive velocities and directions, however, are very different in individual cases, so that there must be other strong modifying influences, and perhaps among these the principal are the distribution of the aqueous vapor and the positions of the general isothermal lines. The equatorial side of a cyclone is generally warmest and contains the most aqueous vapor. As the air containing this vapor is carried around to the east and ascends, it becomes colder both from its ascent to higher altitudes and from its being carried into higher latitudes, so that the vapor which it contains is mostly condensed on the east or northeast side of the cyclone. As the power

of the cyclone is much increased by the latent heat of the vapor given out by condensation, this gives rise to a tendency to form a new center of a cyclone continually in advance of the old one, so that the progressive motion of the cyclone is rather a continual forming of new cyclones, at least so far as the lower part of the atmosphere is concerned, in the direction in which the vapor of the cyclone is mostly condensed. Hence the velocity of the progressive motion is generally much greater than that of the general motion of the atmosphere below. The further around the vapor is carried before it is condensed, the more it will incline the direction of the progressive motion from the east toward the pole. If it should not be carried around as far as the east before it is mostly condensed, it inclines the progressive motion from the east toward the equator. If the isotherms should vary much from the parallels of latitude this also might have considerable effect, so that if they extended from SW. to NE., the progressive motion would be more inclined toward the poles than they would be if they extended from NW. to SE.

These views with regard to the effect of the unequal distribution of vapor upon the direction of the progressive motions of cyclones was first given by Rev. W. Clement Ley. (Laws of the Wind, chap. vi.)

35. In the general motions of the atmosphere the eastward motion, especially in the middle latitudes, is much greater in the upper regions than near the earth's surface, and increases in proportion to the altitude, as may be seen from Table XI, § 43, of the first part of these Researches, and this is especially the case in the northern hemisphere in the winter season, in which, according to the table, the rate of increase with the altitude is more than twice as great as in the summer season. The tendency of this is to carry the upper part of the cyclone eastward faster than the lower part, and as the upper part, in which condensation mostly takes place, carries with it in some measure the power of the cyclone, the progressive motion depends more upon the motion of the upper strata than upon that of the lower ones near the earth's surface, and the lower part of the cyclone tends to lag behind the center of power of the cyclone and to come from under its control, and new portions of the lower strata in front of the cyclone are being continually brought under its power and made to form a part of the cyclone. The cyclone, therefore, so far as the lower strata are concerned, is being continually formed anew in advance from this cause, just as the whole cyclone at all altitudes is, from the effect of the condensation of vapor, because in the middle latitudes of the northern hemisphere it takes place mostly on the east or northeast side of the cyclone. So far as the progressive motion of the cyclone depends upon the general motions of the atmosphere, this motion, in the middle and higher latitudes, should be greater in winter than in summer, since the general eastward motion is greater in the former season than the latter.

From Table XI, § 43, Part I, it is seen that on the parallels of the trade-winds in the northern hemisphere, say 15° or 20°, there is an eastward component of motion in the winter season above the altitude of about one kilometer, which at still higher altitudes soon becomes very strong, so that at that season it would be scarcely possible for a cyclone to move westward unless its power were confined mostly to the lower strata of the atmosphere; for if this power were mostly above, where the general motion is eastward, the whole system must likewise move eastward. In the summer season, on the other hand, it is seen that there is a westward component of motion at all altitudes except in the very high and rare portions of the atmosphere, and hence at this season cyclones in these latitudes must be carried westward.

36. If a regular cyclone existed in an atmosphere in which the general motions of the upper and lower strata were the same, the isobars would be circular, and likewise the rain areas, but where the upper strata move faster in any direction than the lower ones, the whole body of the cyclone becomes of the form of a cylinder inclined forward in that direction, and the effect is to extend the barometric pressure in that direction, and to give the isobars an elliptical instead of a circular form. The effect is the same also upon the rain areas, since the cloud, embracing the upper part of the inclined cylinder, is likewise extended in the direction of the general motions. As the base of the cloud is generally at a considerable altitude above the earth's surface, the center of the cloud and rain area must fall a little forward of that of the barometric pressure, since the latter depends upon the whole inclined cylinder from bottom to top, while the former depends upon the upper part only.

In view of the great extent of a cyclone in comparison with the height of the atmosphere,

neglecting the upper very rare portions, the whole must be regarded as a very thin disk rather than a cylinder. It is probable, therefore, that the forward inclination of the axis of the gyrations is not very great, or at least that the center of gyration of the upper strata does not fall very far in advance of that of the lower strata. For if it did, there would be so great a difference in the velocities of the upper and lower strata in some parts that the friction arising from this cause would tend to counteract this state, and cause the gyrations of the upper and lower strata to somewhat coincide. Still the warm and moist air and cloud of the upper region would be carried far in advance of the center of gyrations below, and this would tend to give the whole cyclonic area an elliptical form, and perhaps to this cause is mostly due the elongation of the areas of low barometer and of the rain areas.

Since the relative velocities of the different strata eastward, in the middle latitudes especially, are greater in winter than in summer, the isobars and the rain areas should be more extended in the direction of progressive motion in the former than the latter season, and the higher the base of the cloud, the further in general should the center of the cloud and rain area fall in advance of the center of the isobars and the point of minimum barometric pressure.

In the latitudes of the trade-winds in summer and autumn, when cyclones occur in these latitudes having a west or northwest progressive motion, it is most probable that the isobars and rain areas are not extended in the direction of progressive motion, and that the center of the cloud and rain area does not fall forward of that of the isobars, since in these latitudes and seasons the upper strata can scarcely have a greater general motion than the lower ones in the direction of progressive motion.

CHAPTER II.

PRACTICAL APPLICATIONS OF THE THEORY AND COMPARISONS WITH OBSERVATION.

38. After giving any theory with regard to natural phenomena, which shows the relations between given conditions and the results which must follow, the next most important step, and the one which naturally follows in regular order, is to compare the theoretical relations or laws with observation where there are observations available for this purpose. And this is especially the case where there is any vagueness or uncertainty in the theory, so that it does not carry absolute conviction with it, but rather merely suggests laws which, although they may be regarded from theoretical considerations as being highly probable, are still not absolutely certain and need to be corroborated by observation. It must be acknowledged that this is frequently the case in meteorological researches, in which the conditions of nature can often be only imperfectly and approximately represented by mathematical equations, and these are then still too complex to admit of a complete solution. For then absolute quantitative results cannot generally be obtained, but merely certain laws or relations depending on the generally unknown laws and constants of friction, or upon constants left undetermined from the impossibility of a complete solution of the problem, so that although the laws may be established by theory, yet the quantitative results depending on these unknown constants, if determined at all, must be determined from observation. Although in meteorological researches theory often needs to be supplemented and corroborated by observation, yet it must not be supposed that the aid derived from theory is not of very great importance; indeed, a very important province of theory often is merely to suggest laws and render them somewhat probable which, but for the theory, would never occur to any one, while the complete establishment of such laws must be left to observation.

In the preceding theory of cyclones certain regular conditions had to be assumed, to which those usually occurring in nature are frequently only very rough approximations. For instance, it is not to be supposed that the conditions of a cyclone in nature consist of regular gradients of increasing or decreasing temperatures, according to any given law, from the center to some definite exterior limit and perfectly symmetrical on all sides, as assumed, for the sake of simplicity, in § 16; for they may often deviate very much from such regular assumed conditions. Besides several sets of such approximate conditions may encroach upon and interfere with one another, and

the conditions of smaller cyclones and tornadoes may be included within those of large cyclones, thus complicating the theory and rendering it more uncertain. In comparisons, therefore, in special cases observation may seem to differ widely from theory, and although this may be in some measure due to the imperfections of the theory, yet it generally arises from the lack of sufficient observations to determine the real conditions which give rise to the phenomena observed. The temperature and barometric gradients of a tornado may be included entirely within the surrounding stations of observation from which synoptic charts are formed, so that theory, from the conditions indicated by such a chart, might give only a moderate wind in a certain direction, while the observed wind, and wind given theoretically by the real conditions, might be that of a hurricane in quite a contrary direction. In the comparisons of theory with observation, therefore, it is necessary to have the averages of a great many observations made under various circumstances and at different times and places, so that the effects of all abnormal disturbances and variations from the assumed regular conditions may be eliminated in the number of observations used.

In the determination of such constants in the theory as are necessarily left to observation, and in the comparisons of theory with observation, it would be impossible, for the want of time and space, to enter here into the examination and discussion of the numerous individual observations made in almost all parts of the world which would be available for this purpose. Fortunately this is not necessary, since there are a number of workers in this field who with great labor have obtained and given to the world many very valuable results which are available for this purpose, so that it will be necessary here mostly merely to enter into the fruits of their labor. For these, investigators generally of meteorological theories, as well as the author, should regard themselves as being under very great and lasting obligations. Among these important results obtained from observation are those of the Rev. W. Clement Ley, published in his "Laws of the Winds," and in various meteorological journals; those of Professor Loomis, deduced, with much labor, from the numerous and important observations and weather maps of the United States Signal Service, and published in a series of meteorological papers in the American Journal of Science and Arts; those of Dr. Hildebrandsson, with regard to the upper currents of the atmosphere, arrived at from an examination of the synoptic charts of the Royal Meteorological Institute of Denmark, and also the results of Captain Toynbee, in ocean meteorology. Many very important observations for our purpose are also found in a work*, recently come to hand, on the hurricanes of the Antilles in September and October of 1876 and 1877. These are especially important on account of their coming from a low latitude from which we have comparatively few observations, and from a region which is peculiarly subject to these devastating cyclones or hurricanes; and also because these cyclones are but little affected by the inequalities of land surfaces, and consequently have more of the regularity of ocean cyclones, while the observations can be made with all the facilities for accurate observation afforded by land stations.

With the laws deduced from the theory of cyclones, corroborated so far as may be, and supplemented by observation, the various inequalities, both local and depending upon the seasons, which are observed in the velocities and directions of the wind and in the atmospheric pressure over the different parts of the globe, may be explained, and the phenomena attending the progressive cyclones over land and sea may not only be explained but often anticipated. Such laws are important to all who are interested in observing and understanding meteorological phenomena, but especially to the mariner, exposed to the destructive cyclones at sea, is a knowledge, not only of the laws of these cyclones, but likewise of the normal states of the winds and of the barometric pressure in all parts of the ocean and at all seasons of the year, unaffected by the abnormal disturbances of these progressive cyclones, of the very greatest importance; since, with a knowledge of the normal conditions of the winds and of barometric pressure at any time and place, he can perceive the first indications of the abnormal disturbances which are the forerunners of these storms, and so can be on his guard, and then, with a knowledge of the laws of these storms or cyclones, he can generally avoid at least their most dangerous part.

In the applications of the theoretical results of the preceding chapter it cannot be expected

* Apuntes relativos á los huricanes de las Antillas en Setiembre y Octubre de 1875 y '76; discurso leido en la Real Academia de Ciencias Médicas, Físicas y Naturales de la Habana por el Socio de Mérito Rdo. P. Benito Viñes, S. J., director del Observatorio magnetico y meteorologico del Real Collegio de Belen de la Compañia de Jesus. Habana, 1877.

that complete rules and sailing directions for the mariner can be given here, for these require not only a knowledge of the normal states of the winds and of the barometric pressure and of the laws of storms, but likewise experience in navigation and a knowledge of coasts, currents, &c., and would require a volume; but it is confidently hoped that the theoretical laws and principles here laid down will be found to be of great advantage to practical navigators who do not rely upon mere routine rules, but also upon correct scientific principles, and likewise to those who furnish rules in detail for seamen who do not concern themselves with theories. Some cases, however, will be pointed out in which, in view of the laws and principles attempted to be established here, it is thought that the rules in general use require to be modified.

39. The first important relations obtained in the preceding chapter are those of § 10, deduced from equation (11), showing the relations, under different circumstances, between the directions of the wind and the isobars, expressed by the angle of inclination i, which is the angle between the direction in which the wind blows and the tangent to the isobars. Here theory gives us no quantitative result, since the value of this angle, as shown by (11), depends mostly upon the unknown function of friction F_s. From what is stated in § 7 this cannot be regarded as a function of the velocity in which we could determine from observation the friction constant, except perhaps at the earth's surface and always under precisely the same circumstances, but it is rather a function of the differences of velocities of the different strata of the atmosphere. We know, however, that it depends upon and vanishes with friction, and that where there is friction there must be a certain amount of inclining of the wind at the earth's surface from the direction of the tangent to the isobar toward the center of a cyclone, as stated in § 10 (a), and hence that the air must move around and toward the center in a kind of spiral; but the amount of this inclination, as required by theory and shown by observation, is very different under different circumstances. So far as testing the theory by observation is concerned, all that can be done in this case is to show from observation that there is a certain amount of inclination at the earth's surface, and that this varies under different circumstances as required by theory, but the amounts can only be determined by observation.

There was at one time a long and animated discussion between the respective adherents to the circular gyratory theory of Redfield, so called, and the radial theory of Espy. Both parties, entirely sincere no doubt, but unavoidably biased in some measure in the support of their theories, thought that they found abundant evidences from observation, after careful surveys and measurements in some cases, to support their theories. If either theory had been the true one, observation would no doubt have satisfied the adherents to the other that it was erroneous, but we now see that the truth lies between the two theories, and hence the vague and uncertain individual observations of the effects of cyclones and tornadoes could be easily construed in many cases to support either theory. In justice to Mr. Redfield, it should be here stated that he did not maintain that the winds were strictly circular, but that there was most probably some inclining of the winds toward the center.

40. Mr. Ley was the first who endeavored to determine the angle of inclination in large cyclones from a great number of observations made at fifteen stations, mostly in the British Isles, from which he deduced the following conclusions (Journal of the Scottish Met. Soc., 1873, p. 66):

I. "The winds commonly incline from the districts of higher towards those of lower pressure. The collective mean result for the fifteen stations is 20° 31′.

II. "This inclination is much greater at inland than at well-exposed coast stations. The collective mean for Brest, Scilly, Farmouth, Pembroke, and Hollyhead is 12° 49′, while for the inland stations, London, Nottingham, Oxford, Brussels, and Paris, it is 28° 53′."

These confirm the part of the theory of cyclones contained in § 10 (a), since they show that at the earth's surface there is a certain amount of inclining of the winds from the tangent to the isobars toward the center, and also that at the inland stations, where friction is greater than at and near the sea, this inclination is greater than it is near the sea.

The average inclination of SE. winds was 35° 11′; of NE. winds, 17° 43′; of NW., 9° 4′, and of SW. winds, 20° 13′. In these results S. winds were taken as SW. winds, E. as SE., and so on. Hence ESE. winds have the greatest and WNW. winds the least inclination toward the center, which winds are very nearly those of the NE. and SW. sides of the cyclone, and consequently of the front and rear sides of it, since the average direction of progress of the cyclones was from SW. to NE.

FIG. 6

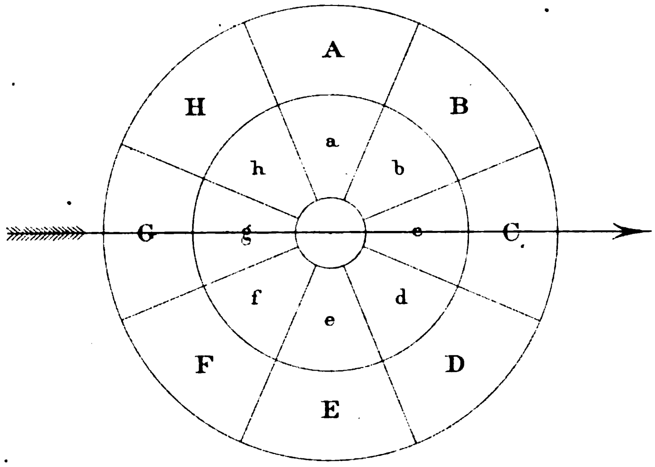


FIG. 7

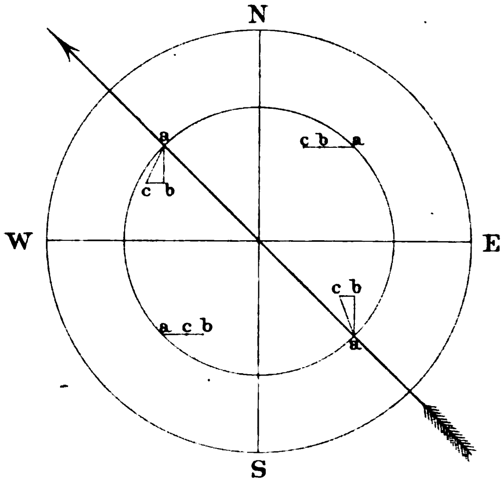


FIG. 8

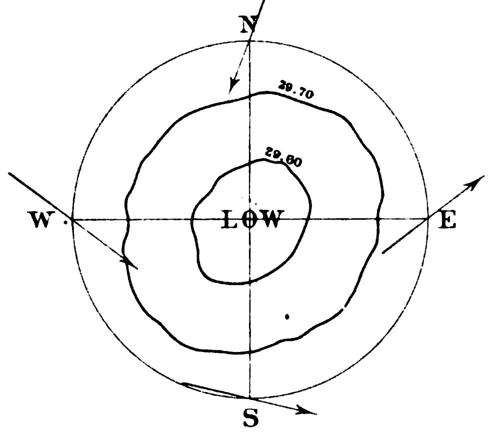


FIG. 9

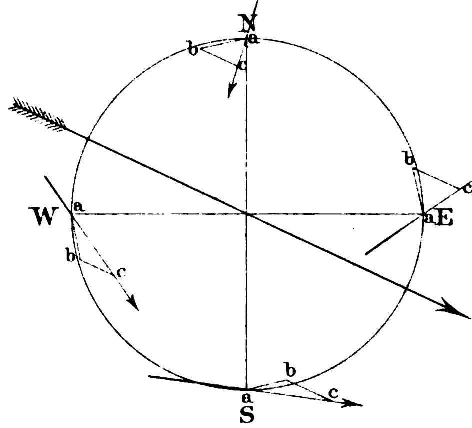


FIG. 10

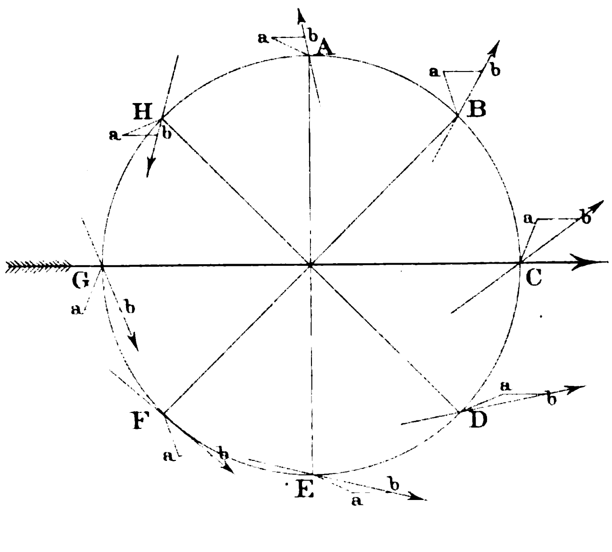
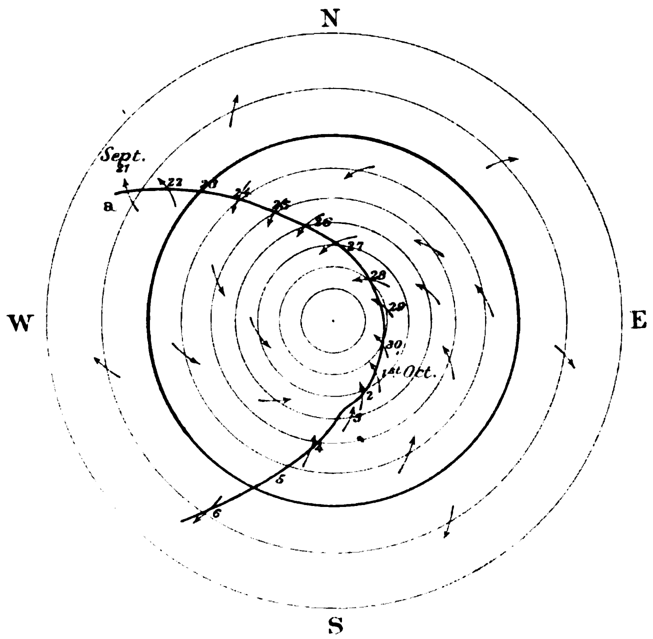


FIG. 11



41. Subsequently Mr. Ley determined the relations between the directions of both the upper and under currents of the atmosphere, in areas of low barometric pressure, and the radius drawn from the center from the averages of a large number of observations in the British Isles, France, Spain, Switzerland, Austria, Turkey, Russia, Denmark, Sweden, and Norway.* His table of results is so important, not only for our present purpose, for which only those results belonging to the surface are needed, but likewise in comparisons further on, that we shall give it here in full, though in a different form and with a different notation. Each area of low depression was divided into sixteen districts, as in Fig. 6, A, *a*, and E, *e* being at right angles on the left and right of the direction of progress of the area of depression.

TABLE.

Exterior districts.					Interior districts.				
Districts.	Surface winds.		Upper currents.		Districts.	Surface winds.		Upper currents.	
	No. of observation.	Mean angle with radius.	No. of observation.	Mean angle with radius.		No. of observation.	Mean angle with radius.	No. of observation.	Mean angle with radius.
		°		°			°		°
A	198	62	51	− 5	*a*	195	65	58	172
B	407	52	173	163	*b*	391	53	104	130
C	511	48	226	152	*c*	426	58	94	135
D	675	54	290	146	*d*	454	55	141	102
E	803	66	328	124	*e*	629	64	135	73
F	378	76	199	101	*f*	402	74	142	61
G	277	79	81	96	*g*	250	77	83	90
H	196	80	43	99	*h*	204	81	46	106

Considering for the present the results only of this table belonging to the surface currents, we find that the average angle with the radius for the exterior districts is 64°.6, and for the interior districts 65°.9, which give for the inclinations respectively 25°.4 and 24°.1, the average being 24°.7. This inclination is a little greater than that obtained by Mr. Ley in his former discussion of observations mostly from the British Isles, but this is what we should expect from theoretical considerations, since in the latter case there was a much larger number of land stations in comparison with the coast stations, and hence friction, and consequently the angle of inclination in the average, should be greater. The mean angle of inclination given by Mr. Ley is greater, and seems to be that given by all the observations without regard to groups or districts, while the averages here given are those of the averages of the several groups. These must give the most probable results, since it is seen from the table that in both the exterior and interior districts the inclinations are greatest in those districts in which there were the most observations, which must be due to some real cause, and the differences cannot be merely accidental ones which may be eliminated by the number of observations.

It is seen that the average inclination of the exterior districts, 25°.4, is very little greater than that of the interior districts, which is 24°.1, and hence these results furnish only a very feeble confirmation of § 10 (*c*), which requires the inclination to be less in the interior than the exterior part of a cyclone, or area of low pressure. It may be seen, however, from an inspection of Fig. 6, that the average distances for the exterior and interior districts was perhaps not very great in comparison with the average radius of the whole districts, and it is mostly near the center that theory requires that there should be much diminution in the inclination.

The districts C, *c*, are on the front side of the cyclone in its progression and G, *g*, on the opposite side, and it is seen from the table that the average inclination of the former is 37°, and that of the latter only 12°. These results agree well with those of the former ones, from observations of the British Isles mostly, the inclinations from the latter for the front and rear sides being, as we have seen, about 35° and 9°, these both being a little less because the general average is less. It seems to be well established, then, from observation, that in European cyclones there is a consider-

* Quarterly Journal of the Metereological Society, 1877, p. 443.

able inclining of the winds toward the center, and that this inclination is greater in the front than the rear part of the cyclone.

42. From the weather maps of the United States Signal Service for the years 1872 and 1873, Professor Loomis obtained the following average inclinations and velocities from a great number of observations in each of the four quadrants of the areas of low barometers (*Silliman's Journal*, July 1874):

	W. quadrant.	S. quadrant.	E. quadrant.	N. quadrant.
Inclination	58° 48′	49° 35′	32° 6′	47° 27′
Velocity in miles	10.1	8.8	8.3	7.6

The average of these inclinations is nearly 47°, which is much greater than that obtained by Mr. Ley for Europe. This, however, is satisfactorily explained by theory. Mr. Ley's observations were mostly of winds of considerable force, but Professor Loomis took in all cases within the isobar of 29.9 inches, which included many observations of velocities of only three or four miles per hour. This is seen not only from the table of results given by Professor Loomis, but also from the small averages of all the observations given above. The average distance from the center, therefore, of the observations used by Professor Loomis was, no doubt, much greater than that of the observations used by Mr. Ley, and hence by § 10 (*c*) the inclination obtained by the former should be greater than that obtained by the latter. Again, the average latitude of the observations used by the former is much less than that of the observations of the latter, and on this account also, by § 10 (*d*), the inclination obtained by Professor Loomis for the United States should be greater than that obtained by Mr. Ley for Europe. All agree in confirming the truth of § 10 (*a*), and in showing that the amount of inclination, on land at least, must in general be considerable, and consequently the cyclonic gyrations not at all circular.

If we now compare the preceding inclinations for the several quadrants with those of Mr. Ley, it is seen that the maxima and minima are nearly opposite, for Professor Loomis gets the greatest inclination in the W. quadrant, which is nearly the rear quadrant on the average in the United States, while Mr. Ley got the greatest inclination in the front districts. This will be considered further on, as we are here merely seeking confirmations of the theoretical deductions of § 10.

The following is the summary of the practical results obtained by Captain Toynbee from a discussion of the observations in the North Atlantic during the great cyclone on the 24th and 25th of August, 1873 (Metereology of the North Atlantic during August, 1873):

1. There is strong evidence that the wind in a hurricane draws in toward its center.
2. The indraft is probably greater in one quarter than in another.
3. The indraft is probably greater near the center than further from it.

The average inclination from all the observations was 29°, the observations being on the average at about the parallel of 50°. This is a greater inclination than that obtained by Mr. Ley for Europe, though much less than that of Professor Loomis for the United States. Theory requires the inclination to be less on the ocean than on land on the same latitudes.

The last item in the summary above is contrary to theory, but the author seems to regard this as only a probable conclusion. The few observations which can be made on the ocean in a great storm, where there are no facilities for making them accurately, cannot be relied upon to give very accurate results.

43. From the observations of Padre Viñes on the hurricanes of the Antilles during September and October of 1875 and 1876, we may deduce some valuable results with regard to the inclination of the winds in cyclones in these low latitudes. Although we do not have in these the averages of many observations, yet we can rely on them with considerable certainty, since the cyclones here, being nearly the same as ocean cyclones, are more regular and not so much affected by irregular abnormal disturbing influences, which require large numbers of observations for their elimination. The following extracts from his book, some in substance merely, but mostly somewhat literal translations, have a bearing on the present part of our subject, which is seeking confirmations of § 10 and the determination of quantitative results which theory does not furnish. In all the hurricanes

of the Antilles it was observed "that the gyrating winds cease to be circular at a long distance from the vortex and are found to deviate from the tangent to the circle with an inclination toward the center, forming a kind of large converging spiral. This converging is likewise said to "vary not only in different hurricanes, but likewise in the same hurricane, with different directions and intensities of the wind and with different distances from the vortex." In the same connection it is stated that the inclination is "especially small at no great distance from the vortex." (Apuntes, &c., p. 90.)

In the hurricane of September, 1875, "the winds of the anterior part of the storm were approximately circular, or with a slight inclination toward the center in some cases." But from observations made at numerous places "the winds of the second (SE.) quadrant, which remained at all these places when the vortex was at a considerable distance, suffered a great deviation toward the center, and in some cases likewise the winds not far from the vortex." (Apuntes, &c., p. 92.)

Similar observations were made of the hurricanes of 1876. In the anterior part of the storm the winds deviated but little from the tangent toward the center, but in the posterior part they blew almost directly toward the center. On the island of Porto Rico "little deviation in the winds of the third and fourth (SW. and NW.) quadrants was noted, some greater convergency in those of the first (NE.) quadrant and a great inclination toward the center in those from the E. and S., especially as they became at a great distance from the vortex." (Apuntes, &c., p. 93.)

The hurricane of the 19th of October, 1876, is presented as a notable example of the great convergency of the winds. "After its passage by Havana the winds, which in the posterior part blew from west to south, suffered a great deviation toward the center, and that not only at a distance from the vortex but even in its vicinity." And "the winds which prevailed from SSE. to S., and which, during the passage of the vortex by Havana blew with force in the different towns situated to the ESE. of Havana, suffered likewise a very notable inclination toward the center. With respect to the winds which prevailed in the first quadrant in the different localities of the anterior part of the cyclone, there was observed in them likewise a notable convergency, though in general in a less degree." (Apuntes, &c., p. 93.)

From the preceding extracts, and many others of the same kind which might be cited, it is evident that there is not only an inclination of the winds in the cyclones of the Antilles, in accordance with § 10 (*a*), but that this inclination is much greater than in high latitudes, as required by § 10 (*d*). This latter deduction from theory, if true, is very important from practical considerations, and although there cannot be any doubt with regard to it where the theory is understood, yet any observations which tend to confirm it must be regarded as being very important, and many more observations of the same kind are very desirable. In the extreme cases, namely, in front and rear of the storm, the angle of inclination in the former is stated to be very small and the direction of the wind to be nearly at right angles to the radius of the cyclone, and in the latter to be so large that the wind blew almost directly toward the center. On other sides of the cyclone the observations seem to give about a mean between these extremes. The angle of inclination, therefore, in the cyclones of the Antilles, and at all places at sea in the same latitudes, is probably about 45° on the average of all cases of great velocities of the wind, and still much greater where the velocities are not very great. But we do not have observations enough yet for determining quantitative results with much accuracy. At sea, in the latitude of England, this angle is most probably only from 20° to 25°. Still nearer the equator than the latitude of the Antilles the inclination must approximate to 90°, and on the equator, if there could be any central point of sensibly lower pressure, we know that the winds would be radial, in accordance with Espy's theory. This theory took no account of the important influence of the earth's rotation, and is therefore true only, so far as the direction of the wind is concerned, for an earth without rotation on its axis, and at the equator, where the effect of rotation vanishes. But this theory would give no violent winds or areas of sensibly lower barometric pressure, as we see is the case at and near the equator.

With regard to the theoretical law of § 10 (*c*) there are many evidences in the observations of the hurricanes of the Antilles, to show that the inclination near the vortex or center of the cyclone is less than at considerable distances. It is not only so stated, as in the cases cited, but it may also be inferred from many other places where, in referring to the great inclinations in special

cases the phrase is often added, "even near the vortex," as though it was considered that the inclination there is usually less than at greater distances.

44. From the deductions of theory, confirmed now and supplemented by numerous observations, it is evident that many of the usual rules and sailing directions must be very much modified, especially in low latitudes. Although the horn cards of Piddington, and all the rules based upon the strictly circular theory of the winds, may still be used at sea in high latitudes without great error, yet nearer the equator they must become more erroneous, and entirely fail at the equator if cyclones could exist there. It is beginning to be pretty generally acknowledged that in sailing directions in a storm, some allowance should be made for a certain small amount of inclining of the winds, but it seems to be thought that this should be the same in all latitudes and at all distances from the center of the storm. If in low latitudes the inclination of the winds may be 60° or more, the determination of the direction of the dangerous center of a cyclone from the usual rules based upon the circular theory, in which the inclination is supposed to be naught, would lead to an error of five or six points of the compass. For this reason the amount of inclination at sea under average circumstances should be determined from observation for different latitudes, so that the navigator could modify his rules to suit the latitude; but unfortunately we have but few observations yet for this purpose at sea, and very few on land for the lower latitudes. Even on the same latitudes considerable allowance should be made for distance from the center. While the center of the cyclone is yet at a considerable distance and the winds have not a great velocity, it should be considered that they are probably, even in high latitudes, considerably inclined toward the center and that nearer the equator they may be nearly radial, but even in these latitudes at places near the center, where the velocities are very great, the gyrations may become nearly circular.

45. So far we have neglected to consider the effect of the progressive motion of the cyclone or air in which the cyclone exists, and consequently have regarded the inclinations as being the same on all sides. Where the cyclone has a progressive as well as cyclonic motion, the former makes the inclinations differ on different sides, as stated in § 34. The progressive motion of the atmosphere at the earth's surface in the United States is very small, as is evident from an examination of the isobars on Chart I, Part I, and as has also been shown by the late Professor Coffin in his "Winds of the Globe," according to which this velocity is only in general two or three miles and even less in places, and from the SW. This, however, with Professor Loomis' small average velocities of the wind, given in § 42, should increase the inclination very sensibly in the west quadrant and decrease it in the east quadrant, while the inclinations of the north and south quadrants should be little affected and have about the average value of all. This is the case with his results, and hence they are in accordance with theory. From the chart referred to above, the progressive motion of the air in the northern part of the Atlantic Ocean, between Europe and America is comparatively great, as shown by the closeness of the isobars, and hence this motion must affect the inclinations in cyclones there very much, especially in such parts of them as have only moderate velocities of the wind, and from this cause the winds should have smaller inclinations in the front than the rear part of the cyclone.

But it now becomes very difficult to explain by the preceding theory why Mr. Ley obtained greater inclinations in Europe for the front than the rear parts of the storms, while Professor Loomis, for the United States, and Padre Viñes, for the Antilles, obtained the reverse. This, however, may be explained somewhat plausibly from the consideration that the cyclones in Europe, especially those of the British Isles and Norway, are cyclones within a great cyclone with its center near Iceland, and this is especially the case during the winter season, as is seen from Chart V, Part I. They are consequently cyclones upon an inclined plane, as it were, and hence the isobars, as usually charted, are very much distorted, and the center of the cyclone as given by the charted isobars is thrown to the left of the real center of the cyclone, as is shown in Fig. 4, and consequently to the NW. of the real center in this case. It is readily seen that this decreases the angle with the radius on the front or NE. side of the storms and increases it on the rear or SW. side, and consequently the inclination is increased in the former and decreased in the latter.

There is also another effect which should be considered here, which is that of the term $D_t s$ in (11), which depends upon the inertia of the air. Since at the earth's surface, if the atmosphere

has a greater progressive motion above than below, a new cyclone is being continually formed and new portions of air in front are receiving an increasing gyratory motion, while the gyratory motion in the rear is being continually diminished for the same reason. The term $D_t s$, therefore, for the front part is positive and for the rear negative, and hence by (11) it tends to increase i on the front side and decrease it in the rear. This may in some cases be considerable, but the effect must generally be small.

46. The effect of the progressive motion of the air on the inclinations of the winds in the cyclones of the Antilles is quite large, since the progressive motion of the trade-winds is comparatively great. Here the trade-winds become nearly east winds, as shown by Professor Coffin in the "Winds of the Globe." Hence, the winds whose cyclonic component of motion is from the N. or S. suffer the greatest deviation from this direction, the inclination being decreased in the former case and increased in the latter. This will be better understood from Fig. 7, in which ab represents the cyclonic component of motion, supposed to have an inclination of 45° on all sides, and bc the progressive component from the east, or nearly, and ac, the resultant of both. The direction of the progressive motion of the cyclones of the Antilles, especially at Havana, may be assumed to be about NW., represented by the arrow in the figure. It is seen that in the front part of the storm the inclination of the resultant ac is much diminished in consequence of the progressive component bc, and made almost perpendicular to the radius, while from the same cause it is very much increased in the rear, and the direction of the resultant motion coincides very nearly with the radius, that is, is very nearly toward the center. On the right side, the direction of the progressive velocity bc coincides with that of the cyclonic motion ab, and consequently the resultant is increased, while on the left side it is in the contrary direction, and consequently the resultant velocity is diminished.

We have now a complete explanation of the observed differences of inclination of the winds in the different quadrants of the cyclones of the Antilles. By a reference to the citations already given in § 43 it is seen that in every instance the winds in the anterior part of the cyclone are said to be nearly circular or to deviate but little from the tangent, while in the posterior part, where the winds are mostly from SE. to SW. the inclination is said to be very great and the winds to blow almost toward the center, as represented by the directions of the resultant ac in Fig. 7, in the anterior and posterior parts of the diagram.

It is further stated with regard to these cyclones generally, "that the winds which have suffered least deviation with respect to the tangent of the circle having for its center the center of the vortex of the cyclone, have been those which have blown from the first and fourth quadrants, and those deviating most with an inclination toward the center, those (which have blown) from the third, and especially those from the second quadrant. In the winds from the E. and S. the inclination is such sometimes at all distances from the vortex that the wind blows directly toward it." (Apuntes, &c. p. 91.) Here it is stated again that the winds which blew from the first and fourth quadrants, which are the winds of the W. or NW. quadrant of the cyclone, and consequently the anterior part of it, had the least deviation from the tangent, and that those winds which blew from the third, and especially those from the second, quadrant, and which are necessarily winds of the posterior part of the cyclones, had the greatest inclination, as they should from what has been shown. This inclination seems to have been so great in winds from the SE. that they blew directly toward the center, as they would according to the posterior part of the diagram of Fig. 7, if the progressive velocity bc were made a little greater in comparison with the cyclonic component ab.

47. From what precedes it is seen that the navigator, on determining the direction of the vortex of a cyclone from the direction of the wind, should, in addition to considering latitude, distance from center, and velocity, likewise consider in what quadrant of the cyclone he is situated, since the direction of the vortex with reference to that of the wind is so different in different quadrants, especially where there is a large progressive motion, as in the trade-wind regions. In the front part of a cyclone, where the mariner is in the greatest danger, the direction of the vortex is generally more nearly at right angles to the direction in which the wind blows—to the left in the northern hemisphere and the contrary in the southern—and consequently the old rules of the circular theory are more nearly correct here than on any other side of the storm. We have seen that this is the case in the front part of the cyclones of the Antilles, where the direction of the vortex is

nearly at right angles to the direction of the wind, while in the rear it is nearly in the direction in which the wind blows. On either side of the direction of progress, the direction of the vortex in cyclones here may be estimated to be about 45° to the left of the direction in which the wind blows.

48. Considering now the directions of the wind in a cyclone at different elevations, and in the upper regions of the atmosphere, it is found that the deductions from theory are generally confirmed by observation in a remarkable manner. According to § 10 (*b*) the inclination is less at some distance above the surface of the earth than at the surface, and at great altitudes it is negative or outward, and hence where the cyclonic motions are not too much disturbed by combined progressive motions, the directions of the clouds, in the northern hemisphere, should be from a point which is to the right of that from which the surface winds blow. The averages of numerous observations in the middle latitudes show that this is the case. Professor Loomis examined in the published volumes of the United States Signal Service the directions of the surface winds and those of the clouds in a great many cases in which there was a well-defined area of high pressure on the east side of the area of low pressure within the limits of the United States. The result at which he arrived was that the surface winds "circulated around the low center, and at the same time moved spirally inward. The upper clouds were in all cases moving away from the low center and towards an area of high pressure on the east and southeast side." (Silliman's Journal, July 1878, p. 17.) Hence the motion was outward above and from a point to the right of that from which the surface winds blew.

Observations on the clouds in the hurricanes of the Antilles gave also a similar result. The clouds in the different positions of the vortex ran almost perpendicular to its direction from the observer, showing that the winds at the altitude of the clouds are less convergent than at the surface. It was likewise observed by Sr. de Gamboa, Director del "Diario de Cienfuegos," in the hurricane of September, 1876, that the winds at the surface had a greater inclination than the lowest clouds. (Apuntes, &c., p. 178.)

Since it has been shown, both from theory and observation, that the air in ordinary cyclones moves in toward the interior in the lower part of the atmosphere, it must of course move out from it in the upper regions, and the inclination be negative, or outward. That this is the case is shown by the table of results given in § 41, obtained by Mr. Ley from his observations of the cirrus clouds. The average of all the angles in this table for the upper currents and for all the districts is 109°, and hence greater than 90°, and consequently the direction is inclined outward 19° and there is a motion of the air above out from the center. The angle of inclination of the surface winds given by the table is 25° inward. Hence the difference between the average directions of the currents above and at the surface is about 44°, those of the upper currents being to the right of those below.

49. The conclusions at which Dr. Hildebrandsson arrived with regard to the upper currents of the atmosphere, from the synoptic charts of the Royal Meteorological Institute of Denmark, are the following:*

"1. Tout près du centre d'un dépression ou minimum barométrique, les courants supérieurs se mouvent à peu près dans une direction parallèle aux isobars et aux vents inférieurs.

"2. À mesure qu'on s'éloigne du centre, ils sont pliés en dehors et déviés à droite des vents inférieurs.

"3. Sur les régions des maxima ils convergent vers leur centre en coupant les isobars à peu près à angles droits."

The first of these confirms that part of the theory which requires a small angle of inclination near the center, for unless this angle is small both below and above, since it must be positive below and negative above, the directions above and below cannot be nearly parallel. The second shows that further from the center the inclinations are greater, in accordance with theory, and hence there is a greater deviation of the currents above to the right of those below. While the third is not in accordance with theory taken generally, it will be shown, further on, that it is in most cases as a result of the effect of the progressive motions of the air in the upper regions.

Mr. Ley from 620 observations upon the motions of cirrus clouds arrived at the following important general law showing the relation between the direction of the higher currents of the

* Essai sur les courants supérieurs de l'atmosphère dans leur relation aux lignes isobarométriques, par H. Hildebrande Hildebrandsson, présenté à la Société Royale des Sciences d'Upsal le 28 novembre 1874.

atmosphere and the distribution of atmospheric pressure at the earth's surface: "*The higher currents of the atmosphere, while moving commonly with the highest pressures, in a general way, on the right of their course, yet manifest a distinct general centrifugal tendency over the areas of low pressure and a centripetal over those of high.*" (Laws of the Winds, *p.* 156.)

According to this law, deduced from observation, the directions of the higher currents incline outward from the center of low pressure and tangent to the isobars, as observed by Dr. Hildebrandsson, and as theory requires.

50. In observing and studying the motions of the upper currents in cyclones it is important to take into consideration the results deduced from theory in § 18, according to which, in ordinary cyclonic systems, the gyratory velocities above are less than below, and at great altitudes may be even reversed over the greater part or the whole cyclonic or low-pressure area below, and become anti-cyclonic instead of cyclonic. This seems to be confirmed by Mr. Ley's observations on the cirrus clouds. He states that "there occur at rare intervals in Western Europe depression systems which affect, but in a very singular way, the directions of the upper currents, reversing them so that they become, on all sides of the area, nearly, or quite, in opposition to Ballot's law, that is to say, there exists a direct upper current circulation above a retrograde circulation of the surface winds." (Laws of the Winds, p. 159.)

Mr. Ley's direct and retrograde circulations correspond to what we have denoted by anti-cyclonic and cyclonic gyrations, respectively, and it is readily seen from an inspection of Fig. 1, in which the directions of the upper currents are represented by dotted arrows, and the dividing line between the cyclone and anti-cyclone indicated by a dotted circle. There is an overlapping of the upper anti-cyclone over the outer part of the cyclone below, and over this belt the winds above and below are very nearly or quite in opposite directions, and hence the former are in opposition to Ballot's law over this belt.

Mr. Ley very justly remarks that "it is important to observe that these examples furnish no proof that Ballot's law *may not be true*, or approximately so, *in every case, for the particular stratum of atmosphere to which it is applied.*" This has been shown, in § 19, to be the case according to the cyclone theory, since the distance of maximum pressure from the center, in strata of different altitudes, decreases with the altitude; so that although the currents above may be nearly in accordance with Ballot's law if we measure the pressure at that altitude, yet if we compare them with the pressures at the surface all the upper currents observed from the belt, Fig. 1, in which the anti-cyclone above overlaps the cyclone at the surface, will be nearly in opposition to Ballot's law, if compared with the pressures at the earth's surface.

51. If we had a cyclone with a cold center the area of cyclonic gyrations would be greatest above, and there would be a belt in which the cyclonic gyrations above would be seen over the gentle anti-cyclonic ones below. Although the conditions for a local progressive cyclone of this kind might exist temporarily, arising from the primary disturbances of temperature, yet they could not give rise to much cyclonic activity, since the currents, downward in the interior and outward from the center at the earth's surface, would be subject to great friction, and the power in ordinary cyclones, arising from the caloric of condensation, would be entirely wanting. Observations, therefore, on local and progressive cyclones of this sort have not been made to establish the truth of this latter law, the reverse of that observed by Mr. Ley in the case of ordinary cyclones. We have, however, two notable examples of stationary cyclonic systems of this sort, in which the law is unmistakably observed. The northern and southern hemispheres of the globe furnish two systems of this sort with the cold centers at the poles of the earth. The cyclonic gyrations at the earth's surface extend to about the parallel of 35° or 30°, but at great altitudes they extend to the calm-belt at the equator, so that in the trade-wind region, where we have below the anti cyclonic gyrations around and from the center, we have above the cyclonic around and toward the center. Hence the motion of the cirrus clouds of this region from the southwest is nearly or quite in exact opposition to that of the trade-winds below.

52. So far, in the consideration of upper currents, we have not taken into account the effect of the progressive motion of the air of a cyclone, which, combined with the cyclonic, gives the observed resultant, and this consequently may be very different from the purely cyclonic; and this is especially the case at great altitudes in the middle latitudes. These motions are generally nearly from west to east, and the eastward components of the velocities of these motions, for the average

of all longitudes, may be approximately computed from the expressions of these velocities in Table XI, Part I. These give for the latitude of the British Isles, say 53° for the eastward component of velocity per hour, at the elevation of 8 kilometers, which may be assumed to be the height of the cirrus clouds—

$$\text{In January, } 6.8^{km} + 8 \times 11.8^{km} = 101.2^{km}$$
$$\text{In July, } \quad 4.2 \; + 8 \times \; 5.3 \; = \; 46.6$$

and for the mean of the year about 74^{km}. These estimates are based upon the assumption that the temperature gradient between the pole and the equator above is the same as at the earth's surface, but it is probably less above, and if so these estimates may be too great. There are also various disturbing causes which may make them vary considerably at different places on the same parallel of latitude. These estimates, however, give at least approximate results, and show that these progressive velocities are quite large, and much more so in winter than in summer, both of which circumstances it is very important to take into consideration.

If we assume that the progressive velocity above is 74^{km} from west to east, and that the gyratory motion at any given distance from the center is circular, and has a velocity of 60^{km} per hour, then on the south side we should have an eastward velocity of $74 + 60 = 134^{km}$ per hour, but on the north side only $74 - 60 = 14^{km}$ per hour, and there would be a motion from some quarter of the west on all sides of the cyclone. At lower altitudes, where the progressive velocity is less, the resultant velocity might vanish on the north side, or there might be a small westward motion, but there would be upon the whole a great preponderance of westerly winds.

This is in accordance with Mr. Ley's observations of the cirrus clouds. He states that "the most elevated clouds not uncommonly traverse a distance of 120 miles an hour, and occasionally much more. The majority of instances in which very high velocities have been observed over the British Isles were in autumn, winter, and spring, and occurred when great but distinct depressions existed in the NE., in Scandinavia or Finland, and the direction of the upper current was from the NW. or NNW. Calms, on the other hand, are extremely uncommon in this elevated stratum, at least in those instances in which it supports visible vapor. I have only once or twice observed an actually motionless cirrus cloud. * * * A condition of baric equilibrium in this stratum is commonly most nearly approached in summer and near the center of areas of high pressure." (Laws of the Winds, p. 163.)

The great velocities over the British Isles were observed when there was a cyclone center in the NE., for then the cyclonic motion coincided somewhat with the progressive motion from west to east. These large velocities did not occur much during the summer season, for then, as seen above, the progressive component is comparatively small, and the magnitude of the resultant depends very much upon this. It is possible, also, that the cyclonic disturbances may be generally less at this season. Calms are said to be rare in these elevated regions, as we would expect from theory, since the progressive motion here is so great that it rarely happens that the gyratory component is sufficient to counteract it and produce a calm on the left side of the cyclone in its progression. Again, this state of baric equilibrium belonging to a calm in these regions, is most nearly reached in summer, because then the progressive eastward velocity is much less, and hence less gyratory motion is required to produce a calm on the left, for unless the gyratory motion equals the progressive, as we have seen above, there must be an eastward motion on all sides. It follows, then, from what precedes, that a stationary, or nearly stationary, condition of the cirrus clouds indicates in the northern hemisphere the existence of a cyclone center at some distance in a southerly direction from the place of observation, while an unusually great eastward velocity indicates the existence of one in some northward direction. In the southern hemispheres the directions are reversed.

53. Professor Loomis's discussion of observations of the United States Signal Service, made on the top of Mount Washington at the times of low barometer, gives the following velocities and inclinations of the wind (Silliman's Journal, January, 1875):

	West quadrant.	South quadrant.	East quadrant.	North quadrant.
Velocity in miles	49	44	37	32
Inclination	55° 7′	— 13° 25′	— 53° 44′	— 69° 54′

A copy of the graphic representation of these results is given in Fig. 8. The progressive motion deduced from these results is N. 65° W. 28 miles per hour, of which the eastward component is 26.4 miles. By Table XI, Part I, this component in open space at the height of Mount Washington would be only about 16 miles, but the current on Mount Washington is increased by the air from lower levels having to pass over the mountain range. Without going into an accurate discussion of the results, it may be readily found approximately that the resultants above are those given by a gyratory velocity of 30 miles with an inclination of 10°, combined with the progressive motion, given above. A graphic representation of the resultants by this assumption is given in Fig. 9, which coincide very nearly with those of Fig. 8, and hence the winds on Mount Washington, for low barometer are satisfactorily explained by the hypothesis that they result from a gyratory or cyclonic motion combined with a large progressive motion.

In order to study the laws of the winds on Mount Washington, Professor Loomis selected, from the published volumes of the United States Signal Service observations, 434 cases in which the wind blew with a velocity of 65 miles or more per hour. As these high velocities must occur mostly when the direction of the gyratory motion coincides somewhat with that of the progressive motion, these velocities occur more frequently in winter than in summer, because the progressive component of velocity, upon which the resultant velocity largely depends, is greatest in the former season, just as in the case of the motions of the cirrus clouds observed by Mr. Ley. Professor Loomis's results from observation give the following proportional numbers for the occurrence of strong winds on Mount Washington in the several seasons of the year: Spring, 14.0; summer, 9.2; autumn, 13.8; winter, 21.2. It is seen that the number of cases of large velocities is more than twice as great in winter as in summer, which shows that at least one of the components of motion, either the cyclonic or progressive motion, is much greater in winter than in summer.

As only high velocities were here considered, it is evident that these must have been mostly from some westerly point, in which cases the two components of motion in some measure coincide in direction. Accordingly the number of cases in which the wind blew from the several points of the compass in these high velocities was "N. 53, NW. 260, W. 63, SW. 27, S. 14, SE. 8, E. 2, and NE. 7. Thus we see that 60 per cent. of all the high winds came from the NW.; 75 per cent. from the W. and NW.; 87 per cent. came from the W., NW., and N., while only 4 per cent. came from the NE., E., and SE." (Silliman's Journal, January, 1879.)

From an examination of 121 cases of high winds on Mount Washington, Professor Loomis deduced the following conclusions:

"1. High winds on Mount Washington circulate about a low center as they do near the level of the sea. 2. The motion of the winds is nearly at right angles to the direction of the low center. 3. The low center at the height of Mount Washington sometimes lies behind the low center at the surface of the earth as much as two hundred miles."

The first two of these conclusions is exactly in accordance with the cyclonic theory, the second one showing that the inclination at considerable altitudes is small, as was observed in the hurricanes of the Antilles, and as required by the theory in § 10 (*b*). For the last of these conclusions theory does not seem to furnish any explanation.

54. A similar examination was made of the observations of the Signal Service on the top of Pike's Peak, and the same general results obtained. (Silliman's Journal, January, 1879, p. 19.) The number of cases of high velocities from November, 1873, to June, 1875, was 363. The proportional numbers of these cases for the several seasons of the year are: Spring, 15.7; summer, 11.3; autumn, 26.5; winter, 31.2. Hence there are nearly three times as many cases in winter as in summer.

The number of cases in which the wind blew from the different directions at the times of these high velocities are: N. 28, NW. 47, W. 154, SW. 111, S. 18, SE. 1, E. 0, NE. 4.

According to these results, the prevailing direction of the winds on the top of Pike's Peak, as on the top of Mount Washington, is from some westerly point, showing that there is a large progressive motion of the wind from the west, or nearly, which is often very much modified by cyclonic motions superadded, which change the direction and velocity of the regular progressive motion, but which rarely so entirely counteract and reverse it as to give rise to a wind from some easterly direction. The total number of winds from SE., E., and NE. during a period of three years, from observations made three times a day, was: Spring, 64; summer, 130; autumn, 86; winter, 30. The greater number of east winds occur in summer, because then the progressive eastward velocity

at the altitude and latitude of Pike's Peak, as given by Table XI, Part 1, is little more than one-third as great as in the winter, and hence in the summer this eastward motion is more readily overcome and reversed, and an east wind produced.

From the great difference in the frequency of easterly winds in the winter and summer season, Professor Loomis infers that "these winds are dependent on difference of temperature more than on difference of pressure." This difference, we have seen, is explained by the greater west component of the winds in winter than in summer, resulting from the greater velocity of the progressive eastward motion in the former than the latter season, and although this depends upon the greater temperature gradient between the pole and the equator in winter than in summer, there is also a corresponding difference in the barometric gradients, if these gradients were determined from the atmospheric pressure at the altitude of Pike's Peak, as they should, as has been stated in § 50, and not at the earth's surface. Although in the northern hemisphere at the earth's surface there is only a small barometric gradient at any latitude between the pole and the equator, yet at the altitude of Pike's Peak, or even of the summit of Mount Washington, the pressure gradient is large, and hence the large eastward progressive velocity of the air. If from the nearly equal barometric pressures of the pole and the equator at the earth's surface we subtract the weight of a column of air of the height of Pike's Peak, and of the temperature of the pole and the equator from the nearly equal pressures at the pole and the equator, it leaves at this altitude a great difference of barometric pressure between the pole and the equator, and consequently there is at this altitude a large barometric gradient, requiring a correspondingly large eastward velocity, and this must increase with the altitude and be much greater in winter than in summer, as given by Table XI, Part 1, which is based upon this principle.

55. We come now to the examination of the results obtained by Mr. Ley from observations of the upper currents already given in the table in § 41. From a mere inspection of the angles given in this table for the several districts it is readily seen that they result from a large progressive motion of the atmosphere in the direction, or nearly so, of the motion of the center of the low-pressure area, combined with a gyratory or cyclonic motion around this center, having a considerable outward or negative inclination. Let A, B, C, &c., in Fig. 10, be the centers of the several outer districts, as represented in Fig. 6, and let A*a*, B*a*, C*a*, &c., represent the velocities of gyratory motion, having an outward inclination of about 30°; and also let *ab* represent the velocity of progressive motion of the upper atmosphere in the direction of the progressive motion of the center of low-pressure area, represented by the arrow, and let us suppose that this latter velocity is very nearly equal to that of the gyratory velocity A*a*, B*a*, &c. The resultants will then be represented by A*b*, B*b*, C*b*, &c. It is readily seen that the effect of the progressive component is to increase the angles with the radius in the front and to diminish them in the rear, just as in the case of the cyclones of the Antilles represented by Fig. 7, except that the progressive velocity in this case is so great in comparison with the cyclonic that the effect upon the angles is very much greater, and is similar to the effect upon these angles on the summit of Mount Washington, as represented by Fig. 8. Upon this hypothesis the largest angles should be those of the districts B, C, and D, and the smallest those of F, G, and H. By a reference to the column of "Mean Angles with Radius" belonging to the upper currents of the outer districts, § 41, it is seen that this is precisely in accordance with observation. In vague observations of this sort, it cannot, of course, be expected that any hypothesis would give any nice agreement with observation in quantitative results. The preceding hypothesis, the angles and ratios of which are represented in Fig. 10, makes the angles a little too large in the rear districts, but this agreement could be improved by assuming a less progressive velocity *ab*. For the interior districts the angles given by observation are less, and hence for these a very small angle of inclination outward would have to be assumed in order to make the resultants represent the observations; and this would be exactly in accordance with the results obtained by Dr. Hildebrandsson for the regions near the center of low area. Also, the gyratory velocity is perhaps somewhat greater in these districts near the center. A diagram similar to that of Fig. 10, with these modifications, would likewise give angles agreeing, as well as could be expected, with those given by the observations, in which, as in the case of the outer districts, the larger angles are in the front and the smaller ones in the rear districts.

It is seen from Fig. 10 that the resultant velocity in the district A is very small, and consequently the average angle from observation very uncertain. In fact the progressive velocity so

nearly counteracts the cyclonic that this must be a district, on the average, of calms rather than of currents. This is in accordance with the observation of Mr. Ley, who with regard to this district remarks that "the upper current, which had previously nearly coincided in direction with the trajectory, presently changes, as a rule, to a nearly opposite point, having just before become very slow. In the interval the cirrus, when visible, which is rarely the case, is sometimes stationary, sometimes moves toward and sometimes from the center of depression, these three instances being nearly equally common, but calms and motions toward the center predominating." (Quar. Jour. Met. Soc., October, 1877, p. 443.)

If these important observations had been analyzed with regard to the seasons, they might have afforded some valuable results for comparison with theory. Mr. Ley, however, makes one remark in his paper which has a bearing on this subject. He says: "I have found that in winter very local depressions, even when deep, scarcely affect the direction of the cirrus currents in their vicinity, the latter continuing to be governed by the more general distribution of atmospheric pressures. Curiously enough, this is not the case in summer." This is completely explained by the result, deduced first from theory, and then confirmed, as we have seen, by the observations on the summits of Mount Washington and Pike's Peak, that the progressive eastward velocity of the upper atmosphere is very much greater in winter than in summer. In winter this velocity is so great in comparison with the cyclonic disturbances that the directions are not greatly affected by them, but this is not the case with the comparatively slow progressive motions of the summer season.

56. In view of what has been shown in the preceding section, it does not seem necessary to resort to the hypothesis of an inclination of the vertical axis of a cyclone backward in order to explain the observed directions of the upper currents of the atmosphere in the areas of low barometric pressure. Inasmuch as we know that the progressive motion of the upper strata is greater than that of the lower ones, this hypothesis of Mr. Ley's is so much at variance with mechanical principles, and with what we would naturally infer would take place under such circumstances, that we should hesitate to adopt it without seeking further for some more plausible hypothesis to explain the observations. These, as we have seen, seem to be satisfactorily explained by combining the large progressive motions of the upper regions of the atmosphere with the cyclonic, and this progressive motion is not a mere hypothesis, but a result deduced from theory and confirmed by numerous observations.

It is true that Mr. Ley's hypothesis receives some support from the somewhat remarkable results obtained by Professor Loomis from the observations of the United States Signal Service on the summits of Mount Washington, Pike's Peak, and Mount Mitchell (Silliman's Journal, Jan., 1879), according to which the maxima and minima of the barometric changes appear to occur later at the summits than at the bases of these mountains, which Mr. Ley explains by his hypothesis. (Quar. Jour. Met. Soc., July, 1879.) But these results of Professor Loomis show too much for this hypothesis, for they show that there is a similar retardation, of just about the same amount, in the times of the maxima and minima of the diurnal changes of barometric pressure at the summits of these mountains, and we cannot reasonably explain this by means of cyclones with reclining axes. When we shall have a satisfactory explanation of this retardation in this latter case, we shall probably have one in the other.

57. Many important practical applications may be made of what we have now learned, both from theory and observation, with regard to the motions of the upper currents of the atmosphere. These, as we have seen, are the resultants of the general motions of the atmosphere, and of those of progressive cyclones arising from local and temporary disturbances. The former change with the seasons, and are considerably modified by the local and permanent irregularities in the distribution of temperature, which is different in different longitudes, and give rise to the approximate conditions of large permanent and stationary cyclones, treated of further on, and these produce some irregularities in the directions of the general permanent currents of the upper regions. With a knowledge of these on the different parts of the globe, and at the different seasons of the year, an observer is enabled to recognize the effect of any cyclonic disturbance, even when the vortex of the cyclone is yet at a great distance, by its effect upon the known regular currents. These are known from theory, and the observations of the bands or streaks of the fine cirrus clouds peculiar to these currents, to be toward some point between N. to E., the direction in low latitudes

being more nearly radial, that is, in the direction of the meridian, than in high latitudes, where it becomes more nearly from W. to E.; but these general directions are not precisely the same on the same latitudes in the different longitudes, owing to the disturbances just referred to above.

The almost universal precursor of a distant cyclone is the appearance of more cirrus clouds than usual, not only differing from those of the general currents in form, but also in the direction of the currents indicated by these clouds. The approach of cyclones from the E. or SE. is always indicated at Havana by the appearance of these clouds when the vortex is yet 500 miles or more distant, and while fair weather is yet prevailing. The same is also observed as they pass off at a distance. Here the directions of the currents as indicated by the cirrus clouds seem to be very nearly radial, as they should be by theory in these low latitudes, and at great distances from the center, and hence the direction from which they come indicates very nearly the direction of the vortex of the cyclone. In these low latitudes the progressive motion of the atmosphere above is small in the cyclone season and does not interfere much with the directions given by regular cyclonic motions.

In higher latitudes the cyclonic motions, referred to the progressive center, are more nearly circular, and the progressive motion, common to the center and the whole of the atmosphere in the upper regions, modifies very much the directions of the currents as they would exist from the effect of cyclonic disturbances alone. Within the cyclonic area, and area of low barometer, they would give the resultant motions as observed by Mr. Ley, and as represented in Fig. 10. But in this area the motions of the upper currents are often hid by the lower clouds, and besides cyclonic disturbances in this area are usually more certainly indicated by surface currents and the currents indicated by the lower clouds which usually exist, so that usually it is only the observation of the cirrus clouds at a considerable distance from the center, beyond the limits of the lower cloud area, that is of advantage in determining the existence and direction of cyclones, and at this distance, it should be remembered, the cyclonic motions of the upper regions, according to theory, may not only become radial, as observed at the Antilles, but even anti-cyclonic. Any derangements, then, in the usual directions of the upper currents, indicated by the cirrus clouds, will indicate the existence of distant cyclonic disturbances before they are indicated by surface winds or clouds in the lower strata, but the exact direction of the cyclonic center will be left somewhat uncertain, though it will generally be found nearly in the direction from which these upper currents proceed, when the observer is in front of the cyclone, and consequently most in danger, since then the progressive motion above, as seen from Fig. 10, combines with the nearly radial cyclonic motion and leaves little uncertainty with regard to the direction. To any one familiar with the usual appearances of the cirrus clouds and the directions of the upper currents indicated by them, and whose duty and interest it is to watch for the first indications of any unusual abnormal disturbances of the atmosphere, a study of the effects of distant cyclonic disturbances upon these upper currents must be of great importance, not only in furnishing an explanation of them, but likewise the first indications of approaching danger.

58. With regard to the existence of an anti-cyclone in connection with every cyclone and the broad annulus of high barometer with its maximum at the dividing limit between the cyclone and anti-cyclone, as shown in § 12, we have many confirmatory observations. But since the maximum height of the barometer is generally but little above the undisturbed normal height beyond the disturbed area, and the gradient of the anti-cyclone is small, and the corresponding winds only very gentle ones, and are very much interfered with and masked by numerous other irregularities, depending upon various causes, the annulus of high barometer and the winds of the anti-cyclone are not readily distinguished on a synoptic chart, and separated from the other irregularities. The charts of barometric pressure and of the winds, given in Part I of these Researches, represent the irregularities of pressure upon which those of the systems of cyclone and anti-cyclone are superimposed, and hence where the inequalities of pressure belonging to both are charted together a regular broad annulus of high barometer is not generally shown by the isobars. As an illustration we may suppose a circular area of uneven land to be inclosed by a wall of the same height on all sides, yet if this height were referred to a level plane, it would not be uniform, but have its maxima and minima. The cyclonic systems, also, are often irregular in themselves, and sometimes two or more interfere with one another, and thus cause still greater confusion.

An unusually high barometer, however, is usually observed a few days before and after the

passage of an area of low barometer, indicating that it is surrounded by such an annulus. Professor Loomis examined the origin and progress of 44 storms in the United States by means of the observations of the published volumes of the Signal Service. He says: "The first stage in the development of each of these storms was an area of several hundred miles in diameter over which the height of the barometer differed but little from thirty inches with an area of high barometer both on the east and west sides and at a distance of about 1,000 miles. * * * On Hoffmeyer's storm charts we frequently find three areas of high barometer, and occasionally four areas of high barometer, surrounding an area of low barometer." (Silliman's Journal, January, 1878, pp. 5–6.) Again, he says: "The areas of high barometer, which uniformly mark the commencement of a storm, invariably attend it during its progress eastward." During the progress of these storms the average height of barometer on the east side was 30.39 inches, and on the west side 30.32 inches. These areas of high barometer around the area of low barometer are simply the annulus of high barometer somewhat broken up and interfered with by the various disturbances already mentioned. The reason that an area of high barometer is especially observed on the east and west sides of an area of low barometer in the United States, is that there is a somewhat regular succession of cyclones following after one another in the middle latitudes, so that the high barometer of the rear of the one and the front of the other fall together and make the height of the areas of high barometer on the east and west sides twice as great as on the other sides, and hence especially prominent and observable. Areas of high pressure were usually found to be accompanied by areas of low pressure on the east and west sides at an average distance of about 1,200 miles. The area of high barometer occurred much more frequently on the east than the west side, and in a direction about 20° S. of E. This is due to an area of high barometer, independent of the local progressive cyclones, lying in the Atlantic in that direction, as is seen from Chart I, Part I, so that this side of the annulus was always, as it were, on higher ground, and this is especially the case in the summer season when the barometer in the interior is about 4^{mm} below the average, which causes a considerable gradient in that direction, as seen from Chart VI, Part I. Loomis also found from Hoffmeyer's charts that in about three-fourths of the cases examined the direction of the area of high pressure from the low is SE. and the average distance about 1,700 miles. In Norway and Sweden there is a considerable ascending gradient in this direction, as seen from the charts referred to above, and hence, for the reasons just given, in the case of American cyclones the highest part of the annulus of high barometer, when referred to a level plane, is in that direction.

59. On the ocean the cyclonic systems are more regular, so that here, if we had a sufficient number of accurate observations, the annulus of high barometer and the winds of the anti-cyclone, could be more readily distinguished from the other irregularities, especially within the tropics where these irregularities are smaller. Hence in the Antilles, where the islands, being mostly small in comparison with the expanse of ocean around them included within the cyclone, do not interfere much with the regularity of the cyclonic system, the annulus of high barometer is readily observed, and the approach of the cyclone center indicated several days in advance of its arrival by the rising of the barometer above its normal level. The approach of the hurricane of September, 1875, was indicated at Havana by a sudden rise of the barometer while the cyclone was yet at the Windward Islands, about 1,200 miles distant. Also, on the 13th of September, 1876, there was a great rise of barometer at Havana while a cyclone was causing great destruction in the island of Porto Rico. (See Apuntes, &c., pp. 117–118.)

We may suppose the situation of Havana with regard to the cyclone in both these cases to have been at *a*, Fig. 11, in which the heavy circle represents the highest part of the annulus of high barometer. Hence there was a rising barometer as the cyclone advanced toward the WNW. and Havana came under the maximum pressure of the annulus. In the first case nothing is said with regard to the direction of the wind, but in the latter the wind was from the south, as required by theory in that part of the cyclone. With regard to the last it is further stated (p. 127) that "during the past night (of the 14th and 15th) we have rapidly departed from the area of maximum barometer, with the wind to the NNE." Havana was then within the dividing limit between the cyclone and anti-cyclone, and hence the barometer was falling and the wind had changed around from S. to NNE., as it should, as may be seen from Fig. 11.

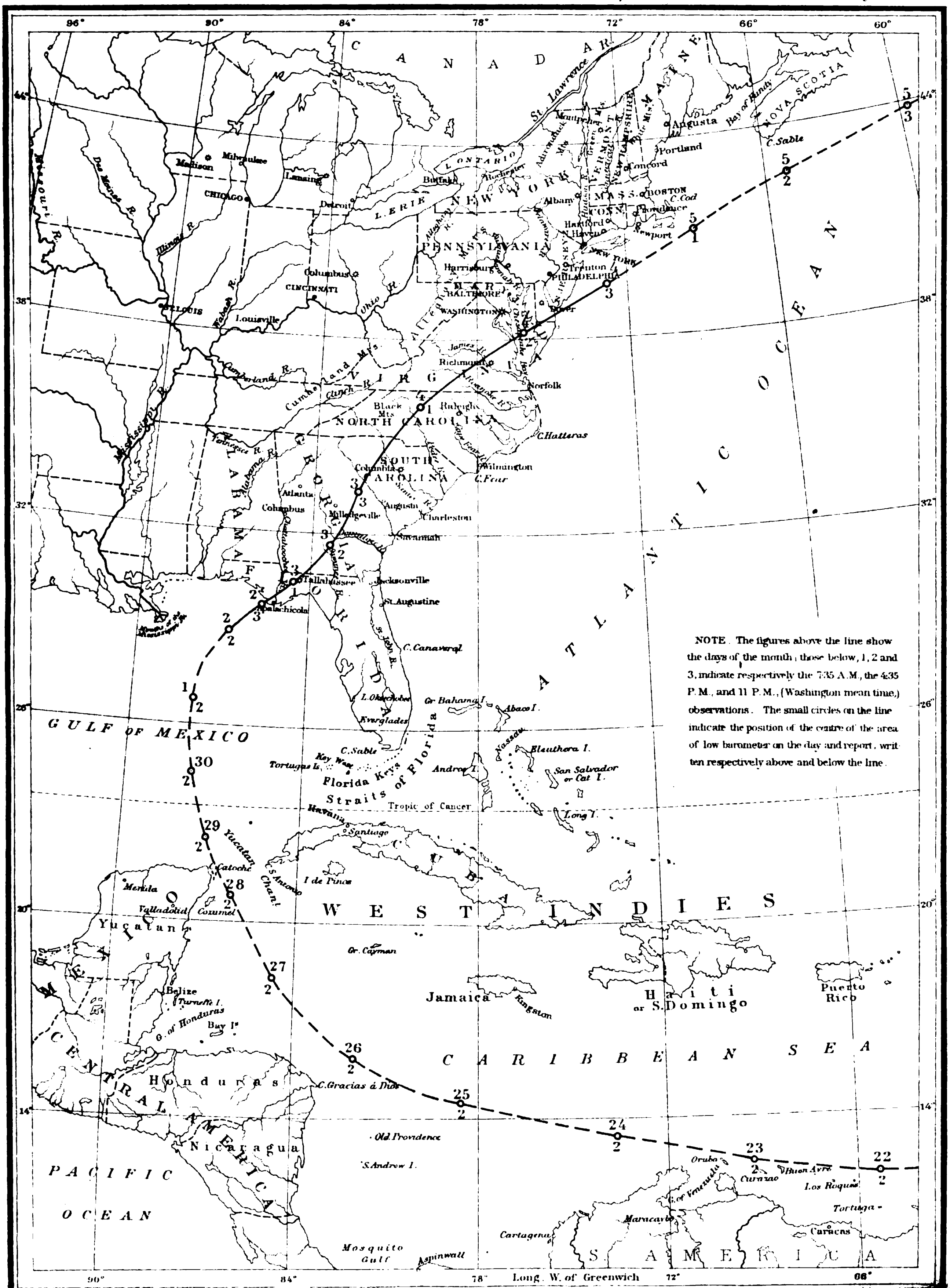
On the 23d of September, 1877, after the announcement of a hurricane on the 21st, among the islands of Barbados, San Vincente, and Grenada, the barometer stood at Havana at the unusual height of 764mm, the wind from the E., and afterward inclining to the NE. In this case we may suppose the situation of Havana in relation to the cyclone to have been at 21, Fig. 11, on the 21st of September, and at 23 on the 23d, under the greatest pressure of the annulus of high barometer. The track of the center of this cyclone, with the positions of its center each day, is given in Fig. 12, which is a copy of a part of the Weather Map of the United States Signal Service for October, 1877. The position of Havana in the cyclone, as obtained from this map, for each of the last days of September and the first part of October, must have been nearly as indicated by the numbers in Fig. 11, corresponding to the dates. As the dimensions of the cyclone gradually increase as it advances, the distances from the center in the figure are not absolute but merely relative to the increasing dimensions, so that we must suppose the figure to represent a larger area, and consequently the same distance from the center in the figure to represent greater distances in the latter days of the cyclone than in the first ones. The arrows in the figure represent the directions of the wind according to theory, making some allowance for the effect of a progressive motion, which would diminish the inclinations on the NW. side and increase them on the SE side.

On the 21st and 22d we have seen that Havana was in the anti-cyclone, and on the 23d under the middle of the annulus of high pressure. We further read (Apuntes, &c., p. 120) that on the 25th the wind was N. and NNE., very nearly as required by theory, as seen from Fig. 11. The barometer continued gradually to fall in Havana until the morning of the 27th, and during that morning there were irregularities in the diurnal oscillation. On the 28th, the barometer continued fluctuating irregularly without any decided tendency to either rise or fall. The wind was at the E. and ENE., fresh, and with variable intensity. By a reference to Fig. 11 it is seen that on the 27th the situation of Havana was very nearly at the shortest distance from the center of the cyclone, and that during this day and the next this distance remained nearly the same, and hence the barometer had no decided tendency to either rise or fall, as was observed. On the 28th, it is stated, the wind was a little N. of E., as required by theory. After this, the wind changed to the south and then the west, and finally, on the 6th of October, after the whole system had passed, and the position of Havana was in the anti-cyclone, it is stated that the wind was from the north. Theory would require the wind here to be rather from the NE., but the directions of the gentle winds of the anti-cyclone are very much changed by slight abnormal disturbances. We see that according to theory the wind, which on the 25th was a little E. of N. should afterwards change around to E., then S., then W., and finally in the anti-cyclone, on the 6th of October, to NE. Accordingly it is stated that from the 25th of September to the 6th of October, when a north wind was prevailing, the wind-vane made a direct complete gyration from N. by E. to S.—W., N. (Apuntes, &c., p. 122.)

If the central path of this hurricane had been on the NE. side of Havana, the line of the relative positions of Havana with reference to the center would have been on the other side of the center of the figure, and it is readily seen that the changes of the wind would have been very different. In all cases in which the track of a progressive cyclone is a straight line, the relative positions of a stationary observer with regard to the center are also a straight line. If this line passes through the center of the cyclone the greatest change of barometric pressure is observed, and at the passage of the center there is a very sudden change of the wind, after the short calm during the passage of the central part, to the opposite direction. If this line passes at either side at some distance from the center, the barometric change is not so great and the veering of the wind is not so sudden nor through so many points of the compass; and the direction in which the wind veers depends upon which side of the observer the center of the cyclone passes. If this line passes near or entirely beyond the limit between the cyclone and anti-cyclone there is only a slight rise of the barometer observed, which is at its maximum when the center of the cyclone is nearest to the observer, and there is no depression observed below its normal position. In all cases in which the position of the observer changes, as in the case of a vessel at sea, of course this must be taken into account also in laying down the positions of the observer with regard to the center of the cyclone, and in determining or explaining the successive changes of the wind. Theory would also give a veering of the wind in the same way, as at and near the center, when the observer passes from the cyclone into the anti-cyclone, but the winds belonging to the anti-cyclone are so gentle, and consequently

FIG. 12

No. 36 *Report of Coast and Geodetic Survey 1878*



so liable to be deranged, or even completely reversed, by slight abnormal disturbances, that this veering is often not observed, at least in cyclones on land.

60. The annulus of high barometer surrounding every cyclone, as deduced from theory, is not only shown from observations of unusually high barometer before and after the passage of cyclones, both in the United States and the Antilles, and from Hoffmeyer's charts, but it is often indicated by the double veering of the wind during the passage of the whole system over the observer. We have also evidence of this annulus from observations in the southern hemisphere. The gales observed by the exploring party of the Challenger at Kerguelen "were preceded by an unusually high barometer, which fell rapidly as the storm began from the north; as the wind shifted to the west the barometer rose." (Quar. Jour. Met. Soc., vol. ii, p. 277.)

So common are these areas of high pressure before and after cyclones at sea that it is thought by some that cyclones often lie between waves of high pressure. Captain Toynbee says he has noticed "that cyclonic winds were formed, as it were, in the hollows of waves of pressure, in which cases the ridges traveled with the cyclones and formed, as it were, part of them. In many cases the ridges, with their corresponding winds, extended over many more degrees of latitude than the cyclones themselves; in fact, they with their intervening hollows frequently passed over us, causing winds veering from S. to NW., which were not cyclones at all." (Quar. Jour. Met. Soc., vol. iv, p. 14.)

It would be difficult to explain, upon any known principles, the existence of mere longitudinal ridges of high pressure; but that there is such a ridge surrounding every cyclone we have seen is strictly in accordance with well-known mechanical principles as developed in the theory of cyclones. Hence, it would seem to be more probable that the observed apparent ridges above referred to are merely parts of the annulus of high barometer, observed on the east and west sides of the cyclone, in its progress from west to east, or perhaps most generally of a succession of such cyclones; in which case the ridges on the east and west sides would be especially prominent, just as in the case of the succession of cyclones passing over the United States, in which, as shown by Professor Loomis, there is apparently a pretty regular succession of such ridges, with low barometer and a veering of the winds between them due to such a succession of cyclones. As the maximum of the annulus of high barometer entirely surrounds the cyclone at some distance from its limit of unusual violence of the wind at the earth's surface, these apparent longitudinal ridges would seem to extend over more degrees of latitude than the cyclones themselves, as stated by Captain Toynbee.

61. It is important to every navigator to know that a cyclone is always surrounded by an annulus of unusually high barometer, and that he cannot get within the limit of its dangerous part without passing through this annulus; for the observation of this on any part of the ocean by means of the barometer may on this account be often a first indication of an approaching cyclone while yet the weather is fair and calms are prevailing, just as in the case of the stationary observer at Havana, who, as he comes under the advancing annulus of unusually high barometer, knows that a cyclone is coming in the distance. But in order to know when the barometer is abnormally high on any part of the ocean, he must know what is the normal height of his barometer at the place where he is and at the time of the year; for from a reference to the charts of barometric pressure in Part I of these Researches, it is seen that this is very different in different parts of the globe, and at the same place at different seasons of the year. Those charts, however, were not intended for that purpose, for they do not have the necessary accuracy over the ocean generally for the want of sufficient observations, and the time has not arrived yet for constructing such charts, but observations are now being collected from almost all parts of the ocean, and the time is perhaps not far distant when such charts can be constructed with the requisite accuracy.

Such a chart might be given for each month from which the navigator could at once get the normal height of the barometer for the time and place; or all the necessary data for obtaining this by a very simple calculation might be given in two charts of the form of Charts I and III, Part I, for the northern hemisphere. It has been shown in Part I that the barometric pressure P may be expressed with sufficient accuracy for practical purposes by the first two terms of (26), which are

$$P = B_0 + B_1 \cos(\varphi - E_1)$$

in which B_0 is the mean barometric pressure and B_1 the amplitude of the annual inequality. A value for E_1 could perhaps be obtained which would be sufficiently accurate for all parts of the ocean. The value of φ could be given in a table, say for each tenth day of the year. With the value of B_0 taken from a chart of the form of Chart I, referred to above, and the value of B_1 from one of the form of Chart III, the value of P would be readily obtained. Such charts for practical purposes of this sort should be given for pressures not reduced to gravity of the parallel of 45°.

The amplitude of the next and neglected term in the expression of P above is shown from observation to be generally considerably less than one millimeter, which will perhaps be only of about the order of the probable errors of the values of B_0 and B_1, which can ever be obtained from observation and given in charts.

62. With regard to the general enlargement of cyclones, in accordance with § 21, this has been fully established by Redfield in the case of the well-developed cyclones originating in the torrid zone and curving around by Florida, and progressing then toward the N. and NE. up into high latitudes. In those in which the progressive motion is very nearly from west to east in the middle and higher latitudes this is not so much the case, for there is a limit beyond which the power of the cyclone cannot make itself felt on account of friction, and this is often reached before they come within the sphere of observation. Also, there is generally in these latitudes a succession of cyclones, as we have seen, which interferes with their continued expansion into very great dimensions.

63. We now come to the consideration of the formulæ in § 14, showing the relation between the barometric gradients and the corresponding velocities of the wind, and the comparison of the results given by these formulæ with observation. It should be borne in mind here that these formulæ are based on the cyclone theory, with certain assumed regular conditions, which would give motions entirely symmetrical on all sides; and hence when the conditions do not have this assumed regularity a nice agreement between theory and observation cannot be expected. Although the temperature disturbances giving rise to winds and barometric gradients may generally have some central point of maximum temperature with gradients of increasing temperature outward from this center, yet this is generally far from completely conforming to the regular gradients assumed in the theory, and this is especially the case in small temperature disturbances with corresponding small velocities of the wind. In well-developed cyclones, in which the temperature and barometric gradients are large and the motions on all sides nearly symmetrical, the regular conditions of the theory are more nearly approximated, and hence in these the formulæ should give relations most nearly in accordance with observation. In all cases, however, for reasons already given in § 38, the formulæ can be best tested by a comparison with observation when the averages of a great number of observations are used, since then most of the abnormal irregularities may be eliminated from the averages.

64. As an example of the application of (18) in a special case of large barometric gradient and velocity of the wind, and also as an imperfect test of the accuracy of the formula, we may take the great Scottish hurricane of the 24th of January, 1868.* From observations of the barometer at noon of Thursday at Aberdeen and Culloden, it was found that there was a difference of one inch in 138 miles, giving a gradient of 0.5 inches very nearly. As this is a case in which the gyration is near the center and very rapid, and also in a high latitude, we may suppose that the value of i is so small that we can put $\cos i = 1$ without sensible error. We may also put for January, $t = 0$, for a large error in the temperature produces only a very small effect. The value of r is not known, but we shall put it at 160 miles. Since the elevation above sea level is small we can put $P = P'$. Putting, therefore, in the formulæ $r = 160$, $\cos i = 1$, $t = 0$, $\psi = 90 - \text{lat.} = 32°$, and $P = P'$, from (22), which must be used in this case, since the unit of distance is one mile and of time one hour, we get $a = 71.24$, $b = 30473$. With these values of a and b, and the value of $G = 0.5$ inches, we get from (20), for the velocity of the wind, $s = 92.9$ miles. The anemometer at Culloden gave a velocity from 90 to 91 miles per hour. This example, however, should be regarded rather as an example of the application of the formula than as a test of its accuracy, for the gradient above, as

* Jour. of Scottish Met. Soc., April, 1868.

determined from the two stations, may not have been the exact gradient at Culloden where the velocity was observed, but rather at some point midway between, and the value of r has been so assumed as to make the result of the formula agree very nearly with observation. With a larger or smaller value of r the result would be different, and the test in the case consists in the reasonableness of the assumed value of r under all the circumstances. We can only infer that in an extreme case like this the formula gives a velocity which is most probably, at least approximately, correct.

From the data given by Padre Viñes, obtained from observations of the cyclone of October, 1876 (Apuntes, &c., p. 112), we get at the distance of 60 nautical miles from the center a gradient of 13.8^{mm} or 0.54 inches. With this gradient, putting $r = 69.5$ miles, $t = 25^\circ$, $i = 30^\circ$, $P = P'$, and latitude $= 22^\circ.5$, we get, as in the preceding example, from (18) and (22), for the velocity of the wind per hour, $s = 78$ miles. There may be considerable uncertainty with regard to the value of i to be used in this latitude and so near the center, and the result, when the value of i is small, depends very much upon this value. The velocities differ some, as we have seen, on different sides of the cyclone, but as the gradient was obtained from observations on both sides, the formula gives a mean of the velocities at the time of observation.

65. In Silliman's Journal for January, 1878, Professor Loomis has given a table of results depending upon the averages of great numbers of observations of velocities, directions, and gradients, obtained from the weather charts of the United States Signal Service for the years 1873 and 1874, from which the observed average velocities in the following table, corresponding to the barometric gradients in the first column, are readily deduced:

TABLE.

G.	r	i	Velocity.	
			Observed.	Computed.
Inches.	*Miles.*	°	*Miles.*	*Miles.*
.06	400	70	10.8	11.8
.07	380	55	19.0	21.6
.08	270	45	27.8	27.8
.09	225	37	31.8	32.1
.10	190	30	34.1	35.1
.11	170	25	36.0	37.4
.12	150	20	37.6	38.9
.13	135	16	39.5	40.2
.14	120	13	42.0	41.0
.15	110	10	46.0	41.7

One-half the observations used belonged to velocities less than 10 miles per hour, and gradients less than 0.6 inches, in the most of which, perhaps, the velocities and directions of the wind, for reasons already given, had only an accidental relation to the isobars, as laid down upon the synoptic charts. These are consequently omitted as being of little value for comparisons with theory. The average distance from the center, represented by r for all the observations, is said to be about 350 miles, and the average inclination, as we have seen, 47°. Hence, for the observations upon which the data in the above table depends, the average value of r and i must be both much less, since the rejected observations of low velocities and gradients must have been mostly in the outer part of the cyclone where r is greater, and likewise i, at least according to theory. The values of r and i in the preceding table are so assumed as to give a near agreement between the computed and observed velocities, and yet be such as are most probably their average values corresponding to the different gradients, judging from all the circumstances. Large velocities and gradients must be mostly near the center of the cyclones, and hence for these the average values of r must be less. And from what we have seen from theory, confirmed, in some measure at least, by observation, the inclinations should be greatest for the smallest gradients and greatest values of r. The average

latitude may be put at 40°, and hence $\psi = 50°$, and we may also suppose that $t = 15°$, and $P = P'$ without material error.

If with the observed values of the gradients G in this table, and the assumed values of r, i, ψ, t, and P, we compute the values of s by means of the formulæ (20) and (22), we get the velocities in the last column. But it should be observed here that the test of the formulæ does not consist in the near agreement between the observed and computed velocities, for with slight changes of r and i these could have been made to agree exactly, but it consists in the reasonableness and probability of the assumed values of r and i, judging from the average values of these for all the observations, and from what theory and observation would require with regard to the values of r and i for the different gradients. While there is considerable uncertainty with regard to these, the result must still be regarded as being confirmatory of the accuracy of the formula, at least for large gradients and velocities. For the smaller velocities and gradients, however, the inclinations required seem to be too large, unless the direction of the wind in the outer part of a cyclone, where the velocities and gradients are very small, are nearly radial. For a more accurate and satisfactory test of the formula, it is much to be desired that we had the average values of r and i corresponding to several classes of gradients from a great number of observations, and this both at sea and on land and in different latitudes.

66. That the inclination of the wind at the earth's surface in the outer part of the cyclone, where gradients and velocities are small should be very great, is entirely in accordance with theory. This is deduced in § 10 (c), from equation (11), upon the hypothesis that the friction at the earth's surface, expressed by F_s in (11), is the same at all distances from the center for the same velocity s; but we have seen in § 7 that the friction depends rather upon the differences of the relative velocities of the different strata, and from § 19 and Fig. 2 it is seen that these relative velocities in the outer part of the cyclone are such as to increase the friction where the velocity s remains the same; for here the cyclonic gyrations at and near the earth's surface have the anti-cyclonic above, so that the cyclonic gyrations are retarded by the friction of the earth's surface below and that of the anti-cyclonic gyrations above, and these latter, and consequently the differences of the relative velocities between the several strata upon which friction depends, are greater over the outer part of the cyclone than at any other distance from the center. We have here, more than anywhere else, the disk of the upper strata gyrating anti-cyclonically, and grinding, as it were, over the disk of the lower strata, gyrating in the contrary direction. Hence the radial component of velocity here has to be great, since the force depending upon the earth's rotation, in a direction at right angles to the direction of the radial component of velocity and in proportion to it, is the force which overcomes the frictional resistances to the gyration, and where the radial component of velocity is required to be great the inclination has to be great.

67. The observed velocities in the preceding table were obtained by means of a cup anemometer, in which, as originally assumed by Dr. Robinson, the velocity of the wind was supposed to be three times the velocity of the middle of the cups. But, according to the recent theoretical researches of Dr. M. Thiesen,* the relation between these velocities is far from being expressed by a single constant, and is not only different for different anemometers, but differs in the same instrument with different velocities of the wind and densities of the air. This is also shown, so far as different velocities are concerned, by the still more recent experimental researches of Dr. Robinson† and F. Dohrant,‡ according to both of which the constant 3 is much too great for great velocities, but is approximately correct for velocities of about 10 miles or less per hour. According to Dr. Robinson the extreme lowest value of this constant is only 2.286, and according to the results obtained by both, the observed velocities of the preceding table should be decreased about one-fifth part to give the true velocities approximately, which would have been given by a correct constant of reduction. With such a decrease of these velocities, it is seen that considerable changes must be made in the values either of r or of i, or of both, in the preceding table, in order

* Repertorium für Meteorologie, 1877. † Phil. Trans., 1878, Part II. ‡ Repert für Met., 1878, Heft. II.

to give an agreement between theory and observation. For the smaller gradients and velocities this agreement could be still had by an increase of 8° or 10° in the values of i, but for the larger gradients and velocities it would require a considerable decrease in the values of r, since when i is small the value of cos i in the formula, upon which the velocity depends, is not much affected unless by a considerable increase of the angle.

68. In order to determine the effect of latitude upon the gradient, Professor Loomis examined 250 cases of high velocities (Silliman's Journal, January, 1878, p. 20), dividing the observations into four groups according to latitude, and obtained the following average distances between the isobars drawn to tenths of an inch: For latitude 29°.3, 98 miles; latitude 37°.5, 66 miles; latitude 42°.7, 62 miles; and for latitude 47°.7, 50 miles. Hence the gradients, which are inversely as the distances between the isobars, increase with the latitude, as theory requires, and a little more rapidly than the sine of the latitude. But in connection with these results we do not have the average velocities or values of s for each class. If we assume that these were the same for each group of observations corresponding to the given averages of latitude, then theory, by (16), would require that the increase of gradient should be a little less than that of the sine of the latitude, or cos ψ, since, according to (11), cos i must increase with the latitude. This increase, however, is but little for high latitudes. In the uncertainties of the data of comparison, and especially in the assumption of equal averages of velocity for the several groups, a very nice agreement could not be expected between theory and observation upon this assumption. With regard, however, to the general law of a decrease of gradient for the same velocity with a decrease of latitude, many observations within the tropics show that the gradient is less here than in higher latitudes with the same velocity of the wind.

69. From (16) it is also seen that according to theory the gradient for the same velocity of the wind must increase with altitude as P : cos i, neglecting the small effect depending on a change of temperature with altitude. Professor Loomis observes that at an altitude of 6,000 feet the gradient is nearly one-third less than it is at the lower stations, but he does not state whether this is the case for the same velocities of the wind. If we suppose the value of P at the lower stations to be 750^{mm}, and the value of $i = 45°$, and at the height of 6,000 feet we put $P = 580^{mm}$, and cos $i = 1$, since the inclinations at great altitudes are small, we get $(580 \times .707) : 750 = .55$ for the ratio between the gradient at the height of 6,000 feet and that of the lower stations, supposing that s is the same for both, which is a greater decrease of gradient than that observed. With an assumed value of $i = 30°$ at the lower stations, the ratio given by theory would agree very nearly with that observed. Of course the observations at the high stations must have been too few to give the observed ratio with much accuracy; and besides it is uncertain whether the average of the velocities above and below were the same. It was probably much greater, and the gradient was increased from this circumstance.

70. It is also seen from (16) that the gradient G must decrease with the increase of temperature t, and hence it should be somewhat less in summer than in winter. With regard to the 250 cases of high velocities examined, Professor Loomis remarks that "the observations indicate that the gradient is less in summer than in winter; but the observations are too few in number to furnish a satisfactory value of the gradient in summer for these high velocities."

Mr. Ley has obtained some definite results with regard to this subject from the examination and discussion of a great number of observations, from which he concludes that "*the mean velocity of the wind corresponding to each gradient is much higher in summer than in winter*,"* consequently the gradients are much less in summer than in winter, other things being the same. The data employed were the daily telegraphic reports issued by the Meteorological Office (London), from August, 1870, to July, 1875. The isobars were in the first place drawn as accurately as could be done for each day, and the gradients instrumentally measured in the direction in which they were the steepest over the station. The stations selected were Stoneyhurst and Kew. There is a

* Suggestions on certain Variations, annual and diurnal, in the relation between the Barometric Gradient and the Velocity of the Winds.—Quarterly Jour. Met. Soc., October, 1876, p. 292.

remarkable conformity in the results for the two stations. The averages for the two stations are given in the following

TABLE.

Gradient per 60 nautical miles.	Winter.		Summer.	
	Number of instances.	Mean velocity.	Number of instances.	Mean velocity.
Inches.		*Miles.*		*Miles.*
.024	142	3.55	274	6.07
.036	245	4.90	402	8.01
.048	317	6.75	406	11.17
.060	305	10.75	274	14.60
.072	191	12.96	133	16.47
.084	170	14.15	101	18.47
.096	149	16.98	77	20.98
.108	93	21.12		
.120	66	24.73		

The results for winter include the observations from October to March, and those for summer the observations from April to September, inclusive. The results in the table show a very decided difference in the velocities for the same gradients in winter and summer, and one which is much too great to be accounted for in full by the formula of (16) with the average temperatures, or values of t, for winter and summer, and this is especially so in the case of the small gradients and velocities.

If we compare the velocities and gradients of the table in § 65 with those of the preceding table, it is seen that the velocities in the latter for the same gradients are much smaller than those in the former, which would require still greater values of i in the formulæ for small velocities, and smaller values of r for the larger ones, in order to make the theory agree with these observations. Mr. Ley has not given the average values of these quantities for the different classes of gradients and velocities, but larger assumed values of i in this case would not be consistent with the average value of i for all gradients and velocities obtained by him for all Europe, as given in § 40.

From all the comparisons which have been made of observations with the results of the formulæ giving the relation between the barometric gradient and the velocity of the wind, we have seen that large values of i, except in the case of large velocities near the center of gyration, are necessary in order to reconcile the formula with observation. In the case of no friction, and consequently completely circular gyrations, the value of i is 0, and the formula with this value gives much larger velocities. Hence the formula of Peslin, based upon the hypothesis of circular gyrations and no friction, gave a velocity much too great in the comparisons which he made with observations.

71. For the same reasons, as in the case of the annual changes of temperature, the formula would give greater velocities of the wind for the same gradient during the day than during the night. But the difference given by the formula accounts for only a very small part of the observed difference. According to a paper* by Dr. Hann, on "The Diurnal Period of the Velocity and Direction of the Wind," it appears that the velocity of the wind by observation during the warmest part of the day, at many places, is from one-third to one-half greater than it is during the coldest part, in the latter part of the night. This is especially the case at inland stations, but at sea there does not seem to be any sensible inequality of this sort, and at great elevations the inequality seems to be reversed, the least velocities occurring during the warmest part of the day.

These diurnal changes in the velocity of the wind cannot be due to corresponding changes in the barometric gradient, as obtained from synoptic charts, for these are known to be very small; but both the annual and diurnal inequalities in the velocities of the winds may be explained, with some degree of plausibility, by supposing that a considerable part of the winds is due to local disturbances of temperature and barometric gradients, producing gyrations and various irregular

* Die Sitzb. der k. Akad. der Wissensch., II. Abth., Jänner-Heft., Jahrg. 1879.

disturbances of the winds, and thus increasing the average observed velocities, while these local disturbances are too small in extent to affect the gradients as obtained from synoptic charts, in which the stations of observation are at long distances from one another.

The effect of numerous cyclonic gyrations upon the velocity of the wind due to the gradients obtained from synoptic charts, may be readily seen by considering a special case. If we suppose a particle of air to have a progressive and also a gyratory motion and that the velocities of both are the same, then the path of the particle of air will be that of a cycloid and hence much greater than that of the progressive motion, corresponding to the observed gradient. The wind in this case would of course blow with variable direction and velocity, as it generally does, but the velocity obtained from the averages of observations without regard to direction would be increased.

From the Table of § 28 it is seen that the atmosphere is frequently in a state of unstable equilibrium during the day and in the summer season, even when not saturated with aqueous vapor, and hence in a state to give rise to numerous cyclonic disturbances of small dimensions, and extending up to no very great altitude. Hence mostly during the summer, and the warmest part of the day, the observed velocities are increased, which give rise to the annual and diurnal inequalities of velocity, while the corresponding changes in the gradients are not observed, because the stations of observation are too far apart to detect them. On the ocean the annual and diurnal changes of temperature are very small, and, besides, the atmosphere, on account of the relative coldness of the ocean, is not often in a state of unstable equilibrium there. The annual and diurnal inequalities of the velocity of the wind, accordingly, is not observed here. If the observations of winds and barometric gradients on the ocean were discussed, as Mr. Ley did those of Stoneyhurst and Kew, they would probably give but little difference between the velocities of winter and summer, corresponding to the same gradients.

As the regular progressive motion of the upper strata of the atmosphere is generally greater than that at the earth's surface, the ascending air arising from the surface on account of small cyclonic disturbances, or from any kind of interchange between the lower and upper strata, has a tendency to retard the greater velocities of the upper currents, and this must occur mostly during the summer season and the warmest part of the day. Thus inequalities are produced in the velocities of the upper currents, the reverse of those below, since the causes which increase the velocities below must decrease them above. A satisfactory explanation of the matter has been given by Dr. Köppen. (Zeit. der Oester. Gesell. für Meteorologie, Band XIV, Seite 333.)

72. The cyclone theory may now be used in the explanation of the various inequalities of barometric pressure which are observed on the same latitude in different longitudes. We shall consider these first with regard to the mean pressures, independent of the seasons of the year, which is that which occurs approximately near the middle of April and October. In Part I of these Researches the effect upon barometric pressure of differences of mean annual temperature between the polar and equatorial regions, taken without regard to differences of temperature in longitude, have been determined from observation, and is shown in the column of B_0 in Table X, p. 37. From this it is seen that for a difference of about 40° (72° F.) of temperature in the northern hemisphere between the pole and the equator, the difference of barometric pressure is very small, being a little less at the pole than at the equator, and having its maximum about the parallel of 35°. In the southern hemisphere, however, this difference, for perhaps about the same difference of temperature, is much greater, for reasons already given in Part I. It is now proposed to consider the deviations of the mean annual pressure in longitude from the mean of all longitudes. This depends mostly on the corresponding deviations of the mean annual temperature in longitude from the mean of all longitudes, but in some measure also on the inequalities of the earth's surface, which interfere with the regularity of the general atmospheric currents and give rise to disturbances of atmospheric pressure.

In the first of the following tables are given these deviations of mean annual temperature for the northern hemisphere, which must be regarded as residuals, or rather disturbances, of temperature, the effects of which are yet to be considered. These, as likewise those for January and July, are only approximate, and are not to be regarded as being very accurate, since they have been obtained from isothermal charts, by interpolating to the latitudes and longitudes of the tables, as well as could be, from a mere inspection of the charts. The results thus obtained were, in many cases, then smoothed off somewhat by means of the differences, so that the numbers in the table

must be regarded as giving the deviations of the general distribution of temperature over the globe, neglecting all local and minor irregularities. This is what is needed for our purpose, rather than numbers giving all the small local irregularities, since it is not proposed here to consider the effects of the latter, but merely those of the more general disturbances of temperature over the globe.

Tables giving the differences between the temperature and the average for all longitudes in the northern hemisphere, for the mean of the year, of January and of July, in degrees of Fahrenheit.

TABLE I.—MEAN TEMPERATURE OF THE YEAR MINUS THE AVERAGE OF ALL LONGITUDES.

North latitude.	Longitude west from Greenwich.																	
	170°	160°	150°	140°	130°	120°	110°	100°	90°	80°	70°	60°	50°	40°	30°	20°	10°	0°
75°	− 6	− 5	− 4	− 3	− 3	− 5	− 7	− 8	− 9	− 9	− 5	0	4	5	7	8	11	14
70	− 6	− 5	− 3	− 1	− 2	− 5	− 7	− 9	−10	−10	− 6	− 1	5	7	9	11	15	18
65	− 4	− 2	0	3	3	− 2	− 6	−11	−14	−15	−12	− 4	3	6	10	14	16	17
60	1	3	4	6	5	0	− 5	−11	−15	−16	−15	− 6	0	5	10	16	16	16
55	5	5	5	7	6	3	− 1	− 8	−12	−15	−14	− 7	0	5	10	14	14	13
50	6	5	5	7	6	4	3	− 3	− 7	−13	−11	− 8	0	6	9	11	10	8
45	4	4	8	4	4	3	3	− 1	− 3	− 8	− 9	− 7	1	7	8	9	8	7
40	1	1	0	0	1	1	2	4	2	− 2	− 5	− 3	3	6	7	6	6	5
35	− 1	− 2	− 3	− 3	− 3	− 2	0	3	3	0	− 1	0	4	5	6	5	5	4
30	− 3	− 5	− 6	− 6	− 7	− 5	− 1	3	3	1	2	3	4	4	4	3	4	3
25	− 3	− 5	− 6	− 6	− 7	− 6	− 4	0	0	0	1	2	2	2	2	2	8	3
20	− 4	− 5	− 6	− 6	− 7	− 7	− 5	− 3	1	0	0	0	− 1	0	0	1	3	4
15	− 4	− 4	− 5	− 5	− 6	− 5	− 4	− 3	1	0	0	0	− 1	− 1	− 1	0	2	4
10	− 3	− 3	− 3	− 3	− 3	− 2	− 2	− 2	− 1	0	0	0	− 1	− 1	− 1	0	2	5
5	− 2	− 2	− 2	− 2	− 2	− 1	− 1	− 1	0	1	1	2	1	− 1	− 1	0	1	3
0	− 2	− 1	− 1	− 1	− 1	− 1	0	1	1	2	2	3	2	− 1	− 1	− 1	− 1	− 1

North latitude.	Longitude east from Greenwich.																	
	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°	130°	140°	150°	160°	170°	180°
75°	16	16	11	7	5	3	3	2	1	0	− 1	− 3	− 4	− 5	− 6	− 5	− 6	− 6
70	21	20	13	5	2	0	0	− 1	0	0	− 2	− 4	− 6	− 7	− 8	− 7	− 7	− 7
65	19	18	12	7	5	2	0	− 1	− 2	− 2	− 5	− 9	−10	− 9	− 9	− 6	− 4	− 5
60	16	14	11	8	6	3	1	− 1	− 3	− 4	− 8	−12	−12	−11	− 8	− 2	0	0
55	12	10	8	5	3	0	− 2	− 4	− 5	− 6	− 9	−11	−11	−10	− 6	− 1	2	4
50	7	4	3	1	− 1	− 3	− 4	− 6	− 6	− 7	− 8	− 8	− 8	− 6	− 3	− 1	4	7
45	6	5	4	2	0	− 2	− 3	− 4	− 4	− 4	− 6	− 8	− 8	− 6	− 4	− 2	1	3
40	5	5	4	3	0	− 2	− 2	0	0	0	− 3	− 7	− 8	− 6	− 5	− 3	− 2	− 3
35	4	4	3	3	2	1	0	0	0	− 1	− 3	− 7	− 7	− 5	− 4	− 3	− 3	− 3
30	2	2	2	3	3	3	3	2	0	− 2	− 4	− 7	− 6	− 4	− 3	− 3	− 3	− 2
25	3	3	3	3	2	2	2	2	1	0	− 1	− 4	− 4	− 3	− 2	− 3	− 3	− 2
20	4	5	4	2	0	1	1	2	2	2	1	0	− 2	− 1	− 1	− 2	− 2	− 2
15	6	8	6	3	1	1	1	1	1	1	0	− 1	− 2	− 2	− 2	− 2	− 2	− 3
10	7	9	7	3	2	1	1	1	0	− 1	− 1	− 1	− 2	− 2	− 2	− 2	− 3	− 3
5	5	8	7	3	2	1	1	1	0	− 1	− 1	− 1	− 1	− 2	− 2	− 2	− 2	− 2
0	2	7	7	3	1	0	0	0	0	0	0	0	− 1	− 2	− 2	− 2	− 2	− 2

Tables giving the differences between the temperature, &c.—Continued.

TABLE II.—MEAN TEMPERATURE OF JANUARY MINUS THE AVERAGE OF ALL LONGITUDES.

North latitude.	Longitude west from Greenwich.																	
	170°	160°	150°	140°	130°	120°	110°	100°	90°	80°	70°	60°	50°	40°	30°	20°	10°	0°
75°	-8	-7	-8	-9	-11	-13	-14	-15	-15	-13	-7	0	7	13	16	18	24	27
70	-8	-6	-7	-8	-10	-12	-14	-16	-16	-15	-8	0	10	18	23	27	33	37
65	-1	2	2	0	-4	-10	-15	-20	-22	-20	-15	-1	12	19	25	31	35	36
60	8	10	10	9	3	-7	-15	-22	-26	-24	-18	-2	10	18	26	34	34	32
55	15	15	15	14	10	2	-7	-18	-22	-23	-18	-7	8	18	24	29	28	26
50	19	19	19	19	15	11	1	-11	-16	-21	-16	-11	5	17	21	23	21	17
45	15	16	16	16	14	9	2	-8	-12	-16	-12	-7	6	16	19	19	17	12
40	9	10	11	12	12	6	2	-4	-8	-10	-8	-2	8	14	16	14	12	8
35	7	7	7	6	8	3	1	-3	-6	-6	3	3	10	13	14	12	10	7
30	4	3	2	1	0	-1	-1	-1	-1	-1	3	7	11	11	11	9	7	5
25	2	1	0	-1	-2	-2	-3	-3	-1	0	2	5	8	8	8	7	5	4
20	0	-1	-2	-3	-4	-5	-5	-5	-1	0	1	3	3	3	3	3	3	3
15	-1	-2	-3	-3	-4	-4	-4	-4	-3	-1	0	0	0	0	1	2	3	5
10	-3	-3	-3	-3	-3	-3	-3	-3	-3	-2	-1	-1	-3	-3	-1	1	3	6
5	-2	-2	-2	-2	-2	-2	-2	-2	-2	-1	0	0	-2	-2	-1	0	1	3
0	-1	-1	-1	-1	-1	-1	-1	0	1	1	1	0	-1	-1	-1	-1	-1	-1

North latitude.	Longitude east from Greenwich.																	
	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°	130°	140°	150°	160°	170°	180°
75°	30	24	18	13	10	10	8	5	1	-3	-7	-12	-14	-16	-16	-15	-12	-9
70	40	33	20	10	5	5	3	0	-3	-7	-12	-17	-21	-23	-23	-21	-15	-10
65	34	29	21	9	5	3	-1	-6	-9	-13	-19	-25	-27	-26	-21	-14	-3	-2
60	28	22	14	8	4	-2	-6	-10	-14	-18	-26	-32	-32	-26	-18	-2	10	6
55	22	16	9	3	-2	-9	-13	-16	-18	-20	-22	-27	-26	-20	-12	0	12	15
50	13	7	3	-3	-9	-15	-17	-21	-21	-21	-21	-21	-17	-11	-3	3	13	19
45	9	5	2	-1	-7	-12	-15	-17	-19	-17	-19	-20	-17	-8	-3	2	10	16
40	6	4	2	0	-5	-10	-10	-10	-10	-10	-14	-20	-18	-10	-4	0	4	10
35	5	3	1	0	-4	-8	-7	-8	-9	-11	-12	-17	-16	-8	-3	0	1	8
30	3	1	0	-1	-2	-3	-4	-5	-8	-11	-13	-15	-11	-5	-2	-1	2	5
25	3	2	0	-1	-4	-8	-3	-2	-3	-5	-7	-8	-8	-3	-1	0	1	3
20	3	3	1	-2	-5	-3	-1	1	1	1	0	-1	-1	1	1	0	0	1
15	6	6	5	-1	-4	-2	1	2	2	1	-1	-2	-2	-1	-1	-1	-2	-1
10	10	10	7	1	1	1	3	3	1	-1	-3	-3	-3	-3	-3	-3	-3	-3
5	7	10	9	4	3	2	2	2	0	-1	-2	-2	-2	-2	-2	-2	-2	-2
0	3	9	9	5	3	1	1	0	-1	-1	-1	-1	-1	-1	-1	-1	-1	-1

Table giving the differences as between the temperature, &c.—Continued.

TABLE III.—MEAN TEMPERATURE OF JULY MINUS THE AVERAGE OF ALL LONGITUDES.

North latitude.	Longitude west from Greenwich.																	
	170°	160°	150°	140°	130°	120°	110°	100°	90°	80°	70°	60°	50°	40°	30°	20°	10°	0°
75°	— 3	— 3	— 1	2	2	0	— 1	— 2	— 3	— 3	— 2	— 1	— 2	— 4	— 5	— 5	— 4	— 1
70	— 4	— 4	2	6	6	2	0	— 2	— 4	— 6	— 4	— 2	— 2	— 6	— 6	— 6	— 4	— 1
65	— 6	— 5	0	4	6	4	1	— 1	— 4	— 7	—10	— 9	— 8	— 7	— 6	— 5	— 3	— 1
60	— 7	— 5	— 3	1	5	5	3	— 1	— 5	— 9	—12	—11	—10	— 9	— 7	— 3	— 2	— 1
55	— 7	— 7	— 7	— 3	1	4	4	2	— 2	— 7	—10	— 9	— 8	— 7	— 5	— 2	— 2	— 1
50	— 7	— 9	— 9	— 5	— 8	3	5	4	2	— 5	— 5	— 5	— 5	— 5	— 3	— 1	— 1	— 1
45	— 7	— 9	—10	—10	— 8	— 2	2	8	7	— 1	— 3	— 4	— 4	— 4	— 3	— 2	— 1	0
40	— 7	— 9	—11	—13	—11	— 5	1	11	11	3	— 1	— 3	— 3	— 3	— 3	— 3	— 1	1
35	—10	—13	—13	—13	—12	— 8	0	10	10	4	1	— 1	— 2	— 3	— 3	— 2	0	
30	—11	—14	—14	—14	—12	—10	0	8	8	4	2	0	— 2	— 3	— 2	— 2	1	2
25	—10	—12	—12	—12	—11	— 9	— 4	2	5	3	0	— 1	— 2	— 3	— 2	— 1	2	4
20	— 8	— 8	— 8	— 8	— 8	— 8	— 8	— 3	0	1	— 2	— 2	— 3	— 4	— 2	0	3	6
15	— 6	— 6	— 6	— 6	— 6	— 5	— 5	— 2	0	1	— 1	— 1	— 2	— 3	— 2	0	2	5
10	— 3	— 3	— 3	— 3	— 2	— 1	— 1	— 1	0	1	1	0	— 1	— 2	— 2	— 1	1	5
5	— 2	— 2	— 2	— 2	— 1	— 1	0	0	1	2	2	3	1	— 1	— 1	— 1	0	2
0	— 2	— 1	— 1	— 1	— 1	— 1	0	1	2	3	3	5	3	— 1	— 1	— 1	— 1	— 1

North latitude.	Longitude east from Greenwich.																	
	10°	20°	30°	40°	50°	60°	70°	80°	90°	100°	110°	120°	130°	140°	150°	160°	170°	180°
75°	1	3	2	0	— 3	— 4	— 3	— 2	0	3	4	5	5	6	5	4	1	— 3
70	2	5	4	0	— 4	— 6	— 4	— 2	2	6	6	8	8	8	7	6	2	— 4
65	3	6	5	4	2	0	2	3	6	7	8	9	8	6	5	3	— 2	— 7
60	3	5	6	7	7	7	7	8	8	8	8	8	7	3	1	— 3	— 5	— 9
55	1	3	5	6	8	8	8	9	9	8	8	7	5	1	— 1	— 4	— 5	— 7
50	0	1	3	5	7	9	9	9	9	7	6	5	3	— 1	— 2	— 5	— 5	— 5
45	1	3	4	5	6	7	8	9	9	8	7	5	2	— 1	— 4	— 6	— 6	— 6
40	3	5	5	5	5	5	7	9	9	9	7	5	1	— 2	— 5	— 7	— 8	— 7
35	3	4	5	7	8	8	8	9	9	8	6	4	0	— 2	— 4	— 6	— 8	— 8
30	2	3	6	8	10	10	10	9	8	7	4	2	0	— 2	— 4	— 6	— 8	— 8
25	5	6	7	8	8	8	8	7	6	5	3	2	0	— 2	— 3	— 5	— 6	— 6
20	7	8	8	7	6	6	5	4	3	3	2	1	— 1	— 2	— 3	— 4	— 4	— 4
15	8	8	8	7	5	4	3	2	1	1	1	0	— 1	— 1	— 2	— 3	— 3	— 3
10	7	7	7	5	3	1	— 1	— 1	— 1	0	0	0	— 1	— 1	— 1	— 1	— 2	— 3
5	4	6	6	4	2	0	— 1	— 1	— 1	0	1	0	— 1	— 2	— 2	— 2	— 3	— 3
0	— 1	5	5	1	— 1	— 1	— 1	— 1	0	1	2	1	— 1	— 3	— 3	— 3	— 3	— 3

73. If we examine the numbers in Table 1 corresponding to the northern part of the Atlantic ocean, and especially if we write these numbers in their proper places on a polar projection of the northern hemisphere, it is seen that there is a considerable area, somewhat circular, with the center a little north and east of Iceland and west of Norway, in which there are large residual temperatures, above the average of all longitudes, amounting at the maximum to about 20° F., which have not been taken into account in Part I, in the general motions and pressure of the atmosphere, and the effects of which we are now to consider. Here, then, we have the approximate conditions of a cyclone with a gradient of decreasing temperature from the center outward, represented approximately by (25). At a considerable altitude, however, the temperature gradient is probably less than at the earth's surface, since the temperature disturbances of the atmosphere are mostly greater at the surface than at considerable altitudes where the temperature remains more nearly the same over all parts of the globe, and at all seasons of the year. The constant C, therefore, in (25) is probably less for the upper strata than the lower ones.

These conditions must give rise to a fixed cyclone, with its center near Iceland, and consequently to an area of low mean annual barometer in the northern part of the Atlantic, and this effect must be superimposed on that belonging to the general motions and pressure of the atmos-

phere treated of in Part I, and consequently we have here an area in which the barometer is lower than the mean of the latitude taken all around the globe. That there is such a depression is shown by Chart I, Part I, on which the isobars are laid down, as well as could be, from observations, and although there is some uncertainty in these lines, from the want of sufficient observations on the ocean, yet observation shows conclusively that there is an area of low barometer such as theory requires. As the distribution of temperature over the hemisphere gives the conditions of a cyclone with a gradient of increasing temperature from the pole to the equator, this is a case of a cyclone within a cyclone; and consequently, for reasons given in § 32, the combined effects of these conditions, treated separately, do not give precisely the true effects of both conditions combined, yet the effects lost are only of a second order, and are of no consequence where only a general explanation of the observed variations of pressure over the globe are aimed at, and not a comparison of exact quantitative results.

74. By the cyclone theory every cyclone must have an annulus of high barometer and its anticyclone surrounding it. As this is a fixed and permanent cyclone, the anti-cyclone may be very broad and the gradient very small, so that the highest part of the annulus may be very little higher than the normal height, and this annulus falls on so many other inequalities that it cannot be recognized on the chart, but the effect of this annulus is seen in the Atlantic, on the parallel of about 35°, where it falls somewhat on, but probably considerably south of, the annulus of high barometer belonging to the hemispherical cyclone, which, as we have seen, is on this latitude, and the effect of the two combined gives an area of barometric pressure a little higher than that of the average of the latitude taken all around the globe.

Since this is a case of a cyclone within a cyclone the chart representing the combined effects of the two should be similar to Fig. 4, in which the annulus of high barometer of the included cyclone falls somewhat on that of the larger one, giving a somewhat elliptical area of high barometer, with winds blowing in some measure all around it, but with much greater velocity on the north and south sides than on the east and west sides. If it were not for the cyclonic system with its center near Iceland, the calm-belt of the parallel of 35° would extend from Africa and the Mediterranean entirely across the Atlantic and the southern part of the United States; but under existing circumstances the anti-cyclone of this disturbing system, with its winds gyrating from left to right, cuts this calm-belt, on the east side of the Atlantic and west coast of Africa, giving rise to gentle northerly winds, and on the west side of the Atlantic and the southern part of the United States, giving rise here to a predominance of southerly winds. It is to this anti-cyclone alone that the gradients on the east and west side of the area of high barometer in the Atlantic, on the parallel of 35°, is due, as well as the corresponding winds of these gradients, while on the north and south sides the gradients and corresponding winds of the two systems combine. Between the center near Iceland and the parallel of 35°, the gradients for this reason become very steep, and hence the west and southwest winds of the Atlantic in the middle latitudes are unusually strong. In the trade-wind region of the Atlantic the anti-cyclones of the two systems combine, and hence the trade-winds here receive additional force from the anti-cyclone of the system, with its center near Iceland. It is from the effect of this latter that the trade-winds on the east side of the Atlantic blow nearly from the north, while midway they are from the northeast, in the Antilles from the east, and in the southern part of the United States from the southeast and south. The effect of the permanent cyclone with its center near Iceland is to cause the prevailing direction of the winds to be more from the southwest and south in the British Isles and Norway, and in Greenland and Baffin's Bay from the northeast. The isobars of Chart I, Part I, for the region of the North Atlantic, laid down from observation are, as nearly as we can judge, exactly in accordance with the preceding deductions from theory. The prevailing directions and forces of the winds of this region, taken for the whole year, are so well known that it is not necessary to refer here, even if space would permit, to the numerous authorities which might be cited to prove that the existing system of winds is in strict accordance with theory. (See Winds of the Globe, Plate 3, by the late Prof. J. H. Coffin, LL. D.)

With regard to the effect of the cyclone of the northern part of the Atlantic on the upper currents of the atmosphere, it should be borne in mind that the cyclonic gyrations of the lower strata, in a system with a warm center, decrease with altitude and become more and more anti-cyclonic, so

that although the surface-winds of the middle latitudes are much increased by the cyclonic gyrations, it is not so much the case at some height above the surface, and in the upper regions of the atmosphere the eastward currents may be much diminished by the anti-cyclonic gyrations prevailing there, especially near the outer limits of the cyclonic gyrations below. It is important to take this into consideration in observing and studying the upper currents of the North Atlantic.

75. There is another effect which should be taken into account here, which coincides in some measure with that arising from the local inequalities of temperature. We have seen in Part I that the general motion of the atmosphere in the middle latitudes is toward the east, and between the parallel of 30° or 35° and the equator it is toward the west. The middle of the current and the maximum velocity eastward are above the parallel of 50°. Where this current encounters the greater friction of the continent, and especially high lands and mountain ranges, it is deflected in part to the right and left, the part to the right passing over the eastern side of the Atlantic and west coast of Africa and joining the general westward current in the lower latitudes, and the part deflected toward the left passing up toward the pole around by Greenland, and joining again the general current eastward in the middle latitudes. The westward current within the tropics is likewise deflected in part by the American continent, and especially the high mountain ranges of Mexico, to the right over the southern part of the United States, and also joins the general eastward current of the middle latitudes. Hence there are produced two great gyrations of the atmosphere from this cause independent of any local disturbances of temperature, the one around some point near Iceland as a center, and the other around some point about the middle of the Atlantic near the parallel of 35°. Since where there is motion in any direction there is a deflecting force arising from the earth's rotation to the right of this direction in the northern hemisphere, the gyration around the center near Iceland must cause a barometric depression in that region, while the gyration around the other center on or near the parallel of 35°, since it is in the contrary direction, must cause an area of high barometer with its center near the center of gyrations.

The effects of these gyrations arising from the deflections of the continents, both on the barometric pressure and on the motions of the atmosphere, coincide so nearly with those arising from the local disturbances of temperature that we cannot separate them and say how much belongs to each cause. The center of the area of resultant low barometer from the two causes, as shown by the chart, seems to be a little further west than it should be if it depended on the temperature disturbance alone, since the maximum of the latter is further east than the center of the area of low barometer. The center of the depression, therefore, arising from the deflections of the continents, is perhaps a little west of Iceland, and therefore the center of the resultant depression arising from both causes is a little east of Iceland, but not so far as it would be if it depended on the cyclone arising from the temperature disturbance alone.

76. From the first of the preceding tables it is likewise seen that there is a temperature disturbance in the northern part of the Pacific Ocean, though less than in the Atlantic, which must give rise to a fixed cyclone and an area of lower mean annual barometric pressure than the normal of the latitude. From Chart I, Part I, it is seen that observation gives such an area of barometric depression. The depression, however, in this case may be due in great part, if not mostly, to the gyrations produced by the deflections of the continents, as explained in the case of the Atlantic under similar circumstances. These deflections are without doubt much greater here than in the Atlantic on account of the high range of the mountains so near the Pacific coast. In this ocean we likewise have a higher mean annual barometric pressure than the average of the latitude extending nearly across the ocean, and a system of winds over the whole ocean, similar to that of the Atlantic, all of which are explained in the same manner as those of the Atlantic.

77. In the interior, or rather on the east side, of Asia, it is seen, from the table of residual mean annual temperatures already referred to, that there is an area of low temperature relatively to the averages of the latitudes. This area is very irregular and furnishes only very imperfectly the conditions of a fixed cyclone with a cold center; besides the area is too large to give these conditions accurately, for reasons given in § 1. If we had the complete conditions of a cyclone, the cyclonic gyrations around the center of this area according to the theory of this kind of cyclones, would probably cause a diminution of the barometric pressure at the central cold area, as in the

cases of the great hemispherical cyclones, in which, especially that of the southern hemisphere, there is a depression of the barometer over the central cold area; but on account of the imperfection of the conditions in this case the greater mean annual pressure due to the less mean annual temperature is but little affected from this cause. It is seen from Chart I, Part I, that the center of the area of high mean annual barometric pressure is much further west than that of the center of the relatively cold area, and it would therefore seem that it must be due, at least in part, to some other cause. It is no doubt caused in part by the overlapping of the areas of high barometer surrounding each of the permanent cyclones of the northern parts of the two great oceans.

In the northern part of North America we have somewhat similar conditions, but the area of relative cold and the amount of temperature disturbance are much less, and consequently the mean annual pressure is less than in Asia, and the area of high barometer relatively to the averages of the latitude is of less extent.

In the northern part of Africa we have a somewhat circular area of relatively high temperature, but the center being very near the equator, it cannot give rise to any cyclonic effect, and the corresponding low barometer of this area, shown by the chart, is due entirely to the greater rarefaction of the air.

78. We now come to the consideration of the local disturbances of barometric pressure in the northern hemisphere and the corresponding disturbances of the general motions of the atmosphere, for January, due to the residuals of temperature given in the second of the preceding tables. It is seen from the tables that the distribution of these residuals for January is somewhat similar to those of the mean temperatures, but that they are in general more than twice as large. The effects of these residuals, then, on the pressure of the atmosphere and the general motions of the atmosphere for January, as treated of and given in the first part of these Researches, must also be similar, but in general about twice as great. Accordingly we have at this season of the year the two great cyclones of the northern parts of the Atlantic and Pacific Oceans very similar to those belonging to the season of mean annual temperatures and pressures, but with a violence and barometric pressure about twice as great. The mean annual pressure at Iceland is about 754^{mm}, while the average of the latitude B_0, Table X, Part I, is 758.2^{mm}, and hence a depression, due to the residual temperatures of Table I, of more than 4^{mm}. In January the pressure at Iceland is only about 750^{mm}, while the average of the latitude for this season, by the table just referred to, is $B_0 + B_1 = 758.2 + 0.6 = 758.8$, and hence the amount of depression due to the residuals of temperature, not considered in the averages for the latitude, is nearly 9^{mm} at this season, and a little more than twice as great as in the case of the mean annual temperatures and pressures. In the Pacific Ocean the barometric depression is not quite so great, but it is about twice as much in January as for the mean of the year.

A part of these barometric depressions, as in the case of the mean annual depressions, is likewise due to the gyrations arising from the deflections of the general motions of the atmosphere by the continents, and perhaps in case of the Pacific the greater part; and since these general motions are much increased in January above the average or mean of the year, the part of these depressions due to this cause should be much increased also in January. On account of these great depressions in January in the northern parts of these oceans, the gradients of the middle latitudes of these oceans, especially of the Atlantic, become very steep, since they combine with the gradient of these latitudes belonging to the general pressure of the atmosphere or that of the great hemispherical cyclone with its center at the pole, and hence the west and southwest winds of these latitudes in January are unusually strong.

As seen from Table X, Part I, the average pressure of the latitude for January is $B_0 + B_1$, in which B_1 has a positive value, except in the extreme north and south latitudes of the hemisphere, with a maximum of 1.66^{mm}. Hence, where the depression caused by these cyclones is equal to B_1 there is no annual inequality though there may be a small semi-annual one. This line of no annual inequality is laid down from observation on Chart III, Part I, designated by 0. The effect of these strong cyclones in January, in depressing the barometer, extends beyond these lines surrounding them, and consequently nearly to the equator. The annuli of higher barometer, therefore, surrounding these cyclones, and superimposed on the various irregularities from other causes, is thrown out to a great distance over the continents, and the area becomes so great, and the cyclonic conditions on

this account so imperfect, that this annulus is very imperfectly formed in this case, especially near the equator, where the forces on which it depends vanish.

79. It is seen from the second of the preceding tables that in January the residuals of temperature over a very large area, with its center on the eastern side of Asia, and over a much smaller area in the northern part of North America, are negative, and hence these areas at this season are relatively cold ones. These give very imperfectly the conditions of a cyclone with a cold interior. If these conditions were perfect, and especially if they were on sea instead of land, the cyclonic gyrations around the center might so depress the barometer in the interior as to nearly or quite counteract the effect on the barometer due to the increased density from cold, as we see in the two great hemispherical cyclones in which the effect of the gyrations is greater than that of the greater density of the air, especially in the southern one, where the barometer in the polar regions is very much lower than at the equator. There is no doubt some cyclonic effect in these regions and some depression of the barometer from this cause, imperfect as the conditions are, but the effect of the greater density from cold is much greater, and hence in these regions, especially in Asia, the barometer stands unusually high in January.

80. From the last of the preceding tables it is seen that the temperature residuals for July are generally small. In the northern parts of the Atlantic and Pacific Oceans, where in January we had the conditions of cyclones, the effects of which caused great barometric depressions, we have now very nearly the average temperatures of the latitude, and hence no conditions giving rise to cyclones and barometric depressions over these regions. Hence the great barometric depressions which prevailed here in January have now very nearly vanished, and the barometric gradients disappeared, as is seen from an inspection of Chart VI, Part I, and the strong west winds which prevail in the middle latitudes on the Atlantic and Pacific Oceans in January are now of very little force, as is indicated by the width between the isobars and consequent smallness of the gradients. The barometric pressure at Iceland, where in January it was very much depressed, is now exactly equal to the average of the latitude, which is $B_0 - B_1$ of Table X, Part I, and hence there is no cyclonic action and depression from this cause, and the gradients of the middle latitudes of the Atlantic are simply those of the average of all latitudes belonging to the great polar cyclone existing at this season, with a small barometric depression over the polar regions. The effect of the great cyclone in the winter season, with its center near Iceland, no doubt extends to the equator and causes the position of the calm-belt there on the Atlantic to be a little further south than it otherwise would be. This effect, added to the general oscillation of this belt, observed all around the globe and theoretically explained in § 23, causes an unusual amount of oscillation of this belt on the Atlantic with the changes of the seasons.

Over Europe, Asia, and North America, now, the temperature is much greater than on the oceans, and considerably above the average of the latitudes, and hence the temperature residuals are positive, and cause a depression of the barometer below the average pressures of the latitudes given by $B_0 - B_1$ by the greater rarefaction of the air mostly, since these residuals are not so distributed as to furnish conditions which can give rise to much cyclonic action.

Over the region of Hudson's and Baffin's Bay, and Greenland, and in the middle of the Pacific Ocean, on latitude 30°, the residuals are somewhat large and negative, indicating that the temperatures of these regions in July are colder than the average of the latitudes, and hence the barometric pressures here should be greater than the averages of the latitude, given by $B_0 - B_1$, and observation shows that this is the case, as may be seen by comparing Chart VI, Part I, with the value of $B_0 - B_1$ given by the table before referred to. These areas of negative residuals are somewhat circular, and hence furnish roughly the conditions of a cyclone with a cold center, and there is probably some cyclonic action and depression of the barometer from this cause, since the barometer over these regions does not differ much from the average of the latitude, while the temperatures differ considerably.

81. We have now seen how little the differences of barometric pressure over the globe in the same season of the year, and at the same place in different seasons of the year, depend directly on mere differences and changes of temperature, and how entirely out of proportion it is to these differences and changes. At the polar regions, especially those of the southern hemisphere, where there is the greatest cold, the barometric pressure is the least, and over the northern hemisphere generally the great changes of temperature from January to July, taking

the averages for the several latitudes, produce at the maximum, on the parallel of about 35°, a change of only about 3mm in the barometric pressure; and near the pole of the northern hemisphere, where this change from winter to summer is still greater, and on certain lines surrounding the northern parts of each ocean of this hemisphere, as laid down on Chart III, Part I, the annual change of barometric pressure entirely vanishes. This disproportion between temperature and barometric pressure is because the barometric gradients in perfect cyclones depend almost entirely on the cyclonic gyrations of the atmosphere, and not upon the temperature gradients, as has been shown by theory in § 19, and the amount of barometric depression in the interior of a cyclonic system with a cold center should be as great as in the case of one with a warm center, except so far as the slight imperfections of the theory with regard to effects of friction are concerned. In the two great hemispherical cyclones with the cold poles of the earth at the center, considering only the average temperature of the latitude, we have a near approximation to a perfect cyclonic system, except so far as inequalities of surface are concerned, and hence there is a diminution of pressure at the poles in proportion to the amount of cyclonic motion produced, and that this depends very much on the regularity and smoothness of the surface is seen from the differences of the gyrations and consequent pressures in the polar regions of the two hemispheres.

When we come to consider the effects of the residuals of temperature, not taken into account in the consideration of the two great hemispherical cyclones, they mostly furnish very imperfectly, if at all, the conditions of a perfect and regular cyclone, so that the cyclone theory cannot be in general applied to them with any degree of accuracy, and therefore the colder areas, with reference to the average for the latitudes, give an increased barometric pressure, since the effect of cyclonic action is not sufficient to decrease the pressure in the central part as much as it is increased by diminution of temperature.

82. In the southern hemisphere there is in general very little variation of temperature in longitude, and hence the residuals of temperature, positive and negative, such as given in the preceding tables for the northern hemisphere, are generally small. The great hemispherical cyclonic system, therefore, with its low barometer in the polar regions and its annulus of high barometer around the globe, having its maximum about the parallel of 30°, is very little disturbed by local irregularities of temperature, the belt of high barometer being only broken up a little, mostly in the summer season, by the greater mean temperature of South America, Africa, and Australia. As in the northern hemisphere, the gyrations arising from the deflection of the general currents of the atmosphere by the continents no doubt produce considerable effect, especially those arising from the deflections of the high range of the Andes in South America, since these general currents in this hemisphere are very strong. We have thus in the South Atlantic, as in the North Atlantic, an area of increased barometric pressure from this cause with its center near the parallel of 30°, and likewise west of the continent of South America on the same latitudes. There is likewise a deflection of the westward currents of the South Indian Ocean by the east coast of Africa down by Madagascar and Mauritius Islands, which passes on down into the general eastward current of the middle latitudes, but a branch of it passes around west of Australia to join again the westward current, thus causing a slight gyration around an area with its center midway between Australia and Africa, and on the parallel of about 30°, and consequently an area with a barometer a little higher than the average of the latitude. Hence on the southern hemisphere, as in the northern, there are currents of atmosphere on the eastern sides of the continents, passing from the lower to the higher latitudes through the belt of generally higher barometer with its maximum about the parallel of 30°.

In this hemisphere the general change of temperature from winter to summer is small, but it is sufficient to produce a proportionate effect upon the strength of the great cyclone system, increasing it in winter and decreasing it in summer, and this in winter increases the barometric gradients and the height of the belt of high barometer, but depresses the barometer in the polar regions, and in summer the reverse. But in winter the amount of atmosphere in either hemisphere is greater, since the opposite hemisphere is then warmer, and the air expands and a portion flows over into the colder hemisphere, so that the general or average height of the barometer is increased from this cause in the winter and diminished in summer. The greater pressure in winter of either hemisphere, arising from this cause, is increased in the latitudes of the belt of high barometer by

the greater activity of the general hemispherical cyclone, but decreased in the polar regions, where the effect of the former cause may not only be destroyed but reversed. Hence the barometer in either hemisphere is in general higher in winter than in summer, not considering inequalities in longitude; but this, according to theory, may be reversed in the polar regions. This is shown by observation to be the case, as is seen from Table X, Part I, in which it is seen that the value of B_1, the coefficient of annual inequality, is reversed at the parallel of about 78° in the northern hemisphere, and on the parallel of 55° in the southern hemisphere.

83. According to § 30 the interior of an ordinary progressive cyclone is generally a region of cloud and rain, especially if the air is moist and the cyclone has continued in action until the moist air of the surface arises by means of the interior gently ascending currents until it arrives at a height at which the aqueous vapor is condensed. That the interior of a cyclone is in general an area of cloud and rain is too well known to ordinary observation to need any proofs from regular series of observations. In a perfect cyclone, in an atmosphere without progressive motion, the area of cloud and rain would be circular, with its center at the center of the cyclone; but when the atmosphere has a progressive motion in any direction, by § 36, this area should be somewhat elliptical with the longer axis nearly in the direction of the progressive motion of the atmosphere, and also nearly in the direction of the progressive motion of the cyclone, since the latter depends very much upon the former. Loomis has shown from the Signal Service observations that the figure of the whole rain area is generally of an oval form. He also ascertained the longer and shorter diameters of a number of rain areas in which the depth of rain was at least one inch during the preceding eight hours, and of others in which the depth was at least a half inch during that time. (Silliman's Journal, July, 1877, p. 3.) With regard to these he remarks: "The form of these rain areas is sometimes quite irregular, but generally it approximates to an ellipse, of which the major axis is not quite double the minor axis."

Since the cloud from which the rain falls is being continually carried forward by the upper stronger currents of the atmosphere's progressive motion, in advance of the gyrations of the lower part of the atmosphere, the center of rain area should fall in advance of the center of the area of low barometer. Loomis found, for the average of 38 cases which he examined, that "the average distance of the center of greatest rainfall from the center of low pressure for the cases north of latitude 36° is 300 miles, but it sometimes exceeds 750 miles." This rainfall was for the preceding eight hours. From this should be deducted the progressive motion of the area of low barometer in four hours, say 100 miles, in order to have the simultaneous distance of rain center from the center of area of low pressure, which would leave it on the average about 200 miles.

The direction of the storm's path coincides very nearly with the direction of the axis of rain area. For the average direction of N. 44° E. of the storm's path, Loomis found, from three years' observations, the average direction of the axis of rain area N. 53° E., and for the average direction of N. 111° E. for the former, he found N. 115° E. for the latter. Hence the directions in all cases are very nearly the same, and no doubt that of the average direction of the progressive motion of the atmosphere, especially of the cloud region in each case.

According to theory, § 36, the forms of the isobars should also have an oval figure, with the longer axis somewhat in the direction of the axis of rain area and of the progressive motion of the cyclone or area of low barometer. Loomis found the forms of these isobars to vary much in form. In some cases they were nearly circular, but 7 per cent. of them were four times as long as wide, 20 per cent. three times, and 47 per cent. double. He remarks: "The average form of the isobars about the storm's center may be said to be an irregular oval, whose length is nearly double its breadth." (Silliman's Journal, July, 1875.)

The preceding results from observations, especially the distance at which the center of rain area falls in advance of the storm center, seem to indicate an enormous inclination of the axis of the storm forward at the top, which is entirely inconsistent with the hypothesis of its inclination backward. But, in accordance with § 37, there can be but little inclination of the real axis of the gyrations, and the center of cloud and rain is carried forward in advance of this.

84. In 101 cases of low barometer in which the amount of rain did not amount to one-eighth of an inch, Loomis found that more than one-half showed a pressure less than 29.70 inches; more

than one-third were below 29.60 inches, and nearly one-fourth of the cases were below 29.50 inches. He says: "There seems to be no room for doubt that barometric minima sometimes form with very little rain, and continue without any considerable rain for eight hours, and sometimes for twenty-four hours and longer. These barometric minima seldom continue stationary for eight hours, but almost invariably travel to the eastward." His final conclusion is that "*rainfall is not essential to the formation of areas of low barometer, and is not the principal cause of their formation or of their progressive motion.*" (Sill. Jour., July, 1877.) This is strictly in accordance with the theory of § 29 In the cases of areas of low barometer without rain, these low areas arise from cyclonic motions due to the temperature disturbances arising from the primary causes, and these, according to the theory, must always exist to a certain extent before the secondary cause arising from the condensation of aqueous vapor can take place. It does not seem, however, that these areas of low barometer without rain continue very long, or that the barometer ever becomes very low, and we could not infer that this would be the case from theoretical considerations. Progressive cyclones of long continuance, and traveling over long distances, must be accompanied by considerable rain, as we always find to be the case; for the temperature disturbance due to the primary causes which first originate the cyclone must soon be lost, and then the cyclone must depend for its support upon the caloric of condensation, and, as we have seen, even this fails when there is not a sufficiently great ratio of diminution of the temperature of the different strata with the altitude. Even when the air is saturated with vapor, if we do not have the latter condition with it, we do not have the condition of a cyclone where the primary causes of disturbance are wanting; we simply have the condition of a very damp and oppressive atmosphere from which no rain falls, because the cyclonic condition of (32) is not satisfied, and hence no upward currents to carry the almost saturated air where the vapor can be condensed.

Although cyclonic areas are generally areas of cloud and rain, yet the reverse of this is not true, for the areas of rarefied air arising from the primary causes, and giving rise to ascending currents and rain, may be so irregular in form as not to satisfy the conditions of a cyclone, except so imperfectly as not to give rise to much cyclonic action, and then we generally have long-continued moderate rains without much wind.

Sometimes there are several rain centers within the area of low barometer, and they are sometimes found in the areas of high barometer, and there are also centers in the area of low barometer where the rain is in greater abundance than elsewhere. This simply indicates that, in accordance with § 32, cyclones may exist within cyclonic systems, and so in the areas of high barometer, and wherever there are isolated spots of rain within the area of low or high barometer, or places where there is greater abundance than elsewhere in the area of low barometer, we have the evidence of the existence of a smaller cyclonic system within a larger one.

85. According to the theory of the progressive motions of cyclones in § 35, every cyclone originating near the equator should first be carried westward and toward the pole; then, after arriving in the middle and higher latitudes, should be carried eastward by the prevailing currents. To be assured that this is the case it is only necessary to glance at the charts which we now have of the numerous tracks of cyclones originating near the equator in all longitudes and in both hemispheres. But on account of the various modifying influences, stated in that section, there is a great diversity in the forms of these tracks and the velocities of progress at different times and places, so that when the mariner is advised of the approach of a cyclone in the distance by any of the usual indications, the best that he can do is to determine the probable cause and velocity by taking the average of all for the locality and the season of the year, which can readily be done from charts upon which the tracks and rates of progress are laid down. But for this purpose he should have charts for all parts of the globe of all the known tracks of cyclones given for the different seasons of the year, for they change considerably with the seasons.

There are certain parts of the globe and seasons of the year in which the conditions seem to be especially favorable for the originating of cyclones. Of the well-developed cyclones which originate near the equator, and usually progress in a somewhat parabolic orbit into high latitudes, a very large and disproportionate number seem to originate in the Atlantic, a little north of the equator, and almost exclusively in the latter part of summer and the first part of autumn. This admits of a very plausible explanation. According to the theory no cyclone can originate at or

very near the equator (§ 4). The region of the trade-winds of the Atlantic are also unfavorable for their origination, since the steady currents, mostly near the surface of the earth, carry the vapor which they contain into or near the belt of calm and cloud before it rises into regions where it is condensed and gives out its latent caloric, upon which the power of the cyclone very much depends. The conditions then favorable for the origination of a cyclone are in or near the calm and cloud belt, where and when this belt is at some distance from the equator. Now, for the theoretical reasons given in § 80, the calm-belt is farthest from the equator at the season, or very nearly, when these cyclones mostly originate, being then 10° or 15° from the equator, and when this belt recedes toward the equator the conditions become so unfavorable that no cyclones originate here.

In the China Sea and North Indian Ocean the equatorial calm-belt is very much deranged and broken up by the great monsoons arising from the wide range of annual fluctuations of temperature in the interior of Asia. According to Dove, (Gesetz der Stürme), the frequency of cyclones in the North Indian Ocean has two maxima, one in May and the other in October. These are the periods of the breaking up and changing of the monsoons, and accordingly then the conditions are more favorable for the origination of cyclones, according to § 83; for these monsoons are simply the exaggeration of the trades in the one case and their complete reversal in the other, by the great temperature disturbances in Asia. In the China Sea the cyclones take place mostly in September and October, when the current arising from the great indraft into Asia ceases; for some reason few occur at any other season of the year.

86. The calm-belts of high barometer around the globe with their maxima on the parallels of 30° or 35° on the average, do not furnish favorable conditions for either the origination or support of cyclones, since it is a belt of descending currents, or rather of a very gradual settling down and flowing out of the air on both sides, near the earth's surface, and hence it is a belt of very dry atmosphere. There is therefore in general very little cyclonic disturbance either in this belt or the belt of the trades, It is within this belt of high barometer and the trades that we have the almost rainless regions of Peru, Nicaragua, the Sahara or Great Desert of North Africa, and of Egypt, Arabia, Persia, &c. Not only very few cylones originate in this belt, but there seem to be only a few gaps where those originating nearer the equator can get through into the higher latitudes, notwithstanding the general tendency they have to move from the equatorial toward the polar regions. The principal reason for this is that this belt in general is too dry to support a cyclone while it is making its way through. They, therefore, pass through, mostly, if not entirely, on the east sides of the two great oceans, those of the North Atlantic in the region of the Antilles, Florida, &c., and those of the North Pacific Ocean in the region south and east of China and Japan. Here the northern tendency of the air on the east sides of the areas of high barometer in these oceans, as observed and explained by theory in § 74, not only aids the cyclones in passing through toward the polar regions, but it carries warmth and moisture with it for their support. It is only at these places that the warm and moist air of the equatorial regions passes into the middle and higher latitudes; for the general returning or anti trade currents of the upper regions are both cold and dry. The warmer water of the Gulf Stream in the Atlantic, and of the corresponding current of the Pacific, is supposed to have considerable influence upon these cyclones, and they no doubt, aid them some, and in some measure determine their routes and directions; but if we examine charts of the tracks of these cyclones, it does not appear that their influence is very great.

At all other places around the globe the cyclones which originate a little north of the equator seem to incline toward higher latitudes, but the tracks laid down on the charts are in all cases short ones, and none of them extend through the belt of high barometer, but it is possible that the cyclones may, under certain unusually favorable circumstances, get through into the higher latitudes.

The progressive velocities of the cyclones, while near the equator, is not nearly so great as it is in the higher latitudes. The reason of this is that this progressive motion at first is nearly westward and depends mostly upon the general westward motion of the atmosphere, which is very small, except near the earth's surface, in comparison with the rapid eastward motion in the middle and higher latitudes, at considerable elevations, upon which the progressive motions of the cyclones there depend. At the vertices of the parabolic storm tracks, where the progressive motion is directly toward the pole, and is independent of the east and west motions of the atmosphere, but depends

entirely upon the general tendency to move from the equator toward the pole, and upon the gentle currents in the same directions on the east sides of the oceans where the cyclones generally if not exclusively make their way through toward the pole, as has been explained, the progressive velocities of the cyclones are the smallest, as theory requires and as observation shows.

87. Excepting the cyclones which in summer and autumn originate near the equator and pass on the east sides of the great oceans toward the pole, the cyclones of the middle latitudes originate in these latitudes, and the strong eastward tendency of the general motions in these latitudes, especially in the upper regions, in which the condensation of vapor and the power of the cyclone is mostly, carries the cyclones forward in this direction, the velocities and directions being somewhat modified by the several other circumstances stated in § 35. That this eastward progressive motion of the cyclones depends, in some measure at least, on these general eastward motions of the atmosphere, is shown by the fact that the progressive motions of the cyclones eastward is greatest in winter and least in summer, just as in the case of the general motions of the atmosphere, as shown by Table XI, Part I. This is shown by Loomis* from the Signal Service observations, according to which the velocities in the United States are nearly one-half greater in winter than in summer. It is true this is not the exact proportion in which the general eastward motions of the atmosphere of winter exceed those of summer, the former being more than twice as great as the latter according to the table referred to, but the progressive motions of the cyclones in summer are increased because the cloud and region of condensation of vapor is generally higher then than in winter, and hence the controlling power of the cyclones being up where the eastward motion is greater, the cyclones move eastward with greater velocity on that account.

If we examine the charts of the United States Signal Service it is seen that while the tracks of cyclones, or areas of low barometer, keep somewhat on the same latitudes across the American continent, and likewise across the Atlantic, in winter it is very different. At this season most of the cyclones originating at, or arriving at the western coast of North America in the middle and higher latitudes, seem to avoid the interior of the continent and to pass in a southeasterly direction on the west side of the continent into low latitudes, passing around by Texas or the Gulf of Mexico, along the eastern side of the United States, up toward Greenland and the northern part of the Atlantic, and crossing it at a higher latitude than they usually do in the summer season.

The interior of North America, in the winter season, is an area of high barometer, very dry and cold. If a cyclone from the Pacific could be carried into this area at this season it could scarcely subsist and get through to the eastern side. Besides, the isotherms at this season run around this area somewhat as the tracks of the cyclones just described, leaving the colder and dryer region on the left, and the warmer and moister one on the right, all the way around. Hence the tracks of the cyclones follow around somewhat in the directions of the isotherms, in accordance with the theory in § 34.

88. In the southern hemisphere, as in the northern, observation shows that the cyclones originating a little south of the equator, likewise tend, in connection with their westward motion, to move also from the equator toward the pole, as theory requires, but that they cannot in general get through the belt of high barometer and dry atmosphere with its middle about the parallel of 30°, except on the eastern sides of the continents where there are slight currents passing from the tropical to the middle latitudes, aiding the progress of the cyclones in that direction and carrying aqueous vapor for their support. Hence all the cyclones originating in the region of the South Indian Ocean, between the equator and the tropic, are carried westward until they arrive where the currents are deflected southward by the continent of Africa by which they are enabled to pass through into the strong eastward current of the middle latitudes, where they are rapidly carried eastward. The vicinity of Mauritius and Madagascar is, therefore, on the great highway of these cyclones, as that of the Antilles and Florida in the northern hemisphere is for those which originate in the Atlantic a little north of the equator.

These cyclones originate mostly during the summer season of the southern hemisphere, because then the great northeast monsoon of India extends considerably south of the equator, becoming there a northwest wind, and meeting the southeast trades 10° or 15° south of the equator, and forcing the equatorial calm-belt down to these latitudes.† Hence the conditions are favorable for

* Silliman's Journal, February, 1880.

† Winds of the Globe, Plate VI.

the origination of these cyclones only at this season, for the reasons given in § 85, in the case of the cyclones originating north of the equator in the Atlantic.

In the Atlantic south of the equator there is another area of higher barometer with winds gyrating somewhat around it, as there is north of the equator, giving rise to a gentle current on the east coast of South America, from the equator toward the middle latitudes; as there is at the Antilles and the southern part of the United States; but we have no account of any cyclones passing from the latitudes near the equator, through here into the great eastward current of the middle latitudes, as at Madagascar and Mauritius and on the east side of the Atlantic in the northern hemisphere. The reason of this is, that the southeast trades here generally extend entirely across the equator, and even during the summer of the southern hemisphere, when cyclones originate in the South Indian Ocean, the trades here extend almost up to the equator, so that the calm-belt, in or near which, only, these cyclones originate when it is sufficiently far from the equator, is too near the equator for cyclones to originate in the part of the Atlantic immediately south of the equator.

89. We have seen that in both hemispheres, all around the globe, there is a belt of high barometer with its maximum about the parallel of 30° or 35°, but that the barometer stands higher on these parallels in the middle of the great oceans than on and near the continents on account of the gyrations of the atmosphere arising from the deflections of the continents, causing a pressure inward from all sides on account of the force arising from the earth's rotation. These areas of high barometer, as is the case in areas of high barometer generally, are dry, and hence cyclones do not originate much within these areas, nor can they generally be supported long if they can get in from without. But since the air gyrates somewhat around these areas, there is no tendency of cyclones to enter them, except on the equatorial sides, since there the general tendency of cyclones is toward the poles. The great calm-belts of the general motions of the atmosphere of these latitudes pass through these areas, and hence they are areas of comparative calm, both as regards the general motions of the atmosphere and likewise cyclonic disturbances. The general tendency of cyclones is to skirt around areas of high pressure, since there are currents on all sides around them, but in these cases this is prevented on the east sides of them by the general tendency of cyclones to move toward the pole, and hence the cyclones originating near the equator at certain seasons only of the year move around these areas of high pressure on the equatorial, western, and northern sides. Over the whole interior of these areas, therefore, in the North Atlantic and Pacific, in the northern hemisphere, and the South Atlantic and Pacific and the Indian Oceans, in the southern hemisphere, there is little danger from any cyclonic disturbances, except around the outskirts during the cyclone season.

The progressive velocities of cyclones around these areas are different in different oceans, and in different parts of the tracks in the same ocean. On the equatorial sides, where the cyclones are carried westward by the general westward current, the velocities are greater than they are where the cyclones curve around into the higher latitudes, since here there is only a small part of the general westward current deflected around, and consequently the velocity of the atmospheric current is very small. After arriving in the middle latitudes, where the general eastward velocity of the atmosphere is comparatively great, the progressive velocities are much increased.

The Southern Indian Ocean is remarkable for the small progressive velocities of its cyclones. According to Piddington, these are often only two or three miles an hour, especially at the vertices of the parabolic tracks where they curve around into the middle latitudes. The reason of this is that there is only a small deflection of the westward currents within the tropic by the continent of Africa, and hence but a slight gyration around the central area and increase of barometric pressure, so that the cyclones are carried around very slowly. The more nearly cyclones can get within the central parts of these areas, the slower their progressive velocities; and it therefore sometimes happens that cyclones in the Southern Indian Ocean are apparently almost stationary.

The cyclones which originate in the middle latitudes at all seasons of the year, and those which come through into these latitudes from near the equator at certain seasons, are carried forward eastward in these latitudes until they die out; but here likewise, as we have seen, they have a tendency to skirt around the area of high pressure and dry air of the interior of North America

in the winter season by curving around toward the equator, and they, no doubt, have the same tendency on the eastern continent, but on account of its great extent they probably never succeed in getting entirely around.

The middle latitudes of the southern hemisphere, with a constant eastward current at the earth's surface of about 30 kilometers per hour, and much greater in the upper regions, with scarcely any interruption from the continents, is a great highway for cyclones, which move with a great progressive velocity, often perhaps entirely around the globe; and touching the continents at Cape Horn and the Cape of Good Hope, give rise to the great storms and barometric oscillations which are experienced at these capes.

CHAPTER III.

TORNADOES, HAIL-STORMS, AND WATER-SPOUTS.

90. Tornadoes are simply special cases of cyclones, governed by the same general principles, and the conditions are in general represented by the same equations. The foundation, therefore, of the theory of tornadoes has already been laid in the theory of cyclones in the first chapter, so that it is not necessary here to go back to first principles, but merely to adopt, by means of slight changes, the equations there given to the special case of tornadoes. The principal feature which distinguishes tornadoes from what are especially called cyclones is the smallness of their extent, and this is due to the difference in the initial temperature conditions which give rise to each. These, in the case of cyclones, consist in gradients of increasing or decreasing temperature from the center to the external part of a somewhat circular area, depending mostly upon the primary causes of temperature disturbance, the range of which is represented by $t'_c - t'_o$ in the first member of (32), and as a disturbance of this kind must generally extend over a considerable area the dimensions of cyclones, even in their initial stages, must be correspondingly large. The conditions of tornadoes depend rather upon the vertical relations of temperature between the different strata of the atmosphere, or values of Δ_c and Δ_o in the second member of (32). When these are such as to give a state of unstable equilibrium of the atmosphere, as heretofore explained, we have a condition in which a tornado is liable to occur at any moment from any slight local temperature disturbance or mechanical impulse, which gives the under strata a little greater tendency to burst up through the overlying strata at any one point than anywhere else. Hence, tornadoes are generally of very small extent, and, although very similar to cyclones in many respects, they form a distinct class, and it can scarcely be said that there is any connecting link between them and cyclones, or, in other words, that the smaller cyclones commence where the larger tornadoes leave off.

91. The general equations of a cyclone (1) are also applicable to a tornado, but, on account of the small extent of the latter, the terms in those equations depending upon the earth's rotation and containing n as a factor may be neglected in the theory of tornadoes without sensible error. At the distance of 1,000 meters from the center of a tornado, on the parallel of 45°, the linear gyratory velocity, due to the earth's rotation, is only about 185 meters per hour, or 0.05 of a meter per second, and at other distances nearer the center less in proportion to the distance. This is so small in comparison with the usual velocities in tornadoes that it may be neglected.

Since in equations (1) the plane of P′ and origin of h are entirely arbitrary, we can put P for P′, and if the quantities in the equation all belong to the same stratum h vanishes. Hence, neglecting the term depending upon the earth's rotation, and putting for ν its value in (2), we get

$$(a) \qquad \left\{ \begin{aligned} \frac{1}{a} D_r \log P &= \frac{v^2}{r} - F_u - D_t u \\ 0 &= \frac{2\,uv}{r} + F_v + D_t v \end{aligned} \right.$$

From equations of (5) and (6), neglecting the term depending upon the earth's rotation, or from the last of the preceding by neglecting the friction term F_v, and integrating, we get

$$r^2\nu = rv = C \tag{b}$$

in which

$$C = \frac{\int_m r^2_0 \nu_0}{m} \tag{c}$$

the notation being the same as in § 3. The value of C, therefore, in this case, depends upon the sum of the initial moments of gyration relative to the earth's surface around the center of the tornado, and is independent of the gyration around this center depending upon the effect of the earth's rotation, represented by the terms $r^2_0\, n \cos \psi$. In cyclones, we have seen, this term determines the manner of gyration from right to left in the northern hemisphere, and the contrary in the southern; but in tornadoes the manner of gyration depends upon the sign of C, and consequently upon the moments of gyration which the air, with reference to the center of the tornado, may have at the time of the origin of the tornado. The sum of these moments can rarely happen to be exactly 0, and the slightest tendency one way or the other gives rise to very rapid gyrations around the center where r is small, as must be seen from (*b*). This is exemplified in the case of a large flat basin of water which is allowed to run out through a small hole in the center. The water may have no perceptible gyration or motion of any kind, yet near the center it will generally run into a gyration the one way or the other. The case is somewhat similar in a tornado, except that, instead of running down, the air of the lower strata, in a state of unstable equilibrium, runs up through those above at the place where it receives its first impulse.

92. In cyclones the gyrations, since they depend upon the earth's rotation, are kept up as long as there is an interchanging motion between the central and exterior part, due to difference of temperature, and they are merely modified, but not entirely destroyed, by friction. They therefore continue as long as the difference of temperature and the radial interchanging motion continue, and hence they may continue for a long time. It is otherwise with tornadoes. Since the gyrations in these depend entirely upon the initial moments of gyration, and the tendency of friction is to constantly reduce these, and consequently the gyratory velocities, the air is soon brought to a state of rest, or nearly so, and the course of the tornado is run.

If the sign of C in (*c*) were entirely accidental, and not determined one way rather than the other by any existing circumstances, the gyrations of tornadoes would be as often in one direction as the other. This, however, does not seem to be the case, but they are generally, if not always, in the same direction as those of cyclones. The reason of this is that the atmosphere is hardly ever entirely free from cyclonic action, which, though almost or quite imperceptible, is yet sufficient to determine the manner of the gyrations of a tornado; for in order to do this, it is not necessary that the center of the tornado should coincide with the center of cyclonic action, but simply that it should be somewhere within the area of the gyrations.

93. In a cyclone, we have seen that the effect of the term depending upon the earth's rotation causes the gyratory velocity at the same distance from the center to be different in the upper and lower strata. In tornadoes, the effect of this term being insensible, the tendency of friction is to reduce these velocities, where different, to the same, except near the earth's surface, where friction is great, and where, consequently, the gyratory velocities are much less. Since the friction term in the last of (*a*) has been neglected in obtaining (*b*), of course this last equation is not strictly applicable in the cases of friction, especially very near the center, where the gyrations are very rapid, and consequently friction much increased. In the case of cyclones we have seen that the effect of friction is very great, and the actual velocities of gyration fall very far short of those given by the hypothesis of no friction. But a cyclone of the usual dimensions is a very broad flat disk of atmosphere, very many times greater in width than in altitude, especially if we neglect the upper very rare part of it, while a tornado may be regarded as a column of gyrating air in which the altitude is several times greater than its diameter. Hence, while the gyrations of such a disk are very much affected by friction, those of a tornado, except near the earth's surface, are affected very little, and in the upper regions they may be regarded as conforming very nearly to the law of (*b*), and consequently the gyratory velocities, at different distances from the center, are very nearly

inversely as the distance. In the initial stages, however, in which the whole energy of the tornado is mostly spent in overcoming the inertia of the air and in producing the gyrations, the velocities of gyration above and below may be, and generally are, very different, and at different distances from the center do not at all conform to the law of (b), since they then depend upon the initial gyrations which the air may have; and it is only after the tornado has been some time in action, in a quiescent atmosphere, or at least one in which the progressive motions of the strata above and below are the same, that the velocities above and below, except near the earth's surface, become nearly the same, and they conform approximately to the law of velocities inversely as the distance.

94. Let us now put P_0, v_0 equal the barometric pressure and gyratory velocity, respectively, at the distance r_0 from the center of the tornado. We then get from (b), after the initial stages of the tornado—

$$(d) \qquad v = \frac{r_0 v_0}{r}$$

With this value of v we get from the first of (a)

$$(e) \qquad \frac{1}{a} D_r \log P = r^2_0 v^2_0 r^{-3} - F_u - D_t u$$

and by integration, regarding a as being sensibly independent of r,

$$(f) \qquad \log P_0 - \log P = \tfrac{1}{2} a v^2_0 \left(\frac{r^2_0}{r^2} - 1 \right) + a \int_r (F_u + D_t u)$$

in which P_0 and P are both at the same altitude and the value of a is given in (3). The last term of the second member depends upon the friction and the inertia of the air in its radial motion toward or from the center, and is generally very small, especially at some distance above the earth's surface. The equation shows the relation between the pressures at the same altitude at different distances from the center. Since the friction of the gyratory motion has been neglected, it is only approximate, but very nearly correct, except near the earth's surface. The value of v_0, therefore, should be that existing at some short distance above the earth's surface.

95. Besides the comparatively small amount of friction in a tornado, as shown in § 92, there is also another circumstance which causes the violence of a tornado to be much greater than that of a cyclone. It is seen from (9), neglecting $n \cos \psi$ in this case, that the force which overcomes the friction and inertia of the air, in the gyrations in a tornado, depends upon u, that is, the velocity of motion toward or from the center. Now, in an ordinary cyclone we have seen that, in consequence of the effect of the earth's rotation, the gyratory velocities are less above than below in proportion to the amount of temperature disturbance; and that the greater part of the force, depending upon the temperature gradient, which keeps up the interchanging motion between the central and external part, is counteracted by this difference of the gyrations above and below, which causes the centrifugal force to be greater below than above. In the tornado, however, in which the term depending upon the earth's rotation is insensible, the velocity of the gyrations is the same above as below, and hence the full force due to the temperature gradient is spent in keeping up the motion toward the center below and from it above. This gives a greater value to u, and hence to the force which overcomes the inertia and friction in the gyratory motion of a tornado.

96. In the theory of the general motions of the atmosphere, and also in that of cyclones, the effect upon pressure of the inertia of the air was shown to be so small that it could be neglected without sensible error, but in tornadoes the velocities are frequently so rapidly accelerated, and consequently the effect of inertia so much increased, that it now becomes necessary to take it into account. From (8), Part I, if we neglect the terms depending upon the earth's rotation, and regard P simply as arising from gravity and the inertia of the air, we readily obtain

$$\frac{1}{k} D_s P = - D^2_t s - gdh$$

in which the direction of s at any point is entirely arbitrary, and hence may be a curved line. Substituting for k its value deduced from (9), (10), and (20), Part I, and putting $u = D_t s$ for the velocity in the direction of s, we get

$$(g) \qquad gl \frac{a + t}{a} d \log P = - udu - gdh$$

In any case in which the air while expanding or being compressed neither receives nor loses caloric by what we have called the primary causes of temperature disturbance (§ —), and hence generally in cases of rapid expansion or contraction, we have from the relation between pressure and temperature first given by Poisson

$$(h) \qquad \frac{P}{P_o} = \left(\frac{a+t}{a+t_o}\right)^{\varepsilon}$$

in which P_o corresponds to the temperature t_o, and $\varepsilon = 3.443$. From this we get

$$\frac{a+t}{a} = \frac{a+t_o}{a}\left(\frac{P}{P_o}\right)^{\frac{1}{\varepsilon}}$$

With this value of the first member substituted in (g) we get by integration

$$(i) \qquad 2gl\varepsilon\frac{a+t_o}{a}\left(1-\left(\frac{P}{P_o}\right)^{\frac{1}{\varepsilon}}\right) = u^2 - u^2_o + 2g\,(h-h_o)$$

in which P_o and h_o are the values of P and h where $u = u_o$. Hence barometric pressure depends upon velocity as well as altitude.

If the initial and final values of P and u belong to the same altitude the last term in (i) vanishes, and the equation is then applicable to horizontal motions. Since in such cases $P_o - P$ is generally very small, we get by development and neglecting very small terms of the third and lower orders—

$$(j) \qquad 2gl\frac{a+t_o}{a}\frac{P_o - P}{P} = 156670\,\frac{a+t_o}{a}\frac{P_o - P}{P_o} = u^2 - u^2_o$$

With $u = 0$, and $u = 20$ meters per second, this gives at the temperature $t_o = 0$, $P_o - P = 1.94^{mm}$. Hence if a horizontal current of air with a velocity of 20 meters per second is stopped by a barrier, where $u_o = 0$, the barometric pressure is increased there nearly 2^{mm}. But if the current is only 14 meters per second, and $u_o = 0$, we get $P_o - P = 1^{mm}$, and a barometer placed before the barrier, where $u = 0$, would stand 1^{mm} higher than in the current. The current, therefore, in varying from 14 meters to 20 meters per second would cause an oscillation of 1^{mm}, and in varying from a calm to a velocity of 20 meters per second would cause an oscillation of 2^{mm}. Hence where the wind blows by blasts and with varying velocities a barometer placed as above, where $u = 0$, is subject to small oscillations corresponding to the blasts and variations of velocity. For a varying temperature it is seen from (j) that the oscillations would be diminished inversely as the absolute temperatures for the same variations of velocity.

Since the line of integration s between u_o and u, or P_o and P, is entirely arbitrary, the final or any intermediate velocity may have any direction determined by the circumstances of the case, and the preceding equation only requires that u_o and u should be at the same altitude, and not that the motions throughout should be horizontal.

Since in horizontal motions the change of temperature is very small we can regard t in (g) as a constant equal to t_o. We thus get by integration, neglecting the last term, since it vanishes in this case—

$$(k) \qquad \frac{2gl}{M}\frac{a+t_o}{a}(\log P_o - \log P) = 360730\,\frac{a+t_o}{a}(\log P_o - \log P) = u^2 - u^2_o$$

in which M is the modulus of common logarithms, and must be used with these logarithms. This gives almost precisely the same results as (j), even for considerable ranges of $P_o - P$. Where, however, $P_o - P$ is large, equation (i) must be used, since in that case the neglected terms in (j) become large, and the temperature t in (g) cannot be assumed to be constant in obtaining (k). This last makes u infinite where $P = 0$, for where temperature is constant the relation between density and tension remains the same, and the rate of acceleration consequently remains the same, and P only vanishes after an infinite duration of time, where the air flows into an infinite vacuum. Where the temperature of the expanding air is that given by Poisson's relation between tempera-

ture and pressure (h), the final velocity in flowing into an infinite vacuum, as given by (i), since in this case the last term vanishes, is

$$u = \sqrt{2\,gl\,\varepsilon\,\frac{a+t_0}{a}}$$

and consequently finite. In the flowing of gases into an infinite vacuum the tension does not vanish in the orifice but only when the velocity of the air ceases to be accelerated, so that the velocity in the orifice is not given by putting $P = 0$ in (i).

97. If we put t' for the temperature of the air where it is quiescent, or nearly so, we get from (g) in this case, since we can put $udu = 0$—

$$(l) \qquad l\frac{a+t'}{a}\,d\log P = -dh$$

Where $t' = t$ for the same values of h, it is seen by comparing this with (g) that we have $udu = 0$ in (g), and we get

$$l\frac{a+t}{a}\,d\log P = -dh$$

Since Poisson's equation (h) is independent of velocities, it is applicable when u becomes infinitely small or vanishes, and consequently in this case. This equation gives

$$d\log P = \frac{\varepsilon}{a}\frac{P_0}{P}\left(\frac{a+t}{a}\right)^{\varepsilon-1}dt = \frac{\varepsilon}{a}\,dt$$

With this the preceding equation gives

$$(m) \qquad D_t h = -\frac{l\varepsilon}{a} = -100.76^{m}$$

or a constant increase of 100.76^{m} in the value of h for each decrease of one degree of temperature.

Let us now put

Δ = the decrease of temperature for each 100^{m} of increase of altitude where it is determined by Poisson's relation,

Δ' = the same in atmosphere quiescent, or nearly so, where it may be different.

We therefore have in general

$$(m') \qquad D_t h = -\frac{100}{\Delta}$$

and this gives by integration, regarding Δ' as being constant for all altitudes,

$$h - h_0 = \frac{100}{\Delta}(t_0 - t) = \frac{100}{\Delta'}(t_0 - t')$$

from which we get

$$t_0 - t' = \frac{\Delta'}{\Delta}(t_0 - t)$$

From (l) and (g) we get

$$udu = g\left(\frac{a+t}{a+t'} - 1\right)dh = -\frac{100\,g}{\Delta}\left(\frac{a+t}{a-t'} - 1\right)dt = \frac{100\,g}{\Delta}\left(\frac{(t_0-t')-(t_0-t)}{a+t_0}\right)dt$$

very nearly; in which t_0 is the temperature where $u = 0$. This, by means of the preceding value of $-t'$ becomes

$$udu = \frac{100\,g}{\Delta}\left(1 - \frac{\Delta'}{\Delta}\right)\frac{t_0 - t}{a+t_0}\,dt$$

The integration of this gives

$$u^2 - u^2_0 = \frac{100\,g}{\Delta}\left(\frac{\Delta'}{\Delta} - 1\right)\frac{(t_0 - t)^2}{a+t_0}$$

With the value of $t_0 - t = \frac{\Delta}{100}(h - h_0)$ from a preceding equation, we get

$$n) \qquad u^2 - u^2_0 = g\left(\frac{\Delta' - \Delta}{100}\right)\frac{(h-h_0)^2}{a+t_0} = .0003592\,\frac{a}{a+t_0}(\Delta' - \Delta)(h - h_0)^2$$

By comparing the value of $D_t h$ in the preceding equation (m') with that of (m) we get for unsaturated air

$$(o) \qquad \Delta = \frac{100}{100.76} = 0^\circ.992$$

In the interior of a tornado with rapidly ascending currents, and where the air is not saturated, we can put Δ equal to this value. If then, in the surrounding parts of the atmosphere we have $\Delta' = 1^\circ.1$, and $t_o = 30^\circ$, putting $u = 0$ at the surface, where $h = 0$, (n) gives at the altitude $h =$ 1,000 meters, $u = 6^m$ nearly, for the velocity per second of the ascending current at that height, and by the formula it increases in proportion to the altitude. In the center of a tornado, however, the region of cloud and saturated air generally comes down to the earth, or nearly so, so that the ascending currents are mostly of saturated air. In this case it is seen from Table I that the value of Δ is much less than in the case of dry air, and varies considerably with temperature and altitude, and the air is in a state of unstable equilibrium especially in the summer season, with a value of Δ' less than one-half of the value of Δ in (o), required in the case of unsaturated air. By comparing the values of Δ in Table I with those of Δ' in Table II, we have, for the altitudes of the principal part of the cloud region in the summer season, $\Delta' - \Delta$ equal to about $0^\circ.15$, and hence the average state of the atmosphere at this season is that of unstable equilibrium when the air is saturated. Tornadoes therefore very frequently, if not generally, originate first in the cloud strata of the atmosphere, and afterward extend down to the earth. With the value of $\Delta' - \Delta$ above, supposing it to be constant for all altitudes from h_o to h, by putting $u_o = 0$, and $t_o = 20^\circ$, (n) gives $u = 7.1^m$ for $h_o - h = 1,000$ meters, and in proportion for other values of $h_o - h$. But the value of Δ' may differ very much from the average values of the table, and give at different times, very different values of $\Delta' - \Delta$. In the winter season it is seen, from the values of Δ in the table, that $\Delta' - \Delta$ is mostly, if not always, negative, and hence we then rarely, if ever, have the conditions necessary for a tornado. It should be borne in mind here that beside the state of unstable equilibrium, it is necessary to have some initial gyratory motion in order to have a tornado. From a comparison of the values of Δ' in Table II, deduced from Glaisher's balloon ascents, with those of Δ in Table I, it is seen that in high and cold altitudes the value of $\Delta' - \Delta$ in (n) is negative, and consequently u is decreased as h is increased, and hence the atmosphere is in a state of stable equilibrium and the ascending currents from the reverse state lower down may be very much retarded if not entirely brought to rest, and the air must consequently flow out from the central part at some intermediate altitude, as in the case of cyclones. Where $\Delta' - \Delta$ is not constant but changes with the altitude by some unknown function, the integration must be performed by mechanical quadratures, as in the case of cyclones in § 28.

98. In considering the motions of solid bodies or particles contained in the atmosphere of a tornado, it is necessary to have some general expression of the amount of resistance which they suffer in their motions through it. Let us confine ourselves to solid or liquid spheres simply, and put—

D = the diameter of the sphere in centimeters.
S = its specific gravity, that of water being unity.
u = its velocity in meters per second relative to the air.
R = the resistance belonging to this velocity.
W = the weight of the body.
s = the specific gravity of the air, that at the earth's surface being unity.

From experiments made by Dr. Hutton with a whirling machine, as given by Professor Loomis*, the resistance of the air to a globe one inch in diameter, for all velocities from 5 up to 300 feet per second, is represented very nearly by $R = .0000672\, u^2$ ounces, u being expressed in feet per second. Assuming that the resistance varies as the square of the diameter and density of the resisting medium, as deduced by Newton from theoretical considerations, and generally very nearly confirmed by observation, we shall have as a general expression, $R = .0000672\, u^2 D^2 s$. But from experiments made at the request of Newton in St. Paul's Cathedral, London, in the years 1710 and 1719, with hollow glass globes five inches in diameter, and also with bladders, Professor Loomis deter-

* Silliman's Journal, second series, vol. 17, p. 48.

mined that the coefficient of resistances is about one-fifth less than that above.* Taking the mean of the two determinations we shall have $R = .0000606\, u^2 D^2 s$. Expressing u in meters, D in centimeters, and the resistance in grammes, we have

$$R = .002859\, u^2 D^2 s$$

The weight of a sphere of water one centimeter in diameter, expressed in grammes, is 0.5238. Hence, the general expression is

$$W = 0.5238\, D^3 S$$

In the case of a body falling through the atmosphere, or any other resisting medium, the velocity of the body is accelerated until the resistance becomes equal to the weight, after which the velocity continues uniform unless the density of the medium changes. Denoting, therefore, this final and uniform velocity by u', we get, by equating the expressions of R and W above

$$(p) \qquad u' = 13.54\sqrt{\frac{D\,S}{s}}$$

For spheres of ice, since in this case $S = 0.865$, we have

$$(p') \qquad u' = 12.60\sqrt{\frac{D}{s}}$$

A hailstone, therefore, one centimeter in diameter would fall through the lower atmosphere at the rate of 12.6 meters per second. At the height of 5,000 meters, where the value of s is reduced to about 0.54, we should have $u' = 17.1$, and hence the final velocity of such a hailstone at that height would be considerably more. From the formula it is seen that in the case of hailstones four times as great in diameter, the values of u' would be twice as great.

In the case of an ascending current, with a velocity equal to u', the body would remain suspended in the air, and if this velocity were greater than u' it would be carried upward with a velocity equal to the excess of the velocity of the ascending current over the value of u' given by the formula above in any case. In the case of a hailstone four centimeters (1.6 inches) in diameter at the height of 5,000 meters, we have $D = 4$, and $s = 0.54$, and with these values (p') gives $u' = 34.2$ meters for the velocity of the ascending current required to keep such a hailstone suspended in the atmosphere at that height, and with an ascending current of 44.2 meters per second it would be carried upward at the rate of 10 meters per second.

The ascending velocity required to sustain a rain-drop in the air is given by (p). If the diameter of the drop is 0.4 centimeter, the formula gives $u' = 2.7$ meters for the velocity of the ascending current required to keep it suspended in the air near the earth's surface where $s = 1$, but in high altitudes it requires considerably more. Mr. Dines has measured rain-drops as small as .0033 inches ($.083^{mm}$) in diameter.† With $D = .0083^{cm}$ (p) gives $u' = 1.2$ meters for the ascending velocity required near the earth's surface to keep the smallest measured rain-drop from falling to the earth. Cloud and fog particles may be many times less, and hence the ascending velocities required to keep them suspended in the atmosphere would be almost imperceptible. Mr. Dines has measured fog particles as small as .00062 inches in diameter,‡ and those not measurable may be very much less. The law of resistance as the square of the diameter may not hold very accurately for particles so very small, but the formula applied in these cases at least shows that the ascending velocities required to sustain them in the atmosphere is extremely small.

99. Where a body near the vortex of a tornado is kept suspended in the air by ascending currents and at the same time gyrates rapidly around the center, the centrifugal force arising from these gyrations may be much greater than that of gravity, and a current of air toward the center is necessary to keep such bodies from being driven by this centrifugal force away from the center, with a velocity much greater in some cases than would be required in an ascending current to keep the same body from falling. The ratio of the centrifugal accelerating force g' to that of gravity g is expressed by

$$\frac{g'}{g} = \frac{v^2}{2\,rg} = \frac{v^2}{19.61\,r}$$

* Silliman's Journal, second series, vol. 18, p. 70. † Quar. Jour. Met. Soc. ‡ Symon's Met. Mag., Jan., 1880.

in which v is the linear gyratory velocity, and r the distance from the center. Calling the lateral pressure arising from this force W′, corresponding to the weight W, in the case of falling bodies, we get by multiplying W by $\frac{g'}{g}$

$$W' = \frac{.0267\ D^3S\ v^2}{r} = \frac{.0267\ D^3S\ r^2_o v^2_o}{r^3}$$

substituting in the last form for the value of v that given in (d). For the resistance of a horizontal current we can use the expression of R above in the case of falling bodies, or of ascending currents. By equating the expression of R with that of W′ we get

$$(q) \qquad u' = 3.06\sqrt{\frac{DS}{s}} \cdot \frac{r_o v_o}{r^{\frac{3}{2}}}$$

for the velocity of horizontal current toward the center which would resist the centrifugal force of gyration, and prevent the body from being driven away.

With a gyratory velocity of $v_o = 3$ meters at the distance of $r_o = 1{,}000$ meters this formula gives at the distance $r = 100$ meters, in the case of a sphere of ice in which $S = 0.865$, and a diameter equal to one centimeter $u' = 8.6$ meters for the velocity of current required to keep such a body from being driven away from the center, at or near the earth's surface, the velocity required being some greater as the altitude increases and the value of s becomes less than unity.

In general the motion of the air relatively to the body is both toward the center of the tornado and ascending, and as the resistance is as the square of this velocity, this resistance, when resolved into the direction of the two components of vertical and horizontal motions, is as the components of velocity and not as their squares, where the resultant velocity remains the same. In such case the expression of u' in (g) must be multiplied into the cosine of the angle between the direction of the resultant velocity relative to the body, and a horizontal line drawn toward the center. The body being supposed to be carried around with the air, the gyratory motion of the air relatively to the body is 0.

100. From (e) it is seen that in the case of gyratory motion around the center the pressure is a function of r, and diminishes very rapidly near the center, where r becomes small. At different altitudes it is also a function of t in the case of ascending air, the relation between pressure and temperature being then expressed by (h).

Let

P'_o and t'_o = the pressure and temperature respectively of air at the earth's surface and at the distance r_o from the center;

t_1 = the temperature of the dew-point at the earth's surface;

P_1 = the pressure of the air when by ascending from the earth's surface and expanding its temperature is reduced to t_1;

h_1 = the height of the stratum at the distance r_1 of which the pressure of the air is P_1 and temperature t_1;

We then get from (h)

$$\log P'_o - \log P_1 = \varepsilon \log \frac{a + t'_o}{a + t_1}$$

Neglecting for the present the very small terms in the second member of (f) representing the friction and inertia of the air, we get

$$(r) \qquad \log P'_o - \log P' = \tfrac{1}{2}\, a'\, v^2_o \left(\frac{r^2_o}{r^2} - 1 \right) = \frac{v^2_o}{2\, gl} \frac{a}{a + t'} \left(\frac{r^2_o}{r^2} - 1 \right)$$

by substituting for a' its value deduced from (10) and (20), Part 1. Subtracting the latter from the preceding equation, we get

$$(s) \qquad \log P' - \log P_1 = \varepsilon \log \frac{a + t'_o}{a + t_1} - \frac{v^2_o}{2\, gl} \frac{a}{a + t'_o} \left(\frac{r^2_o}{r^2} - 1 \right)$$

From the usual barometric formulæ for the determination of altitudes, we have

$$(t) \qquad \log P' - \log P_1 = \frac{h_1}{l\left(1 + \frac{t'^2_o + t_1}{2a}\right)}$$

We also get by development

$$(u) \qquad \log \frac{a + t'_o}{a + t_1} = \frac{t'_o - t_1}{a}\left(1 - \frac{t'_o + t_1}{2a} + \frac{t'^2_o - t^2_1}{3a^2} - \&c.\right)$$

Since the values of t'_o, t', and t_1, differ but little generally, especially when h_1 is not very great, we can put them all equal to t_o' in the small term of the second order, and then neglecting all terms of a third and lower orders in the development, we get from (s) (t) and (u)

$$(v) \quad h_1 = \frac{\varepsilon l}{a}\left(t'_o - t_1\right) - \frac{v^2_o}{2g}\left(\frac{r^2_o}{r^2} - 1\right) = 100.76\,(t'_o - t_1) - \frac{v^2_o}{19.61}\left(\frac{r^2_o}{r^2} - 1\right)$$

The first term of this expression of h_1 gives the height at which the aqueous vapor of the atmosphere at the earth's surface of the temperature t'_o and dew-point t_1 in ascending is first condensed into cloud at the distance r_o from the center where the tension or pressure of the atmosphere is supposed to be not sensibly affected by the forces arising from the gyrations of the air in the tornado.

By substituting v in (r) instead of its expression in (d) we get the same relation between the gyratory velocity v and the barometric pressure which is given between u and this pressure, as may be readily seen from an inspection of the first of (a), from the integration of which these relations are obtained. But (r) is only approximative, and although sufficiently accurate in all practical applications, yet when there is a great difference between P_o and P, (i) should be used, putting the last term equal 0, since in (r) as well as in (j) and (k) it was assumed that the temperature is the same at all distances from the center; but this varies considerably when $P_o - P$ is large. We see from the comparison of these equations we have the same relation between the pressures and the velocities whether these velocities be vertical, radial, or gyratory, or any resultants of all these, and consequently in any direction whatever.

If we put R for the value of r_1 where $h_1 = 0$, we get from (v)

$$(w) \qquad R = \frac{r_o\, v_o}{\sqrt{1796\,(t'_o - t_1) + v^2_o}}$$

Putting V for the value of v where $h_1 = 0$, (d) gives

$$(x) \qquad V = \frac{r_o v}{R}$$

101. If we put h_2 for the value of h where $t_o = 0$, that is, for the height of the stratum of zero temperature, the formula (v) is applicable in this case by putting $t_1 = 0$, provided the air does not become saturated before rising to that elevation. When the air is saturated the value of ε in (v) must be different from that of unsaturated air. From (m) and (m') we get as a general expression in both cases

$$(y) \qquad \frac{l\varepsilon}{a} = \frac{100}{\varDelta}$$

in which the value of $\varDelta$ must be taken from Table I, Chapter I, for each special case, and hence it is not constant for all altitudes. A mean value, however, can always be taken which will give approximate results.

When the ascending air is not completely saturated below, but becomes so at the altitude h_1, we get for the expression of h_2,

$$(z) \qquad h_2 = 100.76\,(t'_o - t_1) + \frac{100}{\varDelta}\,t_1 - \frac{v^2_o}{19.61}\left(\frac{r^2_o}{r^2} - 1\right)$$

The difference consequently between h_1 and h_2 is simply $\frac{100}{\varDelta}\,t_1$ and hence a constant for all values of r. If the temperature t'_o were 30°, and that of the dew-point 27°, we should then have at the distance r_o the value of $h_1 = 302.3$. The average value of $\varDelta$ in this case for saturated air between

the temperature of 27° and zero, by Table I, is about 0°.42. With this value of $\varDelta$, we should have at the distance r_o, $h_2 = 302.3 + \frac{100}{0.42} + 27 = 6730$ meters, which is the value of the first two terms in (z) in this case, the effect of the last term of (z) being supposed to sensibly vanish at the distance r_o.

102. By means of the preceding formulæ, (d), (r), (v), (w) and (x), the results in the following table have been computed for four cases, with the assumed data of gyratory velocity, temperature, and dew-point, in each case given at the top of the table:

$r_o = 1{,}000$ meters; $t'_o = 30°$; $P'_o = 760$ millimeters.												
r.	No. 1. $t_1 = 27°$, $v_o = 10^m$.			No. 2. $t_1 = 22°$, $v_o = 3^m$.			No. 3. $t_1 = 25°$, $v_o = 1^m$.			No. 4. $t_1 = 18°$, $v_o = 2^m$.		
	h_1.	v.	P′.	h_1.	v.	P′.	h_1.	v.	P′.	h_1.	v.	P′.
m.		*m.*	*mm.*		*m.*	*mm.*		*m.*	*mm.*		*m.*	*mm.*
10	....	1,000	2	....	300	443	....	100	716	699	200	685
15	....	667	60	....	200	634	278	67	740	983	133	682
20	....	500	182	....	150	664	376	50	749	1,082	100	716
25	....	400	305	72	120	697	422	40	753	1,127	80	732
30	....	333	402	297	100	716	447	33	755	1,152	67	741
40	....	250	531	520	75	734	472	25	757	1,177	50	749
60	....	167	648	680	50	748	489	17	759	1,195	34	755
100	176	100	716	761	30	756	499	10	760	1,204	20	758
200	272	50	750	795	15	759	502	5	760	1,208	10	759
400	295	25	758	804	7	760	503	2	760	1,209	5	760
700	301	14	760	806	4	760	504	1	760	1,209	3	760
1,000	302	10	760	806	3	760	504	1	760	1,209	2	760
	$R = 127^m$, $V = 77^m$.			$R = 24^m$, $V = 125^m$.			$R = 10^m$, $V = 100^m$.			$R = 13^m$, $V = 308^m$.		

With the assumed values of the constants r_o, v_o, t'_o, and t_1 the values of h_1 have been computed from (v) for each assumed value of r contained in the first column of the table above. The values of v and P′ have been computed respectively from (d) and (r), reducing the logarithms in the latter to common logarithms by means of the modulus, and using the value of P'_o given above. The values of R and v are obtained from (w) and (x). Since the friction terms in the original equations from which the preceding formulæ have been deduced have been necessarily neglected, of course these are merely theoretical results, for which considerable allowances have to be made in some cases for the effects of friction, especially near the earth's surface and where velocities are very great, just as in the case of formulæ for the flowing of liquids and gases through orifices and for the spouting of fluids in which friction is necessarily neglected; for in these cases the observed velocities and heights are often found to fall considerably below the theoretical ones.

103. In No. 1 the assumed gyratory velocity $v_o = 10^m$ at the distance $r_o = 1{,}000^m$ is that of only a very gentle wind, and the data are all so assumed that they may represent those of an ordinary tornado of small extent. The dew-point being only 3° below the temperature of the air at the earth's surface, the value of h_1 or altitude of the base of the cloud at the distance r_o from the center, where it is supposed to be not sensibly affected by the gyrations of the tornado, is only about 300^m, but in consequence of the very rapid gyrations nearer the center it is brought down to the earth at the distance of 127^m, where the gyratory velocity by the formula is 77 meters per second. This, as well as the assumed velocity of v_o, should be understood to refer to strata a little above the earth's surface, and not to the strata very near the surface, and even at all altitudes the values of v near the center, where they are very great, are very much modified by friction.

It is seen from (n) that the condition of unstable equilibrium and of ascending currents, which is one of the conditions of a tornado, requires $\varDelta'$ to be greater than $\varDelta$. From Table I, Chapter I, it is seen that the value of $\varDelta$ in the lower cloud region, with a temperature of about 25°, as in this case, is only about 0°.39, so that if the temperature of the air in the undisturbed state diminished with altitude at a greater rate than 0°.39 for each 100 meters, we should have a state of unstable equilibrium, provided the air is already saturated, that is, that clouds prevail. If not, the tornado

FIG. 14

$t_0 = 30°$, $t_1 = 22°$, $v_0 = 3^m$

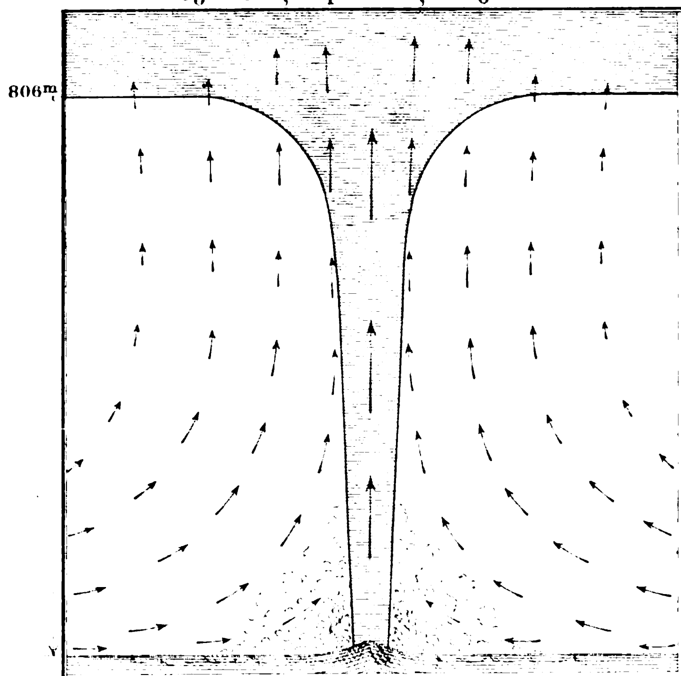


FIG. 15

$t_0 = 30°$, $t_1 = 25°$, $v_0 = 1^m$

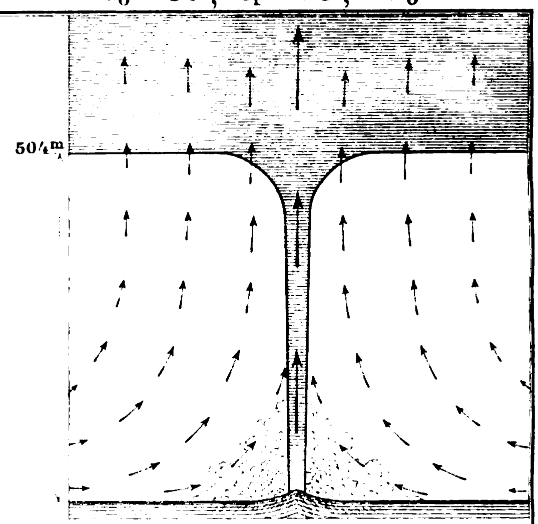


FIG. 16

$t_0 = 30°$, $t_1 = 18°$, $v_0 = 2^m$

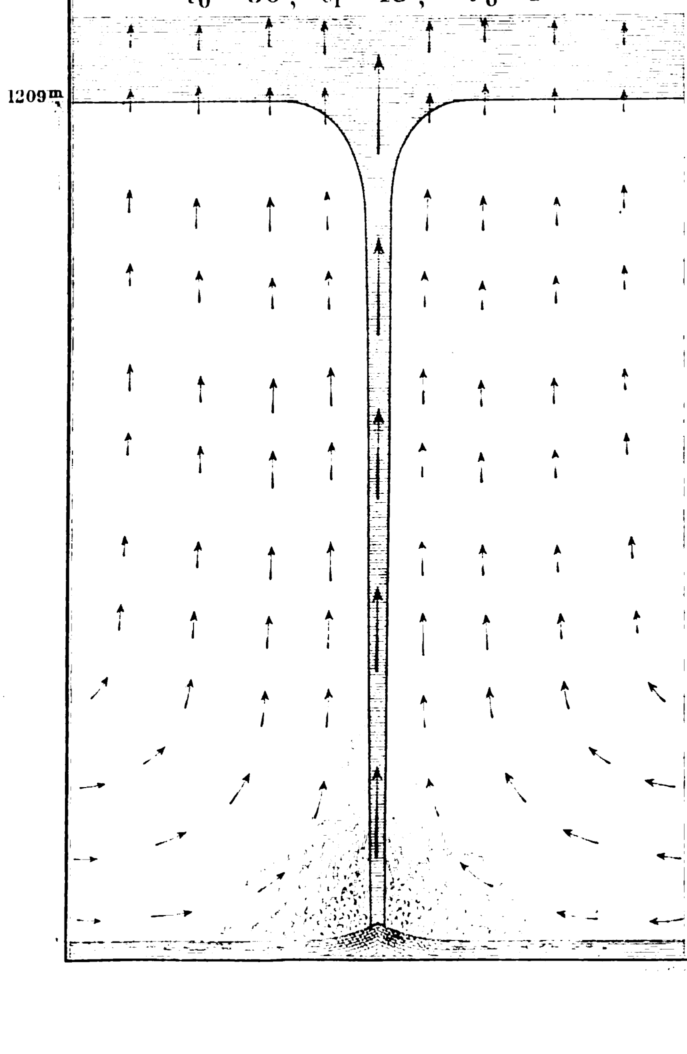
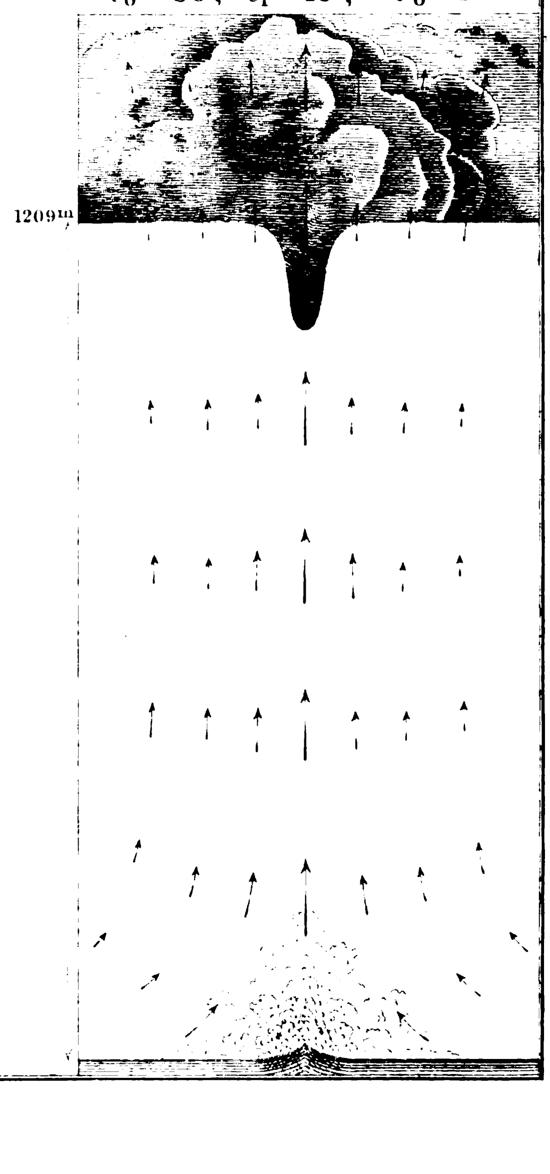


FIG. 17

$t_0 = 30°$, $t_1 = 18°$, $v_0 = 2^m$



cannot commence unless the air is either saturated down to the earth's surface, or the unsaturated air there is in the state of unstable equilibrium, which would require the rate of diminution of temperature or value of $\varDelta'$ there to be more than $0^\circ.992$. But if tornadoes are included in a cyclone, as they generally are, we then have the saturated air up in the cloud region, arising from the ascending current of the interior of the cyclone, and we then have the state of unstable equilibrium with a value of $\varDelta'$, in high temperatures, of only about $0^\circ.4$ and upward, even when the air near the earth's surface is neither saturated nor in the state of unstable equilibrium of unsaturated air. In such a case the gyrations commence first up in the cloud region, but soon extend down to the earth's surface, since the diminution of barometric pressure in the central part arising from the gyrations above causes the air to rush in from all sides toward the center in the lower part of the atmosphere. Of course, with the preceding conditions, we must also have the other condition of a tornado (§ 91), which is an initial gyratory motion relatively to the center.

104. With the assumed values of r in the preceding table, and the corresponding computed values of h_1 in No. 1, used as co-ordinates, the curved line in Fig. 14, representing the stratum of equal tension P_1 and temperature t_1, and which is the base of the cloud, is laid down. The curved lines with arrows represent the lateral and ascending currents of the air, the former, near the earth, being mostly toward the center, in order to supply the draught of the ascending current in the central part, but in the upper regions there must of course be a flowing out from the center. In connection with these motions, it must be borne in mind that the air has also a gyratory motion, and very rapid near the center. Without this we should have simply the currents of Espy's theory. Wherever the currents of air enter within or above the line mentioned above, the invisible aqueous vapor of the unsaturated air is now condensed and becomes visible in the form of cloud. Of course the original cloud particles are extremely small, but are continually increasing in size by aggregation, since the smaller particles are carried up more rapidly than the larger ones, and they are, therefore, constantly coming in contact with one another.* If they become too large to be kept suspended in the atmosphere by the ascending currents, they fall directly back to the earth, but if not, they are carried up where the currents are outward from the center, and are carried out from the center where there are no ascending currents, or only very feeble ones, and where they fall as rain at some distance from the center. The preceding is for the most part true in ordinary cyclones of large dimensions, except that the comparatively feeble ascending currents do not prevent the rain from falling at any part after the drops attain a certain size. In a cyclone also the cloud does not generally come down to the earth in the center, but is only a little lowered.

105. The ascending velocity (u) in the interior of a tornado, as given by (n), depends entirely upon the difference of temperature between the interior and the surrounding parts, and is independent of any gyratory velocity around the center, since the terms in (f) containing v_o as a factor and the friction term F_u, were neglected in obtaining (g) and (n). It is seen that from this source we get only very moderate ascending velocities in the interior of a tornado, upon any reasonable and probable assumption of decrease of temperature with altitude in the surrounding undisturbed atmosphere, as has likewise been shown by Dr. Reye.† With the better knowledge which we now have with regard to the rate of decrease of temperature with increase of altitude, both in ascending currents and in the normal quiescent state of the air, the assumptions by which Espy obtained very rapid ascending currents by his theory, are now known to be not only improbable, but entirely impossible. It becomes necessary, therefore, to seek some other explanation of the powerful ascending currents which, from the observations of their mechanical effects, are known to exist in the central parts of a tornado.

We have seen in the theory of cyclones that all radial and vertical currents depend upon difference of temperature between the internal and external parts, and that when this difference ceases to exist the cyclone soon vanishes. It may also be shown that when a is not a function of the temperature, equation (f) is satisfied with $D_t u = 0$, if v_o has the same value for all altitudes, and it is, therefore, satisfied without radial, and consequently without vertical motions. The radial and vertical motions, therefore, even where there is a gyratory motion, depend upon the difference

* See notice of Professor Osborne Reynolds's paper on Rain-drops and Hail-stones in Nature, December 21, 1876.

† Die Wirblestürme, &c., Anhang, p. 230.

of temperature already referred to, provided these gyratory motions are the same at all altitudes, and the velocity of the ascending currents in the interior is still given by (n). It has been explained that all the strata, through friction and other causes, although at first they may have different gyratory velocities, have a tendency to assume the same gyratory velocity, except those near the earth's surface, where the velocities are much retarded and diminished by the friction of the earth's surface. Let us now suppose we had a tornado with very rapid gyrations near the center at all altitudes, and suffering no resistance at the earth's surface. We might then have a large barometric depression in the center, due mostly to the centrifugal force of the gyrations, and comparatively little of it to difference of temperature between the interior and surrounding air. But notwithstanding the great barometric depression in this case, the velocity of the ascending currents would be only those given by (n), which, we have seen, could in no case be very great.

106. If we now suppose that the barometric depression in the middle of the tornado is 30^{mm}, and that from some cause the gyrations near the earth's surface should cease, so that there would be no force to resist the pressure below toward the center, then by (k), with $t_0 = 30°$, and $u_0 = 0$ in this case, we should have $u = 84$ meters for the ascending velocity per second, caused by this difference of pressure. After the currents arising from this difference of pressure have once set in, of course some of this difference would be spent in overcoming the friction, so that, as in all such cases, the actual velocity would be some less. If we now suppose that the gyrations below are not entirely wanting, but that the velocity is diminished one-half, then the centrifugal force would be one-fourth, and so only one-fourth of the difference of pressure below would be counteracted by the centrifugal force of the gyrations. Let us further suppose that another fourth is used in overcoming friction. We would then still have a difference of 15^{mm} for overcoming the inertia of the air and producing an ascending velocity. With this difference, and with the same value of t_0, (k) still gives an ascending current with a velocity of 60 meters per second. It should be observed here that the direction of the current is not determined in (k), and cannot be vertical at or very near the earth's surface, but from the circumstances of the tornado must become approximately so at a short distance above the earth.

We now see how the enormous velocities of the ascending currents in a tornado may be caused by the differences between the gyratory velocities above and those very near the earth's surface. The former prevent the air mostly from pressing in to fill up the partial vacuum near the center, while the smaller gyratory velocities near the earth allow it to rush in there mostly to supply the draught arising from the diminished pressure within. As these gyrations depend upon the initial moments of gyration of the air in the vicinity around the center, the energy of the tornado depends mostly upon these, the effect of which, by the nature of tornadoes, is mostly concentrated near the center. But the tendency of friction is to constantly use up this energy, so that the tornado cannot continue very long. The initial motions, however, by which the air is caused to run into rapid gyrations around the center depends upon the temperature differences, without which the tornado cannot originate.

107. In order to account for the immense amount of rain which often falls in a tornado in a short time, it is necessary to have an estimate of the amount of aqueous vapor carried by the ascending currents of the interior part up through the base of the cloud into the upper regions, where it is condensed, and also of the depth of rain arising from its condensation if it all falls directly back to the earth. Let

e = the elastic force of aqueous vapor at the base of the cloud;
d = the depth of rain resulting from the condensation in the ascending current, supposing that the current extends so high that the vapor is all condensed.

Then, with an ascending velocity, u, the weight of aqueous vapor carried up per second, measured by the mercurial column, is $0.623\, eu : 7989$. Multiplying this by 13.6, the specific gravity of mercury, we get

$$d = .00106\, eu$$

for the depth of rain given by the condensation per second. With a temperature of 27° at the base of the cloud, as in our assumed example, No. 1, we have from Regnault's table, $e = .0265^{m}$ for the elastic force of saturated air, and with this value the preceding formula gives for an ascending

velocity of 60 meters per second $d = .00168^m$ per second or 0.101^m (about 4 inches) per minute for the depth of rain, if it were to fall directly back.

Our problem is too complex, taking into account friction and all other circumstances, to determine the relation between the radial and vertical velocities; but we know that the radial component must be toward the vortex in the lower part of the atmosphere in order to supply the draught of the ascending current, and greatest very near the earth, where they are least counteracted by the centrifugal force of the gyrations. But somewhere above, it may be at a great altitude or not, according to the rate of diminution of temperature with increase of altitude in the surrounding quiescent atmosphere at the different altitudes, there must be a flowing out of the air above, so that in the ascent of the air there is an inclining toward the vortex below, as represented in Fig. 14, and from it above. The vapor, therefore, above a certain altitude is gradually dispersed, and, if condensed into rain, it falls at some distance from the vortex of the tornado. With an ascending velocity of 60 meters per second the rain would all be carried up to an altitude where it would be carried out from the vortex, and hence none of it would fall directly back. According to (p) such a velocity would keep suspended in the air, at an altitude where the density is diminished one-half, and where consequently $s = 0.5$, a globe of water, if such could exist, with a diameter $D = 1.8^{cm}$ (about 7 inches). With such an ascending velocity, therefore, no rain could fall directly back, but there would be an immense fall of rain around about in the vicinity, especially if the tornado in its irregular progressive motions should remain stationary, or nearly so, for several minutes over the same spot. That an ascending velocity of 60 meters per second, obtained from the assumptions of the preceding section, is no unusual velocity in tornadoes, will be shown from the observed mechanical effects of tornadoes.

108. If the velocity of the ascending current is not so great that the rain is all carried up to where the currents are outward from the vortex, and where consequently the water is dispersed, and yet great enough to prevent its falling back, then in the whole lower part of the cloud, up perhaps to the altitude of 3,000 meters, there may be an accumulation of rain, prevented from falling by the ascending currents, and from being dispersed by the inflowing currents from all sides toward the vortex. Of course the sustaining of this water in the cloud uses up the energy of the tornado and hastens its breaking up. Suppose the gyratory motions and energy of the tornado were such as assumed in § 106, which give an effective force of 15^{mm} for producing ascending currents in the interior; if we now suppose that the interior is so charged with water as to diminish this effective force from 15^{mm} to 3^{mm}, this would require a weight of water equal to 12^{mm} of mercury or a depth of rain equal to $12 \times 13.6 = 163^{mm}$ (about 6.3 inches), and the 3^{mm} remaining, by (j) or (k), would still give an ascending velocity, according to (p), much more than sufficient to keep the rain from falling in drops of the largest size, for by the former we should still have an ascending velocity of about 27 meters per second, and by the latter this would support a drop with a diameter $D = 4^{cm}$. If we now suppose that the energy of the tornado and the velocity of the ascending currents are being gradually diminished, either by the accumulation of water in the cloud, or the effect of friction, this accumulation of water may all fall to the earth in a very short time, and give rise to what is called a *cloud-burst*. This is especially liable to occur in mountainous regions; for if we suppose that a tornado heavily charged with water is moving toward the side of a mountain, M, as represented in Fig. 13, its coming in contact with the mountain would interfere very much with the gyrations of the tornado and the inflowing currents below, and tend to break up the whole system almost at once, and let the whole accumulation of water drop suddenly down. Hence cloud-bursts usually occur on mountain sides.*

56. The water in cloud-bursts does not generally fall as rain, but is *poured down*. Long before the ascending currents are so reduced as to allow the water to fall in drops it seems to collect together at certain places and force its way downward through the ascending current in a stream. This it would naturally do, since we cannot suppose that the water is ever evenly distributed over any given place or that the velocities of the ascending currents are everywhere in the same vicinity

* This agency of mountain sides in causing cloud-bursts was first suggested by Professor Trowbridge, of Columbia College, on hearing my explanation of the manner in which rain can be kept suspended in the cloud, in a paper on this subject read before the National Academy of Sciences, in April, 1878.

the same. A considerable mass having been collected at certain points, it is then enabled to force its way through and draws into its train much more from all sides on its way down, so that it may amount to a continuous stream kept up for several minutes. Of course, having once made an opening for itself, its velocity is gradually accelerated, so that on reaching the earth the velocity may become immense and the stream strike with great force. Each one of these descending streams may make a great hole or basin in the ground; and on a steep mountain side, if the stream continues for a short time, it may give rise to a mountain slide, or at least to a great ravine, carrying rocks and trees with it down the mountain side.

Immediately after the great tornado at Hollidaysburg, Pennsylvania, on the 19th of June, 1838, Espy visited the vicinity and examined the sides of the ridges and mountains. He found a great many holes eight or ten meters in diameter and one or two in depth, according to the nature of the soil and depth to the rock, the sides often cut almost perpendicularly down on the upper side, but entirely washed out below, so as to form the commencement of a ravine. Sometimes the current seemed to strike with so great force that it made a great hole or basin, and rebounded so as not to strike the ground again for a considerable distance. A considerable number of these holes in the earth were often found close together in the same vicinity, indicating that the water was poured down through various openings at the same time. With regard to a ridge a half mile west of Hollidaysburg he says: "On examining the northern side of this ridge, large masses of gravel and rocks and trees and earth, to the number of twenty-two, were found lying at the base on the plain below, having been washed down from the side of the ridge by running water. The places from which these masses started could easily be seen from the base, being only about thirty yards up the side. On going to the head of these washes, they were found to be nearly round basins from about one to six feet deep, without any drains leading into them from above. The old leaves of last year's growth, and other light materials, were lying undisturbed above, within an inch of the rim of these basins, which were generally cut down nearly perpendicularly on the upper side, and washed out clean on the lower. The greater part of these basins were nearly of the same diameter, about twenty feet, and the trees that stood in their places were all washed out. Those below the basin were generally standing, and showed by the leaves and grass drifted on their upper side, how high the water was in running down the side of the ridge; on some it was as high as three feet. It probably, however, dashed up on the trees above its general level." (Philosophy of Storms, p. 375.) In an account of a remarkable storm which occured at Catskill, July 26, 1819, it is stated that "the rain at times descended in very large drops, and at times in streams and sheets." Again, another observer "particularly noticed that he could see no drops of rain, but the water seemed to descend in streams and sheets." It is evident from these, and many other similar paragraphs which might be cited, that the water in such cases is poured down in streams and does not fall as rain, especially since the lightest materials were not washed away close to the margin of these basins on the upper sides referred to above; and the reason of this, evidently, is that the accumulated water in the cloud is poured down in streams while the ascending currents are, as yet, too rapid to allow the largest drops to fall as rain.

110. It may be supposed by some that an ascending velocity as great as 60 meters per second, as deduced from the assumptions in § 106, is not only extremely rare, but one which never occurs in any tornado. This requires a barometric depression in the interior of the tornado of about 30^{mm}. By examining column P′ in the table of § 102, it is seen that with the assumed conditions of our tornado, the theoretical tension of the air near the center nearly vanishes, and the interior becomes almost a vacuum; but of course very great allowances must be made for friction so near the center, where the theoretical gyratory velocities almost become infinite. For since the air of the ascending current in the vortex is mostly supplied by the current near the earth, where friction is great and the gyratory velocity much diminished, this affects the gyratory velocities very greatly all the way up, so that they may differ very much near the center from those given by (*d*), in which the effect of friction is neglected. But that the tension of the air is very much diminished in the middle of tornadoes, much more than the assumed amount of 30^{mm}, is obvious from the explosive effects often observed in the passage of a tornado. The walls of houses are thrown outward in every direction, cellar doors are burst open against the force of a hurricane wind, corks fly from bottles, and explo-

sions occur wherever air is confined, which denote a great and sudden diminution of the outside tension of the air from the diminished pressure in the tornado, so that ascending velocities may, no doubt, be often much greater than that which would arise from a depression of 30^{mm} only in the interior of the tornado. This may also be inferred from the powerful force of the ascending currents in sustaining and carrying in the air large and heavy bodies to a great distance. Of the many examples which might be given it is only necessary to cite one well-authenticated case: During the tornado at Mount Carmel, Illinois, June 4, 1877, we read that "the spire, vane, and gilded ball of the Methodist church were carried fifteen miles to the northeastward." Nothing is stated with regard to the material of which it was constructed, but we must infer that the ascending currents which could keep it suspended in the air for at least fifteen or twenty minutes must have had an enormous velocity. According to the formula (p) the ascending velocity required to keep a sphere one meter in diameter and of the specific gravity of water suspended in the air is 135 meters per second. On account of the probably greater surface, in proportion to the weight, in the case of the church steeple, the velocity required was perhaps much less than this, but this estimate in the case of the sphere enables us to judge better with regard to what was required in the case of the church steeple. One reason why heavy bodies are not more frequently lifted from the earth and carried up is that at the earth's surface the velocities are necessarily horizontal only and the vertical components of velocity are naught. If heavy bodies were generally in the situation of church steeples they would be more frequently carried off.

111. The expression of d, § 107, is applicable to cyclones or to any case of moderate rain in which there is little or no cyclonic action. Wherever there is cloud or rain there are ascending currents, but the velocities in such cases are generally very small in comparison with those in the vortices of tornadoes. With descending currents, however gentle, there can be neither cloud nor rain. If we suppose the velocity u of the ascending current to be only 3 meters per second, and the air saturated at the temperature of 27°, we have in this case $d = .00318 \times .0265 = .000085^m$ for the depth of rain given per second by the vapor carried up if it is carried up so high that it is all condensed. In summer temperatures the mean rate of diminution of temperature with decrease of altitude, as may be seen from Table I, Chapter I, is about 0°.4 per 100 meters in the lower part of the cloud. Hence at the altitude of 3,000 meters the air, which at the earth's surface is 30°, and 27° at the base of the cloud, as in our assumed example No. 1, is cooled down to 16°, and at this temperature, according to Regnault's table, the ascending air has lost half its vapor. Perhaps, on the average, the air does not ascend so high that it loses more than half, except in tornadoes and violent cyclones, and in such cases, with an ascending velocity of 3 meters per second, we have $d = .000042^m$ per second or $.0025^m$ per minute.

With an ascending velocity of 3 meters (p) gives, in the lower part of the atmosphere, $D = .05^{cm}$ for the diameter of the smallest drops which could fall, so that those still smaller would be carried up and kept suspended in the air until by aggregation they should become large enough to fall, or until the accumulation of water in the cloud should diminish the velocity of the ascending current sufficiently to allow them to fall. With an ascending velocity of 3 meters, only the larger drops can fall, so that there cannot be much rainfall until the velocity is diminished. There is therefore a limit to the rate of rainfall beyond which it cannot extend, and this must be considerably less than $.0025^m$ per minute or 0.15^m per hour. In the vortices of tornadoes, where the velocities are enormously great, of course there is a proportionate amount of vapor condensed to rain, but this, as we have seen, must either be poured down, or, if it falls as rain, must be scattered over a large area about the outer limits of the violent part of the tornado, where the ascending velocity is less than 3 meters per second. With very gently ascending currents the rain falls in small drops only, or a mist, before much aggregation takes place, especially if the air is nearly saturated and the base of the cloud low. If the base of the cloud is high these small particles, in their slow fall toward the earth, may be entirely evaporated before reaching it. In such cases the lower base of the cloud has no definite outline, and may fall considerably below the level where condensation first takes place; for these very fine falling particles are simply cloud particles not quite sustained, as usual, by the ascending current.

112. We come now to the consideration of ***hail-storms***. In our assumed example of a tornado, No. 1, the plain or stratum of zero temperature, according to (z) and the illustrating example under

it, which applies in this case, is 6,428 meters above the base of the cloud, and, according to theory, if we take no account of friction, is likewise brought down to the earth in the center. A vertical section of this stratum, passing through the center of the vortex, is represented by the curved line in Fig. 13, between the dark and light colored cloud. Since great allowance near the center, where the gyrations are very rapid, must be made for the effect of friction, the region of freezing and snow is perhaps never brought entirely down to the earth, but, in general, the base is simply depressed a little in the center, as represented by the base of the cloud in Fig. 17. Below this base in Fig. 13 aqueous vapor is condensed into cloud and rain, but above, into snow. The rain-drops below may be also carried up into the snow region by the ascending currents, and, if kept suspended there a little while, frozen into small hail. These may then be kept suspended near the base of the snow-cloud, and increase in size by the rain which is carried up into this region coming into contact with them before it has had time to freeze. In this way compact homogeneous hail-stones of ordinary size are formed. At the height of nearly 7,000 meters, the density of the air in comparison with that at the earth's surface is 0.42. With this as the value of s, (p') gives $u' = 20^{m}$ nearly as the ascending velocity required to sustain a hail-stone at that altitude one centimeter in diameter. This, from what we have seen, is no unusual velocity for ascending currents in tornadoes, and one which must frequently take place, even at great altitudes. It is not necessary that the hail-stones should remain in the freezing region a long time, or remain stationary. They may be carried from the vortex out where the velocity of the ascending current is small, and dropping down some distance, may then be carried in toward the vortex by the inflowing current on all sides, and up again rapidly into the freezing region.

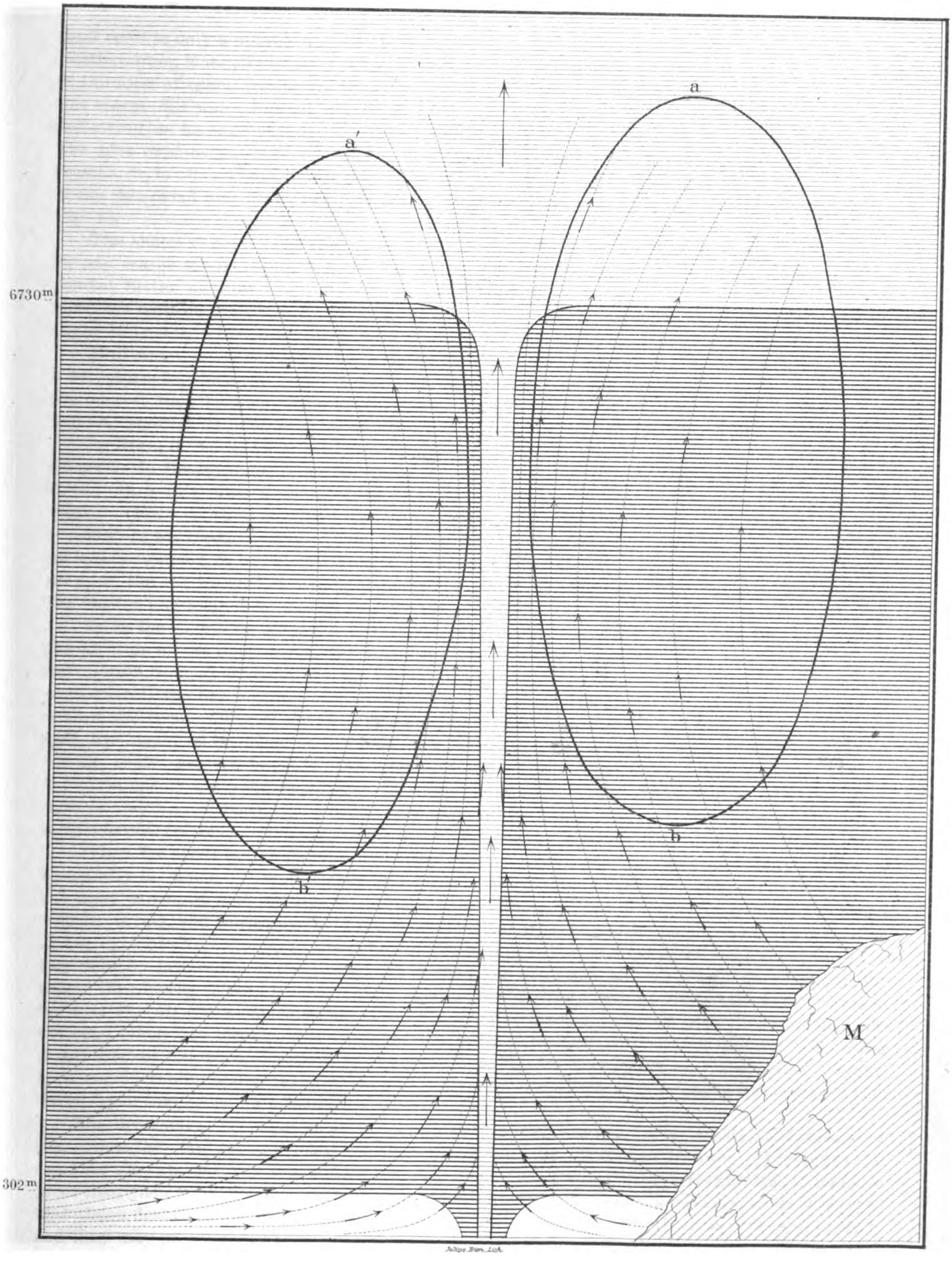
The nucleus of large hail-stones is generally composed of compact snow. A small ball of snow, saturated with unfrozen rain which is carried up into the snow-cloud, is formed in that region and freezes, and being of less specific gravity than compact hail, is kept where it receives a thick coating from the rain carried up, as in the case of the small hail, and afterwards falls to the earth, either at some distance from the center where the ascending currents are weak, or near the vortex after the rapidity of the ascending currents has become sufficiently diminished. As there may be in the case of cloud-bursts a great accumulation of rain and a sudden down-pouring of it all in a short time, so in a hail-storm a great quantity of hail may be collected in the lower part of the cloud, brought in by inflowing currents on all sides toward the vortex, after the ascending currents have become too weak to carry it up and throw it out above again, and are still too strong to permit it to fall. But soon the interior of the tornado becomes so overloaded, and the energy of the whole system so much spent, that the hail is all allowed to fall to the earth almost at once. Hence the almost incredible amounts of hail which are said to sometimes fall in a short time.

113. When a hail-stone is carried up in or near the vortex and carried out above to where the ascending current is too feeble to sustain it in the air, it gradually drops down and the inflowing current draws it in toward the vortex, where it is again carried up, and thus describes a sort of oval orbit, a, b, or a', b', as represented in Fig. 13. It may be thrown up very high into the snow-cloud region, as at a, or but little above its base, as at a'. It may describe a number of such orbits or revolutions before it falls to the earth. While high up in the snow-cloud region it receives a coating of snow, and then while descending very gently where the strength of the currents is not quite sufficient to sustain it, and near the base of the snow region where rain yet unfrozen is carried up, it receives a coating of solid ice, which may be continued for some time after it falls into the rain-cloud, since the hail-stone still continues for some time below zero temperature. After a little the inflowing current below, as at b and b', draws it again into the vortex, where it is again thrown up into the snow region to receive a new coating of snow. It thus receives alternate coatings of snow and ice, and the number of each sort indicates the number of revolutions described, before it is permitted to fall to the earth. When we consider the enormous amount of water which is rapidly carried up in a tornado, and that the lower part of the region of freezing must contain mostly rain, not yet frozen, since the snow there formed is at once carried still higher, we can readily understand how the hail-stone can receive a considerable coating of ice in a short time. While high up in the snow-cloud at its turning-point, as at a and a', it of course remains some time nearly at the same altitude, and, it is reasonable to suppose, long enough to receive its coating of snow.

The accounts of hail-stones composed of a nucleus of snow, and then of concentric layers of ice and snow alternating, are quite numerous, but of the many cases which might here be cited we

FIG. 13

$t_0 = 30°, \quad t_1 = 27°, \quad v_0 = 10^m$



Julius Bien, Lith.

shall refer to only a notable one which has occurred in somewhat recent times. In a hail-storm at Northampton, Mass., June 20, 1870, hail-stones fell weighing over a half pound. Most of them were of the form of concentric layers like the coats of an onion. Mr. Houry, who gave an account of this storm (Silliman's Journal, vol. 50, p. 403), states that in one of them he counted thirteen layers, indicating, as he says, that it had passed through as many strata of snowy and vaporous cloud. The true explanation is that it oscillated as many times between the rain-cloud and snow-cloud regions, or, in other words, that it performed six or seven revolutions with the lower part of its orbit in the rain-cloud and the upper part in the snow-cloud, as described above and as represented in Fig. 13.

The shape of hail-stones varies very much. Some are like a disk or very oblate spheroid. If for any reason the hail-stone becomes the least flattened, the ascending current which keeps it suspended in the air also keeps its shortest diameter perpendicularly to the current, and hence it increases most on the edges. Others are of a pyramidal form. This has been explained by Professor Reynolds in the paper already referred to.

Hail-stones are sometimes of an enormous size. At Olmutz, on the 31st of May, 1868, there was a hail-storm in which the larger stones were mostly of an irregular oval shape with uneven surface, the longest diameter being from 14 to 22 lines. Those from 18 to 22 lines all weighed over 3 drams, and the greatest, 24 lines long, weighed 5 drachms. (Zeit. Oester. Gesell. für Met., Band III, S. 318.)

In Iowa, in April, 1880: "Hail-stones 12 inches in circumference have been measured in Sac County. In Davis County a flattened disk of hail measured 4½ inches in diameter, and was 2 inches thick. In Iowa County a group of ice crystals fell, 2 inches in length, 1¾ inches wide, and 1 inch thick." (Iowa Weather Service Press Bulletin, 82, April, 1880.)

114. In the second example of the table, § 102, the gyratory velocity at the distance of $r_o = 1{,}000$ meters is assumed to be only 3^m, and the dew-point t_1 eight degrees below the temperature of the air t_o, at the earth's surface at the distance r_o, where the influence of the tornado does not affect the tension and temperature of the air at the surface.

From the co-ordinates r and h_1 in the table the curved line in Fig. 14 is laid down, representing the stratum of the tension P and temperature t, where the aqueous vapor of the air in its ascent and motion from all sides toward the vortex begins first to be condensed into cloud, and which is consequently the base of the cloud, as in the case of the tornado in example No. 1. The difference etween the temperature of the air at the earth's surface and the dew point being greater in this case, the height of the cloud at the distance r_o, where it is not sensibly affected by the gyrations of the air, is also greater, but it is nevertheless brought down to the earth at and near the center of the gyrations by the great rapidity of these gyrations there, as represented in Fig. 14, and forms what is called a *water-spout*. Hence *a water-spout is simply the cloud brought down to the earth from a considerable height in a tapering form to a small base by the rapid gyratory motions of the air.*

When the spout occurs on land it is often called a land-spout, but land and water spouts are essentially the same. A water-spout and tornado differ only in their dimensions. It is seen that the same principles and formulæ are applicable in both cases, but in the water-spout the gyratory velocity is less, and consequently the base of the cloud or spout which reaches the earth is smaller. In our example No. 2, (w) gives the radius of the base $R = 24^m$, and (x) gives $v = 125^m$ as the gyratory velocity per second at the outer limit of the spout at or near the earth's surface. Nearer the center the velocity is very much greater. These are the theoretical velocities obtained by assuming $v_o = 3^m$. Of course considerable allowance must be made for the effect of friction, which has not been taken into account in the formula. In order to really have such velocities near the center in any case of nature, and a spout as represented in Fig. 14, it would be necessary to have v_o perhaps considerably greater than 3^m, but still it is readily seen how such a phenomenon can be produced upon the preceding principles.

The destructive effects of so extremely great velocities can readily be imagined, especially at sea, where they are not so much diminished at and near the surface by friction. If, however, we examine the values of v in the table, § 102, it is seen that, although the velocities are enormously great near the center, yet they diminish very rapidly with increasing distances from the center, so that these destructive meteors can be approached within a very short distance with entire safety, and they may pass very near without doing any injury. If, however, they pass directly over a ship

at sea or buildings on land, their destructive effects may be very great and sudden, as is readily seen from theory, and often confirmed by observation. On land they are the same as tornadoes, except that their visible clouded part is not so low or so wide at the base, and the extent of the area of violence is less.

In the center of a water-spout, as in that of a tornado, no rain falls or water descends in any form, though a heavy shower often falls in the vicinity. On land, dust and a great many light substances are carried up in the interior, and as they are being collected from all sides by the inflowing currents toward the vortex below, they often assume the form of a cone, which, in the first formation of the spout, seems to rise up and meet the descending spout, falling apparently from the clouds, and thus the whole phenomenon often assumes the form of an hour-glass. Of the great tornado of West Cambridge, August 22, 1851, it is said:* "To some who watched it closely its form resembled a tall wide-spreading elm-tree. To others it appeared like an inverted cone. Several represent it as a dense upright column, and a few as having the shape of an hour-glass." The average width of destructive violence was about 300 meters. The several observers, no doubt, saw it at different times, and under somewhat different circumstances, in its progressive motion over the inequalities of surface. It was only where the earth contained a great amount of dust or other light materials on the surface that the right cone at the base was observed, and the whole assumed the figure of an hour-glass. This tornado evidently had a good deal of the water-spout feature about it, and it probably fell somewhere between the two forms represented in Figs. 13 and 14.

On the 14th of August, 1847, Professor Loomis observed a water-spout on Lake Erie,† the height of which, by a rough estimate, was about a half mile, and the diameter about ten rods at the base and twenty rods above. This coincides very nearly with our No. 2, represented by Fig. 14. No observations of temperature of the air or dew-point are given, but the difference, by theory, must have been about 8° to give the observed height.

In water-spouts at sea there is a rising up and foaming of the sea water under the spout, on account of the greatly-diminished atmospheric pressure and rapid gyrations within the spout. By theory the water rises 13.6^{mm} for each millimeter of barometric depression, so that if there should be a depression of 100^{mm} the water would rise 1.36^{m}. This heaping up of the water is always observed, and on account of the rapid gyrating currents of the air the water is much agitated at the surface, and much of it may be carried up in the interior of the spout by the ascending current. This sometimes gives the spout a whitish appearance below. The heaping up of the water and rising of the spray, caused by the great agitation, sometimes gives the appearance of a small right cone, as occurs in land-spouts from the collection of dust and other light materials. The water which falls as rain, in connection with a water-spout, is always observed to be fresh water, for the small amount of sea water carried up, when mixed with so much rain water, is not perceptible. That water-spouts do draw up water we know from observation, from which it appears that ponds are sometimes drained, and the fish, frogs, &c., transported to a considerable distance.

From a comparison of v with P' in table, § 102, at the different distances from the center of a tornado or water-spout, it is seen that the barometric pressures are very little affected except within the limits of destructive violence, where observations are not conveniently made, and hence its effect upon the barometer is rarely observed.

115. With the co-ordinates r and h of No. 3 in table, § 102, we get the form of spout represented in Fig. 15, and with those of No. 4 in the same table we get the form of Fig. 16. The former is not very tall because the dew-point of the air does not differ much from its temperature, and the base of the spout is not very large on account of the smallness of the assumed gyratory velocity v_0. In the latter the spout is very high because of the great difference between the dew-point and temperature of the air at the earth's surface. It consequently requires a very great gyratory velocity to bring down a spout with even so small base from so great a height. By assuming different temperature, dew-points, and gyratory velocities, we can obtain spouts of a great variety of forms—from the tornado cloud, with its base covering a considerable area, to a very tall, slender pillar of only one or two meters in diameter.

With regard to the observed dimensions of water-spouts, Horner says that their diameters

* Tornado of West Cambridge, August 22, 1851, by Charles Brooks. † Silliman's Journal, vol. iv, p. 362.

range from 2 to 200 feet and their heights from 30 to 1,500 feet. Dr. Reye states that their diameters on land at base are sometimes more than 1,000 feet. These correspond with our tornado cloud, Fig. 13, and consequently they have little of the appearance of a spout unless they are tall. Oersted puts the usual height of water-spouts from 1,500 to 2,000 feet, but states that in some rare cases they cannot be much less than 5,000 or 6,000 feet. Of course all such estimates are very vague, and may be very much in error. With a high temperature and very low dew-point, differing from the temperature of the air about 16°, a water-spout one mile in height would be obtained by the formula, but spouts of this height probably occur very rarely.

116. The Journal de Physique, vol. 88, contains accounts of a great many water-spouts, with remarks by M. Defranc, from which the following items are extracted:

"On the 23d of June, 1764, a water-spout was seen on the Seine, which had its base on the river and reached up into the clouds. It was judged to be about three feet in diameter at the cloud and still less where it touched the river. There were some parts transparent, which allowed the ascension of the water to be seen. It finally broke at about one-third of its height. The lower part fell in rain, the upper part was drawn up into the cloud in a second of time, and the phenomenon was followed by hail."

This spout is remarkable for the smallness of its diameter. There was, no doubt, water carried up in it from the river, but what appears to be water ascending in water-spouts may be merely the condensed vapor of the atmosphere in the form of cloud or mist. The lower part, without much doubt, had water in it which fell back into the river when the spout broke. Water-spouts are often observed to drop down from the cloud in an incredibly short instant of time and to be drawn up again in the same manner. When the gyrations are such as to not quite reduce the tension and temperature in the center, so as to condense the aqueous vapor, a very slight increase at once reduces the temperature sufficiently and the spout becomes visible from top to bottom almost instantaneously. Just the reverse of this takes place when the spout breaks, and it appears to be drawn up instantly. This is especially the case in small spouts like this one. The hail indicates that the ascending currents of this spout extended up into the region of freezing temperature, but most probably the hail was not formed from the water of the river.

"On May 17, 1763, Captain Cook saw six water-spouts upon Queen Charlotte Sound. In one of these a bird was seen and in arising was drawn in by force and turned around like a spit. Their first appearance was indicated by a violent agitation and elevation of the water. When the tube was first formed or became visible its apparent diameter increased. It then diminished and became invisible at its lower extremity."

The observation of the bird's being drawn in is important in showing that there is a draught and an inflowing of air from all sides to supply the ascending current. The observation also shows that the air has a gyratory motion around the center. At first the gyrations are gradually increased, and after the spout first appears as long as the gyrations are increasing the spout, by theory, must increase in diameter, but the reverse when the gyrations begin to diminish. The violent agitation and heaping up of the water before the spout appears, shows that the gyrations and barometric depression in the center exist before the spout becomes visible, and that the spout only appears after the diminution of tension and temperature necessary to condense the vapor occurs.

"On the 12th of July, 1782, there was a water-spout near the Island of Cuba. The base appeared to occupy the space of four toises, the base of the column four feet, its middle ten feet, and the upper part, in enlarging, formed the cloud. Many discharges of a cannon interrupted the course of the water from the sea, which was elevated in rapid spirals and separated from the base."

This likewise seems to have been a spout of small diameter, but of the usual tapering form toward the bottom and a rapid widening above to form the base of the cloud. That the discharge of cannon may tend to break up water-spouts of small dimensions, is not to be regarded as improbable and entirely imaginary. In such a spout the rapidity of the gyrations, by theory, extends to a very short distance from the center, and the whole comprises a mere column of the air of small diameter, and that the great agitation or oscillating motion of the air arising from the discharge of cannon should sensibly affect the moments of gyration and gyratory velocities, and thus break up the spout, is not at all unreasonable to suppose.

M. Defranc remarks that "he never saw a water-spout before 10 o'clock in the morning nor

after 5 o'clock in the evening"; that "they never appear during the night nor during the winter," and that "there are always two circumstances attending them. The first is the presence of the sun during, or a little before, the phenomenon. The second is the absence of the wind, or only a very feeble one, except always in the space occupied by the water-spout."

Tornadoes and water-spouts originate only in an unstable state of equilibrium of the air, which requires an unusually rapid decrease of temperature with increase of altitude. This can take place only when the strata nearest the earth are unusually heated, and this never occurs in the night or during the winter, and but rarely in cloudy weather. Tornadoes and water-spouts, therefore, take place mostly, if not always, in the summer season and during the day, when it has been clear at least a short time before their occurrence. If any agitation of the air, such as that arising from the discharge of a cannon, tends to break up these meteors, then any considerable disturbance of the air from any cause must tend to prevent their formation, and hence they do not occur when there is more than a feeble wind. This, however, does not apply to land-spouts which are formed in the interior of a tornado, having their origin in a state of unstable equilibrium in the cloud region, and commence there, but rather to water-spouts on seas and lakes, originating in a state of unstable equilibrium of the lower unsaturated strata of the atmosphere.

117. According to theory, without friction there would be a slender thread of spout at the center in all cases after the gyrations had fairly set in and been made to conform to the law of (*d*), but on account of the effect of friction near the center, where the gyratory velocities become enormously great, these velocities are very much diminished there, and do not conform to the law of velocities inversely as the distance from the center. The spout, therefore, is not always brought down to the earth, but the base of the cloud is simply brought down a little in the center in the shape of a funnel, as represented in Fig. 17. This funnel-shaped cloud is to be seen at the first formation of a spout, before fully formed, and also at the end after the spout is broken up, and is often observed in land tornadoes in which the whirl first commences up in the cloud. In this latter case the spout may not be brought to the earth at all by the gyrations, or these gyrations may so increase and extend themselves as to draw the cloud down over a considerable area of the earth's surface, as represented in Fig. 13, and then the cloud does not have the appearance of a spout.

Water-spouts at sea, first appearing some distance off, can generally be avoided or, if not, preparations can be made to guard against the danger; but where they are suddenly formed on the spot, and drop down, as it were, from above, they give little forewarning of approaching danger. This is exemplified in a case of recent occurrence. The British bark Bel Stuart, Captain Harper, on the evening of November 14, 1878, one hundred and sixty miles off Cape Sable, "was struck by a white squall in a comparatively smooth sea and clear sky, which swept her decks and created consternation on board. At 6 p. m. of the same day, all hands being on deck after supper, a strange sighing of the wind was observed by the watch, and the sky became suddenly threatening, without a corresponding indication of the barometer, which showed a rising tendency. Captain Harper and his first officer were on the deck at the time. All hands noticed the peculiar change in sea and sky and were discussing it, when, without a moment's notice, the sea forward seemed to swell up to meet the lowering sky and swept the bark across her bows, carrying away her foretopmast, foretopgallant-mast, jib, jib-boom, foretopmast-stays, and the maintopgallant-mast, with all their accompanying sails. In a moment, as it seemed, the bark, with all sail set and in a fair wind, with a moderate sea, was left a comparative wreck to wallow in the trough of the tremendous seas which had followed the spiral volume of water. Two minutes before the fatal catastrophe Captain Harper says there was no indication of the water-spout." (N. Y. Herald, December 10, 1878.)

It seems from this account that the bark ran into the spout as it was being formed and before it became visible. The sighing of the wind was caused by the rapid gyrations of the air, and the threatening sky by the incipient condensation of the aqueous vapor carried up by the ascending current. The barometer remained unaffected because, as we have seen, the barometric pressure is diminished only at and very near the center, and the observed rising of the sea was caused by this depression, as has been explained in the theory of water-spouts. The injurious effects prove the great rapidity of the gyrations near the center given by theory.

118. When we have the conditions of a water-spout in fair weather with little moisture in the air, we have what are called *white squalls* or *fair-weather whirlwinds*. In such cases the dew-point

is so low, and consequently the cloud, when formed, is so high, that the gyrations may not be able to bring it down, in the form of a spout, to the sea. But still the gyrations and the rapidly ascending current in the central part are there, and the rising and boiling of the sea. High up in the air, also, directly over the boiling of the sea below, is a patch of white cloud, formed by the condensation of the vapor in the ascending current when it arrives at the height at which it begins to be condensed. This cloud may eventually extend over a considerable portion of the heavens, but at first it is a small cloud in a clear sky, as represented in Fig. 17, and is white because of the great amount of reflected light. It has, no doubt, the funnel shape beneath, but being high up, and the observer being generally nearly under it, this feature is not generally observed. If the air be not too dry it may be followed by a shower of rain.

Peltier says: "White squalls are very rare, but they are sometimes met with between the tropics, especially near elevated lands; they are generally violent and of short duration. They often take place when the sky is clear and without any atmospheric circumstances giving notice of their approach. The only thing which indicates their proximity is the boiling of the sea, which is very much agitated by the violence of the winds. Many of these squalls, which commence either by a little cloud, or even without any visible cloud, are soon accompanied by violent rains and thick clouds."

The *bull's-eye squalls* on the west coast of Africa are of precisely the same nature. According to Piddington, the Portuguese describe these squalls as "first appearing like a bright white spot at or near the zenith, in a perfectly clear sky and fine weather, and which, rapidly descending, brings with it a furious white squall or tornado."

By comparing these descriptions with the account of the water-spout in the preceding section, it is readily seen that all these squalls are precisely the same as water-spouts, except that the air is too dry, and the gyrations not quite sufficient to bring the cloud down in the form of a spout. They may occur almost anywhere, but are much more frequently met with in some parts of the earth than in others. They are very common on and near the west coast of Northern Africa. This is because the atmosphere there is frequently in a state of unstable equilibrium. The tendency of the lower strata of the atmosphere there, as in the trade-wind region generally, is toward the west, except the surface sea-breeze currents in summer. This is proved by the immense amount of dust which is carried out from the continent over the ocean, so that ships and the whole surface of the sea at times become covered with it. This air, coming from the warm and dry deserts of Africa, is very warm. In the upper strata the cold ocean air from the west overlaps this warm air below, and thus is produced a very rapid diminution of temperature with increase of altitude, and consequently a state of unstable equilibrium, which is one of the conditions of a tornado or water-spout. The other condition of initial gyratory motion, as explained in § 91, can rarely be entirely wanting. Hence these squalls abound here, but not generally in connection with water-spouts, because the dry air from the continent contains too little moisture.

119. In very hot, dry climates, where there is a sandy soil, we have *sand-spouts*, or pillars of sand. The dry air in such climates is frequently in the state of unstable equilibrium, and there are all the conditions of a water-spout except the vapor to condense. The gyrations of the air and the ascending currents are the same as in the water-spout, but instead of carrying up vapor, they collect the sand or dust from the vicinity and carry it up. The inflowing current from all sides toward the vortex, up to a considerable height often, keeps it near the center. According to (q), however, there is a distance from the center where the force arising from the inflowing current acting upon the particles of sand, and the centrifugal force of the gyrations, are in equilibrium, and this distance depends upon the diameter D of the sand particle, unless we suppose that the velocity u of the inflowing current toward the center is inversely as $r^{\frac{1}{2}}$, which is impossible, for this would make u infinite at the center, whereas we know that it must vanish there. We know that u' must be such a function of r that it will increase from the center, and also that it cannot increase as rapidly as the first power of r, for that would require the ascending current to be the same at all distances from the center, but we know it must be greatest at the center. Let us therefore put $u' = cr^{\frac{1}{2}}$ between these extremes. With this expression (q) becomes

$$cr^2 = 3.06\, r_o v_o \sqrt{\frac{DS}{8}}$$

This shows that for different values of D there is a corresponding value of r, and hence the sand-particles are arranged at different distances from the center, according to their size, the largest ones being farthest off. On account of the unknown value of c in any case we cannot determine this value of r, but we know each size of particles must have its status at some certain distance from the center. The same result would be obtained with either extreme of $u' = c$, or $u' = cr$, except that the status of the particles would not be precisely at the same distance.

If there was no limit to the size of the sand particles there would be no limit to the distance from the center at which they would gyrate, provided the ascending current were sufficient to carry them up, but with a limit to their size there would be a limit to this distance, and this would determine the radius of the sand-spout or pillar of sand. But where there is no limit to the size of the particles, and they are of all sizes from the smallest up, even in this case there is a condition which determines the size of the spout, and that is that the ascending current must be strong enough to keep them suspended in the air. As the largest particles must gyrate at the greatest distances from the center, there is a certain distance at which the ascending current is not sufficient to sustain these larger particles, and this determines the limit of the spout, for beyond this distance the larger particles cannot be sustained and the smaller ones are driven nearer the center, where the ascending current keeps them from falling.

The height of the spout in this case depends upon the height at which the ascending currents incline outward from the center, for of course it is there carried away and dispersed. This height depends upon various circumstances. Many of these pillars of sand are observed to be very tall. As the air in the interior is collected mostly from very near the earth's surface, it is often very hot. They generally have a progressive motion with the prevailing general current of the wind.

120. Water-spouts and sand-spouts are *hollow*. By the equation in the preceding section, there must be a limit within which water or sand particles cannot enter on account of the centrifugal force of the gyrations, unless we suppose the smallest of these particles are infinitely small. And since r diminishes as the fourth root of D, even in that case there would be sensibly a vacuum of particles, if we suppose the sizes to be equally distributed. In the case of the water-spout the tension and temperature diminish very rapidly toward the center, so that before the vapor arrives near the center it is nearly all condensed, and the centrifugal force tends all the time to drive the water or cloud particles away from the center, even beyond the distance where condensation first takes place, as represented in the figures of the water-spouts. But as the ascending current is very rapid, and the spout is larger above, the particles are perhaps always kept within that limit, and the spout is not enlarged by the driving of the particles away from the center, beyond the limit where condensation first takes place. In the case of both water-spouts and sand-spouts, therefore, if there is not an absolute vacuum of particles in the central part, the particles are so exceedingly rare there that it must be regarded as being sensibly a vacuum.

The circumstances under which observations of such a vacuum can be made are very rare. M. Boué, in the year 1850, observed three water-spouts at the same time on Lake Janina, from the top of a high mountain. The weather was entirely clear, without clouds or wind, but very oppressive and hot. The spouts seemed to rise up from the lake, and he could look down into the top of them and see that they were hollow in the middle. (Bulletin Soc. Géologique de France, t. viii, p. 274.)

Of a whirlwind observed at Schell City, Mo., in the summer of 1879, Professor Nipher says: "There were no surface winds strong enough to bear dust along the surface of the ground, but the dust carried up in the vortex was collected only at the vortex of the whirl. The dust column was about two hundred feet high, and perhaps about thirty or forty feet in diameter at the top. The direction of rotation was the same as of storms in the northern hemisphere. Leaving the road the whirl passed out on the prairie, immediately filling the air with hay, which was carried up in somewhat wider spirals, the diameter of the cone thus filled with hay being about 150 feet at top. It was then observed also that the dust column was *hollow*. Standing nearly under it, the bottom of the dust column appeared like an annulus of dust surrounding a circular area of perfectly clear air. The area grew larger as the dust was raised higher, being about fifteen or twenty feet wide when it was last observed." (Nature, Sept. 11, 1879.)

The water-spout connected with the great tornado of April 18, 1880, at Marshfield, Mo., seems also to have been hollow. In this case the cloud was high, and, as frequently happens in land

tornadoes, the spout was not brought down to the ground, but simply appeared as a funnel-shaped cloud. The following testimony is taken from an account of this tornado given by Professor Nipher: "Mr. W. R. Steel, who heard it from a distance of half a mile, heard it roaring distinctly; the cloud was high in air and was cone-shaped. The bottom was, however, high in air and was entirely disconnected from the dust-whirl at the ground. The bottom of the funnel seemed to sway somewhat as well as to move up and down. It looked like dark smoke, and he could occasionally see up into the bottom of the funnel, which seemed to be hollow. The inside seemed to be somewhat lighter colored than the outside. He saw also the effects of wind at the surface of the ground, branches being twisted from the trees. This evidence was given to Professor Shepard without any leading questions being asked." (Missouri Republican, May 14, 1880.)

121. The wind is often observed to blow in *blasts*, with an oscillating vane and unsteady barometer. This arises from the air running into numerous whirls, or gyrations, while it at the same time has a progressive motion. These are especially liable to occur in connection with general cyclones extending over a considerable area, for then, especially in the cloud region, the air is often in a state of unstable equilibrium, and on account of the gyrations of the cyclone the moments of gyration, with regard to any point in the cyclone where there may be a rushing up of the air of the lower strata through the upper ones, cannot generally be naught, and hence we have both conditions which are required for the formation of these whirls in the atmosphere. These may form small secondary cyclones, or tornadoes and water-spouts, or simply a slight whirling in the atmosphere of not much violence. The progressive motion of the air in which these gyrations occur may be, and is mostly, that of the larger cyclone in which the small gyrations occur.

When the directions of the gyratory and progressive motions coincide, the resultant velocity becomes unusually strong, and we have a strong blast. The area of the whirl passes on and we have the average velocity, or it may be succeeded immediately by the side of another whirl in which the gyratory motion is in the contrary direction, and the average velocity then is very much diminished, or if the gyratory and progressive velocities are nearly the same, there will be sensibly a perfect calm for several moments. It is readily seen, also, that with the passing of each one of these whirls, or gyrations, the resultant direction of wind must at the same time change with the passing of different portions of the whirl, and this causes an oscillation of the wind vane. If the gyratory velocity is greater than the progressive, it may be whirled entirely around. These blasts and oscillations of the vane are especially observable on the clearing-up side of a storm. This is the clear cold side, and as the upper part of this is carried forward eastward faster than the lower, it overlaps the more central warmer part of the air below before it has time to cool down by radiation, and thus there is produced a more rapid diminution of temperature with increase of altitude, and a state of unstable equilibrium in which these gyrations of the air readily originate. It is similar to what takes place on the west coast of Africa, as explained in § 118, giving rise to the bull's-eye squalls. As a patch of white cloud is seen vertically over the place of each one of these squalls, so on the clearing-up side of a storm a cumulus cloud, white if the sky generally is clear, indicates the existence of one of these gyrations of the air which give rise to blasts and oscillations of the wind vane. The cloud, generally high up because there is not much vapor remaining in the air, is produced by the condensation of the vapor in the ascending current in the central part of these whirls, which mostly are not of much violence, but sufficient to produce considerable variations in the velocity and direction of the wind.

122. The oscillations of the barometer, sometimes called "pumping," and which are known to be associated with gusts of wind, are not only accounted for by (j) or (k), but the exact amount of barometric oscillation, corresponding to a given change in the wind's velocity, can be computed. It will be remembered that this formula gives the effect of the small term in (f) depending upon the inertia of the air, which was neglected in the general theory of cyclones and tornadoes, but has to be considered where there are any sudden changes of velocity. By the formula (j) or (k) a change of velocity from 10 meters per second to a calm affects the barometer placed on the side of a wall or any barrier against which the wind blows, and where consequently it is in a calm, and the value $u_o = 0$, and produces a corresponding change in the pressure of 0.05^{mm}. A change from a velocity of 10 meters to 20 meters per second produces a change of 0.15^{mm}, and from a velocity of 20 meters to 30 meters a change in the pressure of 0.25^{mm}, the whole change being as the

squares of the velocities. From this it is seen that these small oscillations of the barometer, for the same changes of velocity, are much greater for large progressive velocities than for small ones, and hence this unsteadiness of the barometer becomes particularly observable on the approach of and during storms.

These oscillations in northern latitudes rarely exceed 0.02 in.* (0.5^{mm}). With a progressive velocity of 20 meters per second, changed by a blast to 36 meters, there would be a change in the barometer of 0.5^{mm}, and the same with a change from 30 to about 42 meters per second. These are about the greatest changes which occur in the velocities of the wind in gusts or blasts, and hence the oscillations rarely exceed 0.5^{mm}. Much, however, depends upon the position of the barometer. This must be placed where the air is entirely obstructed, and the velocity reduced to naught. If placed in the current there is no change produced by a change of the velocity. On the windward side of a barrier there is a slight rise of the barometer with an increase of the wind's velocity, but on the lee side there is a slight decrease due to the "drag" of the air. A barometer placed in an air-tight room of course would not be affected, and perhaps not sensibly in any room with doors and windows closed, especially where the blasts are sudden and of short duration.

In the hurricanes of the Antilles observation shows that these small oscillations of the barometer are closely connected with and dependent upon the blasts of the wind, and that oscillations of the vane always accompany the blasts, showing that the latter are due to small gyrations of the air. The amplitudes of the oscillations here seem to be, in general, a little greater than those given above, arising perhaps from the greater violence of the winds generally in these hurricanes. "Under the influence of the blasts the barometric column is so agitated and so irregular that it renders the reading of it very difficult, since it is scarcely possible to take an exact medium. The amplitude of the oscillations is usually from four to eight-tenths of a millimeter, and sometimes more. The agitation is fitful and violent, just as the impulses which the anemometer receives and the oscillations made by the vane." (Apuntes, &c., p. 181.)

123. A considerable rise of the barometer, amounting sometimes to several millimeters, is often observed during a sudden squall, accompanied by an abundance of rain or hail. The rise in such cases depends mostly upon a sudden condensation of a great amount of aqueous vapor into rain where there is a rapid ascent of saturated air, for this rain, so far as it is retarded in its fall by the resistance of the air, adds to the pressure of the air at the earth's surface. The rain or hail in such cases usually falls from a great height, and as soon as the drops in falling arrive at their maximum velocity, where the *resistance is equal to their weight*, the pressure of the air is increased by the whole weight of the rain in the air above. Of course there has to be a barometric depression to give rise to the ascending currents, but when the rain is rapidly generated, the accumulated weight of it soon amounts to more than the barometric depression, and causes, for a short time, a rise of the barometer. When the ascending currents also are very rapid, we have seen that the rain and hail are carried out from the center where there is a barometric depression, and falls and adds its weight to the air where there is no barometric depression.

In a tornado at New Harmony, Ind., April 30, 1852, an account of which has been given by Chappelsmith (Silliman's Journal, vol. xxiii, p. 17), the rise of the barometer for a short time was nearly one-tenth of an inch, and this was accompanied by a great fall of rain and hail. For several days before there was a gradual fall of the barometer amounting to four or five tenths of an inch, and a gradual rise again after the tornado, so that the tornado seems to have been at or near the center of a cyclone of considerable extent, but the small rise of the barometer seems to have been very sudden at the commencement of the shower, and the fall back again to about the same level at the end of the shower.

Two very sudden storms of this sort were experienced by the submarine cable expedition in the Persian Gulf on the 1st and 2d of November, 1869. (Quarterly Journal Met. Society, vol. i, p. 117.) In both cases the violence of the storm continued about two hours, while torrents of rain swept the decks of the vessels employed in laying the telegraph cable. It is stated that "the barometer remained on both occasions unaffected up to the last moment; but as soon as the storm arrived it rose about two-tenths of an inch, and fell again when it passed over, thus showing that the propel-

* See papers on certain small oscillations of the barometer, by Hon. Ralph Abercrombie, F. M. S. Quarterly Journal Met. Soc., vol. ii, p. 435. Also, p. 450.

ling power was pressure from behind, produced by the weight of the falling rain or some other cause, and not vacuum in front, as in ordinary storms." There is nothing in the theory of storms to account for such a rise of the barometer, except upon the hypothesis that it is caused by the weight of rain; but the theory requires that there should be a small barometric depression somewhere in the vicinity, at least in the initial stages of the storm, to give rise to the ascending currents upon which the rain depends. That no such depression was observed only shows that the observer and the rain-fall were not precisely where the depression was, but only in its vicinity, for from the results of the table, § 102, it is seen that within the limits of considerable violence of the wind there is bu little barometric depression when there is no rain, and this may be surpassed by the increase o pressure from rain where the rain falls, unless it should be exactly at the center, where in violent storms it has been shown the rain cannot fall.

That the rise of barometer during heavy rain-falls is due to the weight of rain is evident from comparisons of rain-fall with the curve of the barogram, from which it is seen that heavy rain-falls coincide with these small sudden rises of the barometer, as has been shown by Hon. Ralph Abercrombie in the paper already referred to.

END OF THE SECOND PART.

ERRATA IN PART I.

In § 8, equation (10), for "0.3663" read "0.003663."
In § 9, equation (14), for "α" read "α'."
In § 13, for "the year 1860" read "June, 1859."
In § 35, equation (40), for "sin θ" read "sin $^2\theta$."
In § 42, for "0.00000032775" read "0.0000032775," and for "0.0035^{mm}" read "0.035^{mm}."

APPENDIX No. 11.

DISCUSSION OF TIDES IN PENOBSCOT BAY, MAINE, BY WILLIAM FERREL.

U. S. COAST AND GEODETIC SURVEY OFFICE,
December 31, 1878.

SIR: I have the honor to submit the following results of the harmonic analysis and discussion of the tide observations made in Pulpit Cove, on North Haven Island, in Penobscot Bay, Maine.

This site was selected by the Superintendent as best fulfilling the conditions for favorably exhibiting the characteristics of the tides in the Gulf of Maine, being as near to deep water and as secure from storms as could be found on the coast of Maine.

Sheltered from the immediate effect of great storms, the Pulpit Cove Tide Station, situated in lat. 44° 09′, long. 68° 53′, lies within 25 miles of the deep water of the gulf, with a free access of the tide wave through not less than 25 fathoms, and distant only 1¼ miles from the limit of 30 fathoms, in the deep basin of Penobscot Bay. Through that distance the depth gradually diminishes without any bar to 3 fathoms, at low water, within about 100 yards from the tide-gauge. (See accompanying illustration.) Since the range of the tide here is nearly ten feet, the sectional area of the inlet near the gauge is at low water about four-fifths of what it is at high water.

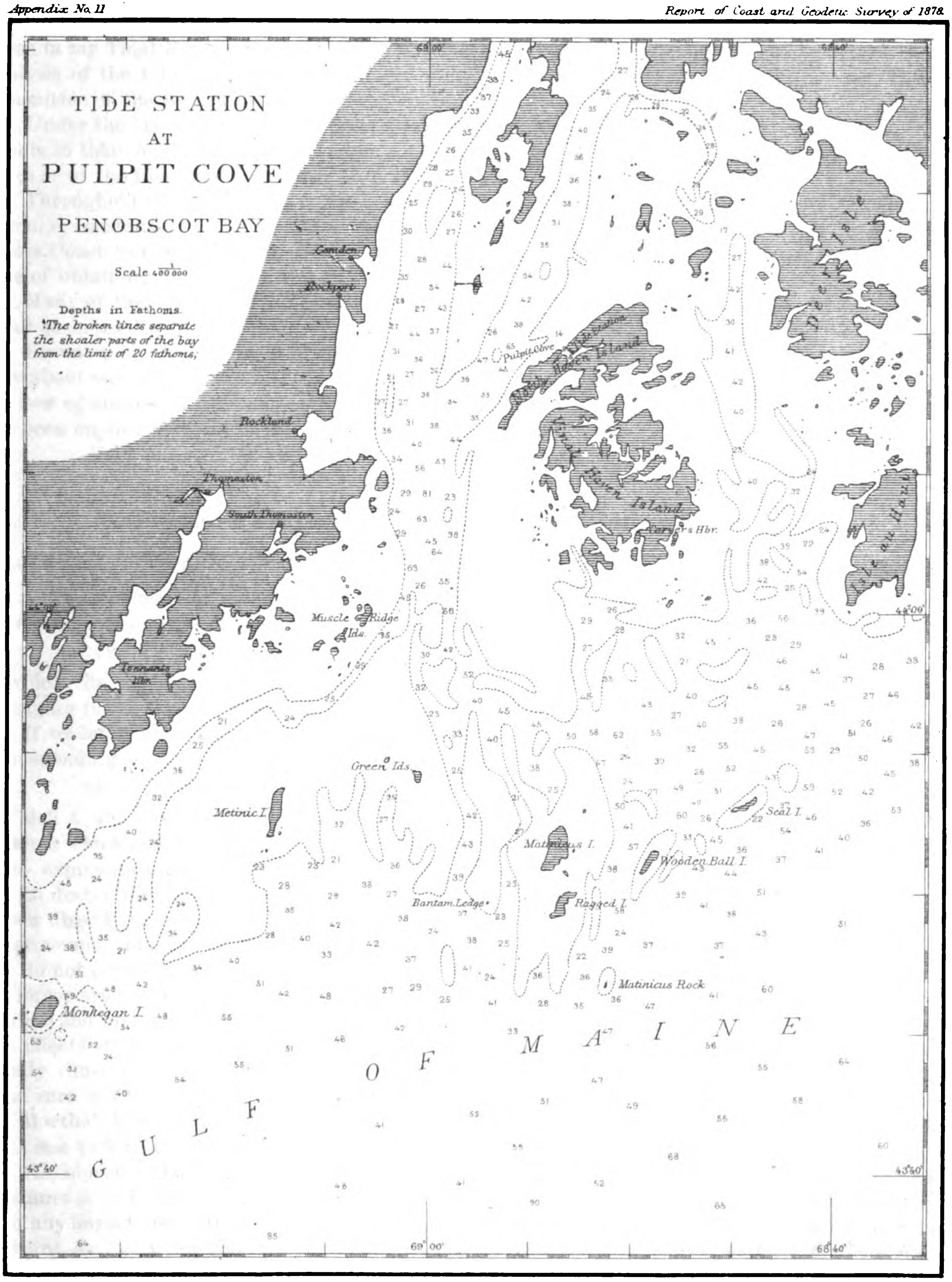
The observations used in this analysis were commenced January 22, 1870, and extend to the end of the year 1875, embracing a series of nearly six years. These were made by means of a self-registering tide-gauge, from the curves of which the hourly co-ordinates were measured and recorded, and the records were furnished me by Mr. R. S. Avery, chief of the Tidal Division. The series is wonderfully perfect, every hourly co-ordinate being given for the whole of the six years, and it rarely happened that a co-ordinate was given by interpolation or otherwise, as is indicated by a differently colored ink, and not from actual measurements of the curves. The observations appear to be entirely reliable.

The principles and general plan adopted in analyzing these tides are the same mainly as those adopted by the Tidal Committee of the British Association, of which Sir William Thomson is chairman, and used in analyzing the tides of Liverpool, Portland Breakwater, and other places. There are, however, many variations in the details of carrying them out.

The preliminary investigations necessary for the commencement of the work, together with the results of the analysis, and their discussions and applications, are contained in the following pages under four general heads: I. The general principles of the harmonic analysis and discussion of tide observations. II. The results of the analysis of the tide observations of Pulpit Harbor. III. The comparison of the results of the analysis with theory. IV. Practical applications of the results.

Under the first head are given the object of the harmonic analysis, an explanation of the general principles, and an investigation and preparation of all the formulæ and tables necessary in carrying out the analysis and in reducing the results. There is also given a more thorough investigation of the subject of shallow-water tides than was given in my Tidal Researches, which seemed necessary in the study of the various tidal relations and the further improvement of the tidal theory. This preliminary part required much labor and study, and the results form the principal part of the following pages. This, however, is work now done, once for all, and need never be repeated, being needed, however, in future for constant reference, if the work is continued on this plan, but the work in any future analysis of the tides would be comparatively small after all this preliminary work has been done.

Under the second head are given the results of the analysis of the tides of Pulpit Cove, with all the necessary reductions required to make them convenient, either for theoretical investigation and study of the tidal theory, or for practical purposes.


TIDE STATION
AT
PULPIT COVE
PENOBSCOT BAY
Depths in Fathoms.
The broken lines separate the shoaler parts of the bay from the limit of 20 fathoms.
Camden
Rockport
Pulpit Cove
Rockland
Thomaston
South Thomaston
Muscle Ridge Ids.
Green Ids.
Metinic I.
Matinicus I.
Wooden Ball I.
Seal I.
Ragged I.
Bantam Ledge
Matinicus Rock
Monhegan I.
Carvers Hbr.
G U L F O F M A I N E
69° 00'
68° 40'
43° 40'

Under the third head is contained a comparison of the results obtained from the analysis of the tide observations of Pulpit Cove, with the various theoretical relations and tidal expressions given in my Tidal Researches, in the same manner that I compared in that work the results of the analysis of the tides of Liverpool, Portland Breakwater, Kurrachee, &c., obtained by the Tidal Committee of the British Association.

Under the last head are given directions and formulæ for making practical applications of the results in tidal predictions or the computations of tidal ephemerides, either of the hourly co-ordinates or of the times and heights of high and low water.

Throughout the whole work it has been necessary to make references frequently for principles, formulæ, constants, &c., to my Tidal Researches, published as a separate Appendix to the United States Coast Survey Report of 1874. It would have been impossible to go over the whole work here of obtaining them again, and they are to be found nowhere else.

Many of them depend upon complex developments, and in revising these I have found a very small term used with the wrong sign, which affects the constants of the two nodal lunar terms of the diurnal tide. The reversal of this term causes an error which in the Atlantic tides amounts to only about one-eighth of an inch, and its correction is only important inasmuch as it affects slightly my new equations for determining the moon's mass from the diurnal tide. This small correction has been applied here to the two constants affected.

Very respectfully, yours,

WM. FERREL.

C. P. PATTERSON,

Superintendent United States Coast and Geodetic Survey.

I.—GENERAL PRINCIPLES OF THE HARMONIC ANALYSIS AND DISCUSSION OF TIDE OBSERVATIONS

1. The potential of the tidal-disturbing forces of the moon and sun, denoted by V, may be developed into a series of harmonic terms, which may be expressed by

$$V = \Sigma_n P_n \cos (i_n t + c_n) \tag{1}$$

in which the angle increases in proportion to the time t, and in which P_n, i_n, and c_n are constants belonging to the term of which the characteristic is n.

If we let H represent the height of the tide above some assumed horizontal plane, we have a corresponding similar tidal expression.

$$H = \Sigma_n A_n \cos (i_n t + c_n - \epsilon_n) \tag{2}$$

in which A_n and ϵ_n, called amplitudes and epochs, are constants not found in the expression of V, but between which and the constants P_n and i_n, respectively, there are certain theoretical relations. In these expressions a constant of integration is implied, which, in the latter, may be denoted by A_0.

In deep-water tides, for each term in H, there is a corresponding term in V, upon which the former directly depends, but in shallow waters there is a numerous class of terms, called shallow-water components, for which there are no corresponding terms in the tidal forces, and which therefore do not depend directly upon these forces. While the sensible terms depending directly upon the forces are mostly only diurnal and semidiurnal components, there are often quite large quarter-diurnal, and also one-sixth diurnal, and even one-eighth diurnal components in shallow-water tides. The effects of the shallow-water components are especially observable in river tides, where they usually cause great distortions in the form of the regular semidiurnal tide of deep waters. In some rare cases, where the semidiurnal tide is very small small, as in the vicinity of Nantucket and Martha's Vineyard, the lunar quarterdiurnal shallow-water component is the principal tide, and gives rise to four maxima and four minima in 24 hours.

The object of the harmonic analysis of the tides is to determine from observation the unknown constants A and ϵ for each of the numerous terms in the expression of H, which are supposed to be of any importance. If it were not for the abnormal disturbances of winds, changes in barometric pressure, &c., very few observations would suffice for this purpose, but on account of these disturbances it is necessary to use the averages of a sufficient number of observations to eliminate all sensible effects of these disturbances, which, when great accuracy is aimed at, requires the observations of a series of years.

2. The terms in the expression of H (2) may be divided into classes each of which may, in

general, contain several sensible terms, called diurnal, semidiurnal, &c., components, since the period of the first one, either real or only supposed, differs but little from that of a solar day. If we let e denote the characteristic of the class, and s that of any term in the class, we can put the expression of H into the following form:

$$H = \Sigma_{(e,s)} A_{(e,s)} \cos (s i_e t + s c_e - \varepsilon_{(e,s)}) \tag{3}$$

in which i_e and c_e have the values of i_n and c_n respectively in (2) belonging to the diurnal component.

The principal class of terms in this expression is that which depends upon the *mean moon*, that is, upon a moon moving uniformly in the equator at the same distance from the earth, giving rise to a semidiurnal tide, and a very small terdiurnal tide, and in shallow waters to several shallow-water components. The next most important class is that which depends in the same way upon what may be called the *mean sun*, giving rise in deep waters to a semidiurnal tide only, but in shallow waters to quarterdiurnal, &c., tides.

As each of the other classes of terms in H depends upon terms in the expression of V in (1) similar to that of the mean moon, each one may be said to depend upon a fictitious moon, differing merely from the mean moon in magnitude and the apparent velocity with which it moves around the earth.

If we put M_e as a generic expression of these moons and let $e = 1$ be the characteristic of the class of components depending upon the mean moon, then this class may be denoted by the M_1-tides or components. In like manner the class depending upon the mean sun may be called the M_2-tides, and so on for all the others. The angle $i_e t + c_e$ in (3) is the hour angle of the moon M_e, c_e being its value at the time $t = o$, which may be assumed at pleasure. The constant ε is usually put with the negative sign in tidal expressions, so that its value divided by si gives the time by which the maximum of the tide follows the passage of the fictitious moon over the meridian.

3. In the harmonic analysis of the tides it is necessary to know, at the outset, the values of i_e and also those of c_e in (3) for each of the three components of which we wish to determine by observation the amplitudes and epochs. In the subsequent discussions of the results and comparisons with theory it is also necessary to know the values of the constants P_n in (1), or at least their relative values, since there are certain theoretical relations between the constants P_n and the amplitudes A_n, and likewise between E_n and i_n. These constants are only obtainable from a complete development of the function V, which is found in the second chapter of my Tidal Researches, in which the constants in the following schedule are either found, or quantities from which they are easily deducible.

SCHEDULE I.

Class of tides.	Values of P_n. Diurnal components.	Values of P_n. Semidiurnal components.	i_e	c_e	Description of components.
			°		
M_1 or M...	....	1.0000	14.492052	$\lambda' - \lambda$	Mean lunar.
M'_1	....	0.0359	14.489846	$\frac{1}{2}(\omega + \pi) - (\lambda - \lambda')$.	Lunar nodal.
m_1........	.052	....	14.496694	$\lambda' - \lambda + \bar{\omega} - \frac{1}{2}\pi$....	Lunar elliptic and declinational.
n_1........	.011	....	14.487410	$\lambda' - \lambda - \bar{\omega} + \frac{1}{2}\pi$....	
M_2 or S..	....	$0.4582 - 36.2\ \delta\mu$	15.000000	0..................	Mean solar.
M_3 or K..	$.5306 - 13.1\ \delta\mu$	$0.1256 - 3.2\ \delta\mu$	15.041069	λ'..................	Lunar and solar declinational.
M'_3	.071	0.0359	15.043275	$\lambda' - \omega - \frac{1}{2}\pi$........	Lunar nodal.
M_4 or L..	....	0.0286	14.764239	$\lambda' - \lambda + \frac{1}{2}v_1 + \frac{1}{2}\pi$..	Lunar elliptic.
M_5 or N..	....	0.1922	14.219865	$\lambda' - \lambda - \frac{1}{2}v_1$.......	
M_6 or θ..	.3813	....	13.943036	$\lambda' - 2\lambda + \frac{1}{2}\pi$......	Lunar declinational.
M'_6	.084	....	13.940824	$\lambda' - 2\lambda + \omega + \frac{1}{2}\pi$..	Lunar nodal.
M_7 or P..	$.1730 - 13.6\ \delta\mu$	....	14.958932	$-\lambda + \frac{3}{2}\pi$.........	Solar declinational.
M_8 or λ..	....	0.0078	14.727813	$-\frac{1}{2}v_1 + \frac{1}{2}\pi$.......	Evection.
M_9 or ν..	....	0.0375	14.256291	$\frac{3}{2}v_1 + 2(\lambda' - \lambda)$....	
M_{10} or μ..	....	0.0240	13.984104	$2(\lambda' - \lambda) + \frac{1}{2}\pi$....	Variation.
M_{11} or R..	....	0.0039	15.020534	$\frac{1}{2}v' + \pi$..........	Solar elliptic.
M_{12} or T..	....	0.0266	14.979466	$-\frac{1}{2}v'$............	
M_{13} or J..	.011	....	15.585443	$\lambda' + v_1 - \frac{1}{2}\pi$.......	Lunar elliptic and declinational.
M_{14} or Q..	.052	....	13.398661	$\lambda' - 2\lambda - v_1 + \frac{1}{2}\pi$..	

The first column of this schedule contains the notation adopted to designate the different classes of components, and, in connection, that used by the Tidal Committee of the British Association. The former corresponds with the general form of notation and is preferable for some reasons, but it will be more convenient in some cases to use the latter. For instance, the lunar quarterdiurnal component may be denoted by $M_{(1,4)}$, or by M_4, and the solar semidiurnal component by $M_{(2,2)}$, or by S_2, and so on, and hence the latter notation is most convenient in such cases, since there is only one subscript number.

There are several components introduced into the schedule which are not used in the analysis of the tides, and hence are not included in the regular notation, which is designed to be used merely in the analysis of the tides. These components, it will be observed from the corresponding values of i, have periods so nearly the same as those of other larger components that it is impossible to analyze them except with a long series of observations. In the analysis of a single year's observations the effect of the smaller components is to cause a change from year to year in the amplitudes and epochs of the larger components of nearly the same periods. In the discussions of the results of the analysis, however, and in comparisons with theory, it is necessary to have the constants belonging to these components, and hence they are included in the schedule.

The values of P_n are copied from page 44 of my Tidal Researches, the small correction being applied to the two lunar nodal diurnal components which has been referred to. The values of i_e, given in degrees per hour, have been obtained from the data given at the bottom of the next page in the work already referred to. The expressions of c_e are readily deducible from those of ut, §§ 28, 29, Tidal Researches, by subtracting $(\lambda - \lambda')$, the difference between the mean longitudes of the moon and sun, in the case of the diurnal components, and from the half of those expressions, in § 29, in the case of the semidiurnal components. Where the signs of P_n have been changed in copying, a corresponding change has been made by adding $\frac{1}{2}\pi$ to the expression of c_e, equal to π to $2c_e$. In the development of the forces for theoretical purposes it was necessary for these expressions to be arranged with reference to the mean moon, and the predominating tidal component, but in the analysis of the tides these expressions are necessarily required with reference to the mean sun. Hence the necessity for the reduction above. The following are the definitions of the quantities given in the expressions of c_e:

λ = the mean longitude of the moon;
λ' = the mean longitude of the sun;
v_1 = the mean anomaly of the moon;
v' = the mean anomaly of the sun;
ω = the longitude of the moon's ascending node;
$\tilde{\omega}$ = the longitude of the moon's perigee;
v_2 = the argument of variation.

The values of c_e in the expression of H in (3) depend upon the assumed epoch of $t = 0$, and the values of the quantities in the expression of c_e must be taken for that epoch. It must be understood here also that the expressions of c_e, given in the schedule, are those which are to be used when the epoch is assumed at noon, but if t, or the hours, are reckoned from midnight, π, or 180°, must be added.

4. In addition to the short-period components of the preceding schedule, there are also a few of comparatively long period, which may be sensible to observation, especially in high or low latitudes of the earth. The expression of these components in feet is (Tidal Researches, § 94)

$$Y_0 = 0.230\,(a - 3\cos^2\theta)\,\Sigma_e H_e \cos(u_e t + k_e) \qquad (4)$$

in which, with the constant k_e added, the origin of t is the same for all the components. We shall give here the constants of only the principal components, which will be all that we shall have occasion to use. They are

$H_2 = 0.163$, $u_2 = 13°.06500$, $k_2 = v_1 = -2c_8$, Lunar elliptic component.
$H_3 = 0.312$, $u_3 = 26°.35280$, $k_3 = 2\lambda = 2(c_3 - c_1)$, Lunar declination component.
$H_6 = 0.023$, $u_6 = 0°.98560$, $k_6 = v' = 2c_{11}$, Solar elliptic component.
$H_7 = 0.141$, $u_7 = 1°.97130$, $k_7 = 2\lambda' = 2c_3$, Solar declination component.

In the expression of Y_0 above, θ is the polar distance of the tide-station, and in the case in which the earth is supposed to be entirely covered by the ocean, the value of α becomes unity, but in the real case of nature it is generally a little different. The difference cannot be conveniently determined on account of the irregular outline of the continents, and the whole amount is generally too small to be of any importance.

The values of u are given in degrees per day, and the values of c in the expressions of k are the same as in the preceding schedule for short-period components, that is, equal to v, 2λ, &c., when $t = 0$.

5. Besides the tide-components in the preceding schedule, which depend directly upon the tidal forces, there is a very large class of components which do not depend so directly upon these forces, and for which there are no corresponding terms in the development of the forces having the same periods. These components are sensible to observation in very shallow waters only, and are hence called shallow-water components. These components are of so much importance in the theory of shallow-water tides that it is necessary to give here a fuller development of them than that given in my Tidal Researches.

In the fundamental tidal equations there are found in the equation of continuity two terms of the form $D_\theta(\gamma u)$ and $D_{\tilde\omega}(\gamma v)$ in which θ and $\tilde\omega$ are the polar distance and longitude, respectively, and u and v the motions in latitude and longitude, and γ is the depth of the water reckoned from the variable oscillating surface. If we put γ_o for the part of γ, independent of the vertical tide-oscillation and a for the vertical oscillation we get—

$$D_\theta(\gamma u) = D_\theta(\gamma_o u) + D_\theta(au)$$
$$D_{\tilde\omega}(\gamma v) = D_{\tilde\omega}(\gamma_o v) + D_{\tilde\omega}(av)$$

Laplace took into account only the mean depth, neglecting the part depending upon the vertical oscillation as being inconsiderable in comparison with the whole depth. His theory, therefore, is not strictly applicable to tides in shallow waters.

6. A first approximate solution of the tidal equations, in which the last terms of the preceding equations are neglected, would give expressions of a, and also of u and v, of the form of that of H in (2), and hence we can put—

$$(5) \qquad a = \Sigma_n A_n \cos(i_n t + c_n - \varepsilon_n)$$
$$u \text{ or } v = \Sigma_n B_n \cos(i_n t + c_n - \varepsilon_n + \varepsilon'_n)$$

in which the constants A and ε for each term are obtained from the analysis of the tide-observations, and in which E'_n is an unknown function of i_n. The products, therefore, of au and av introduce into the tidal equations a very numerous class of terms, which may be expressed by

$$R \cos(ut + c' + E' - q)$$

in which $R = A_n B_n$ where the term arises from the product of two components in (5) of different periods, but equal to $\frac{1}{2}AB$ where the two components have the same periods, and in which u, c' and q are either the sums or the differences of i_n, c_n, and ε_n respectively belonging to the two components multiplied into each other. The constant ε' is a function of ε'_n and consequently of i_n.

Each one of the terms of the preceding form in the tidal equations must give rise to a similar tide-component of the same period, which may be expressed by

$$(6) \qquad a' = CR \cos(ut + c' + E' - q - e)$$

in which C and e are functions of u, and hence vary in the different terms.

Since the components in (5) are either diurnal or semidiurnal, those in (6) are either components of long period, or are diurnal, semidiurnal, terdiurnal, or quarter-diurnal components, in each of which classes the values of u differ but little.

In each of these classes considered separately, the constants C, ε', and e are functions of the

same quantities, of which none of them vary from term to term, except i_n in E′ and u in C and e, and the variations of these are very small. We may, therefore, regard these constants as approximately constant for each class of components, and for such class (6) may be put into the form of

$$h = \mathrm{C\,R} \cos (ut + c' + \mathrm{E} - q) \tag{7}$$

in which C and E may be regarded as constant for all the terms.

Since $\mathrm{A}_n : \mathrm{B}_n$ in (5) are functions in which none of the quantities vary in the different components except i_n, and this varies very little in each of the classes, instead of putting $\mathrm{R} = \mathrm{A}_n \mathrm{B}_n$ or equal $\frac{1}{2}\mathrm{A}_n \mathrm{B}_n$, we can put it equal to $\mathrm{A}_n \mathrm{A}_n$ or $\frac{1}{2}\mathrm{A}_n \mathrm{A}_n$, in the various combinations of the n components, and the preceding expression (7) will still hold, except that the value of the unknown constant C will be different. We shall then have the values of R in terms of quantities known from observations.

7. From the three most important diurnal components in the expressions of (5), we get, from what precedes, two classes of shallow-water components, of the form of (7), the first arising from all the resultant terms in the products of the components in which the angles are the sums of the angles in the components, and hence semidiurnal components, and the other arising from those in which the angles are the differences of the angles in the components multiplied, and hence components of long period. We thus get the components in the following schedule, and the values of the constants R, u, c', and q in the expression of (7):

SCHEDULE II.

Designation of component.	R	Semidiurnal components.			Components of long periods.		
		u	c'	q	u	c'	q
$K_1\ K_1$	$\frac{1}{2}A_3^2$	$i_3 + i_3 = 2i_3$	$2c_3$	$2\epsilon_3$	$i_3 - i_3 = 0$	0	0
$K_1\ O_1$	$A_3\ A_6$	$i_3 + i_6 = 2i_1$	$c_3 + c_6$	$\epsilon_3 + \epsilon_6$	$i_3 - i_6$	$c_3 - c_6$	$\epsilon_3 - \epsilon_6$
$K_1\ P_1$	$A_3\ A_7$	$i_3 + i_7 = 2i_2$	$c_3 + c_7$	$\epsilon_3 + \epsilon_7$	$i_3 - i_7$	$c_3 - c_7$	$\epsilon_3 - \epsilon_7$
$O_1\ O_1$	$\frac{1}{2}A_6^2$	$i_6 + i_6 = 2i_6$	$2c_6$	$2\epsilon_6$	$i_6 - i_6 = 0$	0	0
$O_1\ P_1$	$A_6\ A_7$	$i_6 + i_7$	$c_6 + c_7$	$\epsilon_6 + \epsilon_7$	$i_6 - i_7$	$c_6 - c_7$	$\epsilon_6 - \epsilon_7$
$P_1\ P_1$	$\frac{1}{2}A_7^2$	$i_7 + i_7 = 2i_7$	$2c_7$	$2\epsilon_7$	$i_7 - i_7 = 0$	0	0

The designation $K_1\,K_1$ of the first component indicates that it arises from the diurnal component of the K or M_3 tides; that of the second, $K_1\,O_1$, that it arises from the diurnal components of the K and O tides, and so on. The values of i and c in the expressions of u and c' are contained in Schedule I. The values of A and ϵ in the expressions of R and q are obtained from the analysis of the tide-observations for the component depending directly upon the forces contained in the first schedule. This leaves only the constants C and E in (7) for each class of shallow-water components unknown, so that if these are determined from the analysis of the tide-observations for any one of these components, they are then known for all, and consequently all the components of the class are known quantities, without any further analysis of tide-observations.

From the values of u in the semidiurnal components it is seen that the periods of three of them coincide with the periods of three semidiurnal components depending directly upon the tidal forces, and hence the shallow-water components cannot be separated from the others directly by the analysis of the tide-observations. If, however, we can determine C and E by the analysis for any component whose period does not coincide with that of any of the other components, we then know the values of the shallow-water components combined with those depending directly upon the tidal forces.

8. In the same manner as in the preceding section, we get from the principal semidiurnal factors combined with those of the diurnal, two more classes of shallow-water components; the one being diurnal, having angles which are the differences of those in the original components, and the other, angles which are the sums of those angles.

These, together with the values of the constants R, u, c', and q in (7) belonging to them, are contained in the following schedule:

SCHEDULE III.

Designation of component.	R	Diurnal components.			Terdiurnal components.		
		u	c'	q	u	c'	q
$M_2\ K_1$	$A_1\ A_3$	$2i_1 - i_3 = i_6$	$2c_1 - c_3$	$\epsilon_1 - \epsilon_3$	$2i_1 + i_3$	$2c_1 + c_3$	$\epsilon_1 + \epsilon_3$
$M_2\ O_1$	$A_1\ A_6$	$2i_1 - i_6 = i_3$	$2c_1 - c_6$	$\epsilon_1 - \epsilon_6$	$2i_1 + i_6$	$2c_1 + c_6$	$\epsilon_1 + \epsilon_6$
$M_2\ P_1$	$A_1\ A_7$	$2i_1 - i_7$	$2c_1 - c_7$	$\epsilon_1 - \epsilon_7$	$2i_1 + i_7$	$2c_1 + c_7$	$\epsilon_1 + \epsilon_7$
$S_2\ K_1$	$A_2\ A_3$	$2i_2 - i_3 = i_7$	$2c_2 - c_3$	$\epsilon_2 - \epsilon_3$	$2i_2 + i_3$	$2c_2 + c_3$	$\epsilon_2 + \epsilon_3$
$S_2\ O_1$	$A_2\ A_6$	$2i_2 - i_6$	$2c_2 - c_6$	$\epsilon_2 - \epsilon_6$	$2i_2 + i_6$	$2c_2 + c_6$	$\epsilon_2 + \epsilon_6$
$S_2\ P_1$	$A_2\ A_7$	$2i_2 - i_7 = i_3$	$2c_2 - c_7$	$\epsilon_2 - \epsilon_7$	$2i_2 + i_7$	$2c_2 + c_7$	$\epsilon_2 + \epsilon_7$
$K_2\ K_1$	$A_3\ A_3$	$2i_3 - i_3 = i_3$	$2c_3 - c_3$	$\epsilon_3 - \epsilon_3$	$3i_3$	$3c_3$	$2\epsilon_3$
$K_2\ O_1$	$A_3\ A_6$	$2i_3 - i_6$	$2c_3 - c_6$	$\epsilon_3 - \epsilon_6$	$2i_3 + i_6$	$2c_3 + c_6$	$\epsilon_3 + \epsilon_6$
$K_2\ P_1$	$A_3\ A_7$	$2i_3 - i_7$	$2c_3 - c_7$	$\epsilon_3 - \epsilon_7$	$2i_3 + i_7$	$2c_3 + c_7$	$\epsilon_3 + \epsilon_7$
$L_2\ K_1$	$A_4\ A_3$	$2i_4 - i_3$	$2c_4 - c_3$	$\epsilon_4 - \epsilon_3$	$2i_4 + i_3$	$2c_4 + c_3$	$\epsilon_4 + \epsilon_3$
$L_2\ O_1$	$A_4\ A_6$	$2i_4 - i_6 = i_{13}$	$2c_4 - c_6$	$\epsilon_4 - \epsilon_6$	$2i_4 + i_6$	$2c_4 + c_6$	$\epsilon_4 + \epsilon_6$
$L_2\ P_1$	$A_4\ A_7$	$2i_4 - i_7$	$2c_4 - c_7$	$\epsilon_4 - \epsilon_7$	$2i_4 + i_7$	$2c_4 + c_7$	$\epsilon_4 + \epsilon_7$
$N_2\ K_1$	$A_5\ A_3$	$2i_5 - i_3 = i_{14}$	$2c_5 - c_3$	$\epsilon_5 - \epsilon_3$	$2i_5 + i_3$	$2c_5 + c_3$	$\epsilon_5 + \epsilon_3$
$N_2\ O_1$	$A_5\ A_6$	$2i_5 - i_6$	$2c_5 - c_6$	$\epsilon_5 - \epsilon_6$	$2i_5 + i_6$	$2c_5 + c_6$	$\epsilon_5 + \epsilon_6$
$N_2\ P_1$	$A_5\ A_7$	$2i_5 - i_7$	$2c_5 - c_7$	$_5 - \epsilon_7$	$2i_5 + i_7$	$2c_5 + c_7$	$\epsilon_5 + \epsilon_7$

In this schedule the designation $M_2\ K_1$ of the first component denotes that it depends upon the semidiurnal lunar or M tide and upon the diurnal declinational or K tide; the designation $S_2\ P_1$ denotes that it depends upon the semidiurnal solar or S tide and upon the diurnal P tide, and so on. In the two factors in the expressions of R, the first is the amplitude of the semidiurnal, and the second the amplitude of the diurnal component of the tide denoted by the characteristics or subscript numbers. In the expressions of q, also, the first epoch is that of the semidiurnal component or factor, and the second that of the diurnal one.

From an inspection of the several values of u in the diurnal components of this schedule, it is seen that there are three in which the value of u, and consequently of the period, is the same as that of the diurnal declinational component in Schedule I, and hence these three shallow-water components, forming in reality only one resulting component, are combined with the latter, from which it cannot be separated by the analysis of the observations. There is also one, the $M_2\ K_1$ tide, affecting the diurnal M_6 or K tide of Schedule I, and another, the $S_2\ K_1$ tide, affecting the diurnal P tide.

Among the terdiurnal components there are two whose periods are so nearly identical with that of the lunar terdiurnal component that in the analysis of only one year's tide-observations they cannot be separated, and appear to give only one resulting component. These two components are those designated by $L_2\ O_1$ and $N_2\ K_1$.

9. The largest shallow-water components arise, in general, from the semidiurnal components of Schedule I, especially in the Atlantic Ocean, where the diurnal components are small. By the multiplication of the semidiurnal terms in (5) into each other, two and two, there arise two classes of terms giving rise to corresponding shallow-water components, the one in which the angles are the sums of the original angles, being quarter-diurnal components, and the other, in which the angles are the differences, being components of long period. We thus get the following schedule of shallow-water components, with the corresponding values of the constants R, u, c', and q in (7):

SCHEDULE IV.

Designation of component.	R	Quarter-diurnal components.			Components of long period.		
		u	c'	q	u	c'	q
M_2 M_2	$\frac{1}{2}A_1^2$	$2i_1+2i_1=4i_1$	$4c_1$	$2\epsilon_1$	$2i_1-2i_1=0$	0	0
M_2 S_2	A_1 A_2	$2i_1+2i_2$	$2c_1+2c_2$	$\epsilon_1+\epsilon_2$	$2i_1-2i_2$	$2c_1-2c_2$	$\epsilon_1-\epsilon_2$
M_2 K_2	A_1 A_3	$2i_1+2i_3$	$2c_1+2c_3$	$\epsilon_1+\epsilon_3$	$2i_1-2i_3$	$2c_1-2c_3$	$\epsilon_1-\epsilon_3$
. .	. .	. .	. .	. .	. .	. .	. .
S_2 S_2	$\frac{1}{2}A_2^2$	$2i_2+2i_2=4i_2$	$4c_2$	$2\epsilon_2$	$2i_2-2i_2=0$	0	0
S_2 K_2	A_2 A_3	$2i_2+2i_3=4i_{11}$	$2c_2+2c_3$	$\epsilon_2+\epsilon_3$	$2i_2-2i_3$	$2c_2-2c_3$	$\epsilon_2-\epsilon_3$
S_2 L_2	A_2 A_4	$2i_2+2i_4$	$2c_2+2c_4$	$\epsilon_2+\epsilon_4$	$2i_2-2i_4$	$2c_2-2c_4$	$\epsilon_2-\epsilon_4$
. .	. .	. .	. .	. .	. .	. .	. .
K_2 K_2	$\frac{1}{2}A_3^2$	$2i_3+2i_3=4i_3$	$4c_3$	$2\epsilon_3$	$2i_3-2i_3=0$	0	0
K_2 L_2	A_3 A_4	$2i_3+2i_4$	$2c_3+2c_4$	$\epsilon_3+\epsilon_4$	$2i_3-2i_4$	$2c_3-2c_4$	$\epsilon_3-\epsilon_4$
K_2 N_2	A_3 A_5	$2i_3+2i_5$	$2c_3+2c_5$	$\epsilon_3+\epsilon_5$	$2i_3-2i_5$	$2c_3-2c_5$	$\epsilon_3-\epsilon_5$
. .	. .	. .	. .	. .	. .	. .	. .
L_2 L_2	$\frac{1}{2}A_4^2$	$2i_4+2i_4=4i_4$	$4c_4$	$2\epsilon_4$	$2i_4-2i_4=0$	0	0
: :	: :	: : :	: :	:	: :		

The amplitudes and epochs, A and ϵ, in the values of R and q, are those of the semidiurnal components in Schedule I. From the components given in each division of this schedule the law is seen by which all the others, with the corresponding constants, may be obtained.

10. All the preceding shallow-water components arise from using in (5) the components of Schedule I, as has been explained, and hence they may be called shallow-water components of the first order. In the same manner and upon the same principles we can now, in a third approximation, introduce the shallow-water components of the first order into (5), and thus obtain a numerous class of shallow-water components of a second order. For instance, if we use, in (5), the quarter-diurnal components of the preceding schedule, together with the components of Schedule I, we get, from the semidiurnal and the quarter-diurnal factors, terms which give rise to two classes of shallow-water components of a second order, the one being semidiurnal and the other one-sixth-diurnal component. These, together with the values of the constants R, u, c', and q of (7), are contained in the following schedule:

SCHEDULE V.

Designation of component.	R	Semidiurnal components.			One-sixth-diurnal components.		
		u	c'	q	u	c'	q
M_2 $(M_2\ M_2)$	$\frac{1}{2}A_1^3$	$2i_1$	$2c_1$	ϵ_1	$6i_1$	$6c_1$	$3\epsilon_1$
M_2 $(M_2\ S_2)$	$A_1^2\ A_2$	$2i_2$	$2c_2$	ϵ_2	$4i_1+2i_2$	$4c_1+2c_2$	$2\epsilon_1+\epsilon_2$
M_2 $(M_2\ K_2)$	$A_1^2\ A_3$	$2i_3$	$2c_3$	ϵ_3	$4i_1+2i_3$	$4c_1+2c_3$	$2\epsilon_1+\epsilon_3$
. .	. .	. . .	. . .	. .	. . .	. . .	. .
M_2 $(S_2\ S_2)$	$\frac{1}{2}A_1\ A_2^2$	$4i_2-2i_1$	$4c_2-2c_1$	$2\epsilon_2-\epsilon_1$	$4i_2+2i_1$	$4c_2+2c_1$	$\epsilon_1+2\epsilon_2$
M_2 $(S_2\ K_2)$	$A_1\ A_2\ A_3$	$2i_2+2i_3-2i_1$	$2c_2+2c_3-2c_1$	$\epsilon_2+\epsilon_3-\epsilon_1$	$2i_1+2i_2+2i_3$	$2c_1+2c_2+2c_3$	$\epsilon_1+\epsilon_2+\epsilon_3$
M_2 $(S_2\ L_2)$	$A_1\ A_2\ A_4$	$2i_2+2i_4-2i_1$	$2c_2+2c_4-2c_1$	$\epsilon_2+\epsilon_4-\epsilon_1$	$2i_1+2i_2+2i_4$	$2c_1+2c_2+2c_4$	$\epsilon_1+\epsilon_2+\epsilon_4$
. .	. .	. . .	. . .	. .	. . .	. . .	. .
S_2 $(M_2\ M_2)$	$\frac{1}{2}A_2\ A_1^2$	$4i_1-2i_2=2i_{10}$	$4c_1-2c_2$	$2\epsilon_1-\epsilon_2$	$4i_1+2i_2$	$4c_1+2c_2$	$2\epsilon_1+\epsilon_2$
S_2 $(M_2\ S_2)$	$A_1\ A_2^2$	$2i_1$	$2c_1$	ϵ_1	$2i_1+4i_2$	$2c_1+4c_2$	$2\epsilon_1+2\epsilon_2$
S_2 $(M_2\ K_2)$	$A_1\ A_2\ A_4$	$2i_1+2i_3-2i_2$	$2c_1+2c_3-2c_2$	$\epsilon_1+\epsilon_3-\epsilon_2$	$2i_1+2i_2+2i_3$	$2c_1+2c_2+2c_3$	$\epsilon_1+\epsilon_2+\epsilon_3$
. .	. .	. . .	. . .	. .	. . .	. . .	. .
S_2 $(S_2\ S_2)$	$\frac{1}{2}A_2^3$	$2i_2$	$2c_2$	ϵ_2	$6i_2$	$6c_2$	$3\epsilon_2$
S_2 $(S_2\ K_2)$	$A_2^2\ A_3$	$2i_3$	$2c_3$	ϵ_3	$4i_2+2i_3$	$4c_2+2c_3$	$2\epsilon_2+\epsilon_3$
S_2 $(S_2\ L_2)$	$A_2^2\ A_4$	$2i_4$	$2c_4$	ϵ_4	$4i_2+2i_4$	$4c_2+2c_4$	$2\epsilon_2+\epsilon_4$

It is seen from the values of u in this schedule that two or more of the components, as written out in detail, have the same period as some component in Schedule I, depending directly upon the forces, and hence all combine to form really only one component, which is brought out by the

analysis of the observations, and the different parts are inseparable by the analysis. For instance, the components M_2 (M_2 M_2) and S_2 (M_2 S_2) both have the same value of u, and hence the same period as that of the lunar semidiurnal component of Schedule I. In like manner there are two components affecting the solar semidiurnal component. The amplitudes and epochs A and ϵ in this schedule are those of the semidiurnal components.

11. By using the components of the preceding schedule in connection with the semidiurnal components we get in precisely the same manner and upon the same principles a still lower order of shallow-water components. The most important of the semidiurnal components in this schedule have been so used; and since their periods are the same they are combined with the semidiurnal components of Schedule I. From the one-sixth-diurnal components, so used, we get, giving those only depending upon the mean lunar and solar tides, the following:

SCHEDULE VI.

Designation of component.	R	Quarter-diurnal components.			One-eighth-diurnal components.		
		u	c'	q	u	c'	q
M_2 M_6	$\frac{1}{2} A_1^4$	$4 i_1$	$4 c_1$	$2 \epsilon_1$	$8 i_1$	$8 c_1$	$4 \epsilon_1$
M_2 S_6	$\frac{1}{2} A_1 A_2^3$	$6 i_2 - 2 i_1$	$6 c_2 - 2 c_1$	$3 \epsilon_2 - \epsilon_1$	$6 i_2 + 2 i_1$	$6 c_2 + 2 c_1$	$3 \epsilon_2 + \epsilon_1$
S_2 M_6	$\frac{1}{2} A_2 A_1^3$	$6 i_1 - 2 i_2$	$6 c_1 - 2 c_2$	$3 \epsilon_1 - \epsilon_2$	$6 i_1 + 2 i_2$	$6 c_1 + 2 c_2$	$3 \epsilon_1 + \epsilon_2$
S_2 S_6	$\frac{1}{2} A_2^4$	$4 i_2$	$4 c_2$	$2 \epsilon_2$	$8 i_2$	$8 c_2$	$4 \epsilon_2$

12. Only a few, perhaps, of the shallow-water components in the preceding schedules are sensible to observation at most tide stations; but at Liverpool and a few other places the amplitudes of some of them have been found, from actual analysis of the tide-observations, to be quite large, that of the quarter-diurnal component at Liverpool being nearly 9 inches. Even the one-eighth-diurnal lunar component of the third order is quite clearly brought out from the analysis of the observations of many stations. At river stations far from the sea many of these components must be quite large, since it is owing to them that the semidiurnal tide is so distorted frequently and made to vary so much from that of deep waters, the form of which does not deviate from what is called "the law of sines and cosines."

A few of the preceding relations between the amplitudes and epochs of the shallow-water components were first published in my Tidal Researches, and comparisons made with those obtained by the Tidal Committee of the British Association from the analysis of the tide-observations of Liverpool and other places; and although these theoretical relations claim to be only approximately correct, yet it was surprising to see how nearly they coincided with those obtained from observation.

In studying the theoretical relations of the tides, and making comparisons between them and the tidal forces producing them, it is not only necessary to have a complete schedule of all the shallow-water components which may, in any case, affect the components depending directly upon the forces, but also to have from theory as many relations between the amplitudes and epochs of the various components as possible; hence the importance of the preceding schedules.

13. It is now necessary to devise a method of obtaining the amplitudes and epochs of the components in (2) from averages of observations taken at regular intervals for a certain period of time. From (2) we get—

$$(8) \qquad H = \Sigma_n A_n \cos (\varphi_n - \epsilon'_n)$$

in which

$$(9) \qquad \begin{cases} \varphi_n = i_n t \\ \epsilon'_n = \epsilon_n - c_n = \epsilon'_{(s,\epsilon)} - sc_s; \text{ or } \epsilon_{(s,\epsilon)} = \epsilon'_{(s,\epsilon)} + sc_\epsilon \end{cases}$$

This expression of H may include all the shallow-water components of the form of (7), since the latter can be reduced to the form of the expression of H above.

If we now take the average of all the observations made at equal and short intervals of time throughout the time $t = \tau$, which fall within certain limits $\varphi'_{n'}$ and $\varphi''_{n'}$ of any argument $\varphi_{n'}$, of which

the characteristic is distinguished by an accent, excluding all multiples of 2π, the values of $\varphi_{n'}$ are arbitrary and independent of the time, and the corresponding relative values of the other arguments may be expressed by—

$$\varphi_n = \varphi_{n'} + (i_n - i_{n'})\,t \tag{10}$$

and the observations which fall between the limits φ'_n and φ''_n of any other argument will be evenly distributed between these limits.

If we put—

$a =$ the average of all observations of H between the limits $\varphi'_{n'}$ and $\varphi''_{n'}$, and let φ'_n and φ''_n denote generally the limits of the other arguments, we shall have from (8), and from Tidal Researches (310)—

$$a = \Sigma_n \frac{2 \sin \frac{1}{2} (\varphi''_n - \varphi'_n)}{\varphi''_n - \varphi'_n} \mathrm{A}_n \cos \left(\tfrac{1}{2} (\varphi'_n + \varphi''_n) - \varepsilon'_n\right) \tag{11}$$

If the limits $\varphi'_{n'}$ and $\varphi''_{n'}$ are taken at equal distances before and after $\varphi_{n'}$, we shall have $\frac{1}{2}(\varphi'_{n'} + \varphi''_n) = \varphi_{n'}$, and where the limits of the observations in time are $t = 0$ and $t = \tau$, we get from (10)

$$\varphi'_n = \varphi_{n'}$$

$$\varphi''_n = \varphi_{n'} + (i_n - i_{n'})\,\tau$$

Hence we get, in all cases in which the limits are functions of the time—

$$\frac{2 \sin \frac{1}{2} (\varphi''_n - \varphi'_n)}{\varphi''_n - \varphi'_n} = \frac{2 \sin \frac{1}{2} (i_n - i_{n'})\,\tau}{(i_n - i_{n'})\,\tau}$$

$$\tfrac{1}{2} (\varphi'_n + \varphi''_n) = \varphi_{n'} + \tfrac{1}{2} (i_n - i_{n'})\,\tau$$

The limits φ'_n and φ''_n for the argument of the component of which the characteristic is n', are entirely arbitrary, and if we denote the coefficient of A_n in (11), in this case, by k, we shall have—

$$k = \frac{2 \sin \frac{1}{2} (\varphi''_{n'} - \varphi'_{n'})}{\varphi''_{n'} - \varphi'_{n'}}$$

This expression of the coefficient of A_n applies to all the components in which the arguments are multiples of $\varphi_{n'}$, since the limits for all such arguments are independent of the time and depend upon the limits assumed for $\varphi_{n'}$.

If we now put $\varphi_n = s\varphi_e$ and $i_n = si_e$, the expression of a above can be put into the form—

$$a = \Sigma_s (\mathrm{M}_s \cos s\varphi_{e'} + \mathrm{N}_s \sin s\varphi_{e'}) = \Sigma_s \mathrm{A}'_s \cos (s\varphi_{e'} - \varepsilon''_s) \tag{12}$$

in which

$$\begin{cases} \mathrm{M}_s = k_s \mathrm{A}_s \cos \varepsilon'_s + m_s \\ \mathrm{N}_s = k_s \mathrm{A}_s \sin \varepsilon'_s + n_s \end{cases} \tag{13}$$

$$k_s = \frac{2 \sin \frac{1}{2} s\ (\varphi''_{e'} - \varphi'_{e'})}{s\,(\varphi''_{e'} - \varphi'_{e'})} \tag{14}$$

$$\begin{cases} m_s = \Sigma_e \dfrac{2 \sin \frac{1}{2} s (i_e - i_{e'})\,\tau}{s\,(i_e - i_{e'})\,\tau} \mathrm{A}_e \cos \left(\tfrac{1}{2} s\,(i_e - i_{e'})\,\tau - \varepsilon'_e\right) \\ n_s = -\Sigma_e \dfrac{2 \sin \frac{1}{2} s\,(i_e - i_{e'})\,\tau}{s\,(i_e - i_{e'})\,\tau} \mathrm{A}_e \sin \left(\tfrac{1}{2} s\,(i_e - i_{e'})\,\tau - \varepsilon'_{e'}\right) \end{cases} \tag{15}$$

In these expressions of m and n the term arising from putting $e = e'$ must not be included, and all terms having $si_e - s'i_{e'}$ in the denominator in which the integral value of s differs from that of s' have been neglected, since in these cases the denominator becomes very large and the term of no importance. The amplitudes and epochs A and ε, therefore, are those of the diurnal, semidiurnal, &c., components, according as the value of s in (12) is one, two, &c. The values of m and n, therefore, belonging to any one class of components, diurnal, semidiurnal, &c., are not affected by the components of any of the other classes.

From (13) we get—

$$(16)\qquad \begin{cases} A_{,} = \dfrac{M_{,} - m_{,}}{k_{,} \cos \varepsilon'_{,}} = \dfrac{N_{,} - n_{,}}{k_{,} \sin \varepsilon'_{,}} \\ \tan \varepsilon'_{,} = \dfrac{N_{,} - n_{,}}{M_{,} - m_{,}} \end{cases}$$

These give the true amplitudes and epochs from the values of the co-ordinates M and N, and m and n, the latter being simply very small corrections of the former.

14. In the second form of expression of a, (12), the amplitude and epoch, A′ and ε'', require corrections, generally small, depending upon m and n, to reduce them to the true amplitude and epoch of the component having the same argument. If we put $\delta\varepsilon'$ for the correction of ε'', we have

$$(17)\qquad \varepsilon' = \varepsilon'' + \delta\varepsilon'$$

Since the epoch, or time $t = 0$, in the angle $\varphi_e = i_e\, t$ in (12), is entirely arbitrary, we can suppose it to be such as to make $\varepsilon'' = 0$, and we then have $M = A'$ and $N = 0$. Upon this supposition it would be necessary to diminish the epoch ε' in (13) and (15) by the angle ε'', and hence we should have—

$$(18)\qquad \begin{cases} M_{,} = k_{,} A_{,} \cos \delta\varepsilon'_{,} + m_{,} \\ N_{,} = k\; A_{,} \sin \delta\varepsilon'_{,} + n_{,} \end{cases}$$

in which

$$(19)\qquad \begin{cases} m_{,} = \Sigma_e \dfrac{2 \sin \frac{1}{2} s\,(i_e - i_{e'})\,\tau}{s\,(i_e - i_{e'})\,\tau} A_e \cos\left(\frac{1}{2} s\,(i_e - i_e)\,\tau + \varepsilon''_{e} - \varepsilon'_{,}\right) \\ n_{,} = -\,\Sigma_e \dfrac{2 \sin \frac{1}{2} s\,(i_e - i_{e'})\,\tau}{s(i_e - i_{e'})\,\tau} A_e \sin\left(\frac{1}{2} s\,(i_e - i_e)\,\tau + \varepsilon''_{e} - \varepsilon'_{,}\right) \end{cases}$$

From (18), with the values of $M = A'$ and $N = 0$, we get—

$$(20)\qquad \begin{cases} A_{,} = \dfrac{M_{,} - m_{,}}{k_{,} \cos \delta\varepsilon'_{,}} = \dfrac{A'_{,} - m_{,}}{k_{,} \cos \delta\varepsilon'_{,}} \\ \tan \delta\varepsilon'_{,} = -\,\dfrac{n_{,}}{M_{,} - m_{,}} = -\,\dfrac{n_{,}}{A'_{,} - m_{,}} \end{cases}$$

From these and (17) we get the corrected amplitudes and epochs directly from the uncorrected ones, which is more convenient than obtaining them from (16), in which the corrections are applied to the co-ordinates M and N.

In the expression of (15) and (19) the uncorrected values of A_e and ε'_e are necessarily used in computing the corrections $m_{,}$ and $n_{,}$, but these values are more than sufficiently correct, since these corrections are generally very small.

15. The terms in the expressions of (15) and (19) vanish when the value of τ is such as to satisfy the condition of

$$\tfrac{1}{2}\,(i_e - i_{e'})\,\tau = n\,\pi$$

in which n is any integral number. Hence if the periods were all commensurable a value of τ could be assumed which would make the whole expression vanish. Since A_1 of the semidiurnal component is usually much larger than any of the others, it is best to assume such a value of τ as will make the term in the expressions of m and n vanish, which contains A_1 of the semidiurnal component, and this is done by assuming such a value of τ as will satisfy the condition

$$(21)\qquad (i_1 - i_{e'})\,\tau = n\,\pi$$

In the harmonic analysis of the tides it is usual to take one year's observations at a time, and therefore for each class of components of which the characteristics are e it is best to take such a value of τ, which is the time of duration of the series of observations used, as will satisfy (21), and

at the same time be the nearest to the time of one year. With the values of i_e in Schedule I, these conditions give for each of the M_e tides—

	d	h		d	h
$\tau_1 =$	369	3	$\tau_8 =$	381	18
$\tau_2 =$	369	3	$\tau_9 =$	381	18
$\tau_3 =$	368	20	$\tau_{10} =$	369	3
$\tau_4 =$	358	5	$\tau_{11} =$	369	0
$\tau_5 =$	358	5	$\tau_{12} =$	369	7
$\tau_6 =$	368	20	$\tau_{13} =$	363	14
$\tau_7 =$	369	12	$\tau_{14} =$	363	14

In the same manner we obtain the most suitable values of τ to be used in analyzing the shallow-water tides, the values of u in (7) being used in (21) instead of i_e.

With these values of τ_e, and the values of i_e in Schedule I, and putting m_e for the value of m belonging to the M_e component, and neglecting all terms which are very small, either on account of the largeness of the denominator $s(i_e - i_{e'})\tau$, or the smallness of the numerator, or of the value A_e, we get from (19), by putting $s = 1$, the following:

DIURNAL COMPONENTS.

$$m_1 = .0236\ A_3 \cos(91^\circ.9 + \varepsilon''_1 - \varepsilon'_3) + .0236\ A_6 \cos(268^\circ.1 + \varepsilon''_1 - \varepsilon'_6) + .0277\ A_7 \cos(88^\circ.1 + \varepsilon''_1 - \varepsilon'_7)$$

$$m_3 = .0099\ A_7 \cos(3^\circ.6 + \varepsilon''_3 - \varepsilon'_7)$$

$$m_4 = .0452\ A_3 \cos(110^\circ.0 + \varepsilon''_4 - \varepsilon'_3) + .0610\ A_7 \cos(116^\circ.8 + \varepsilon''_4 - \varepsilon'_7) + .0152\ A_6 \cos(250^\circ.0 + \varepsilon''_4 - \varepsilon'_6)$$

$$m_5 = .0152\ A_3 \cos(110^\circ.0 + \varepsilon''_5 - \varepsilon'_3) + .0452\ A_6 \cos(320^\circ.1 + \varepsilon''_5 - \varepsilon'_6)$$

$$m_7 = .0115\ A_3 \cos(4^\circ.2 + \varepsilon''_7 - \varepsilon'_3)$$

$$m_{11} = .6296\ A_3 \cos(91^\circ.0 + \varepsilon''_{11} - \varepsilon'_3) + .2099\ A_7 \cos(267^\circ.2 + \varepsilon''_{11} - \varepsilon'_7)$$

$$m_{12} = .2099\ A_3 \cos(93^\circ.0 + \varepsilon''_{12} - \varepsilon'_3) + .6296\ A_7 \cos(269^\circ.0 + \varepsilon''_{12} - \varepsilon'_{11})$$

$$m_{13} = .0138\ A_3 \cos(325^\circ.0 + \varepsilon''_{13} - \varepsilon'_3) + .0045\ A_6 \cos(214^\circ.5 + \varepsilon''_{13} - \varepsilon'_6) + .0106\ A_7 \cos(329^\circ.3 + \varepsilon''_{13} - \varepsilon'_7)$$

$$m_{14} = .0045\ A_3 \cos(145^\circ.5 + \varepsilon''_{14} - \varepsilon'_3) + .0138\ A_6 \cos(35^\circ.0 + \varepsilon''_{14} - \varepsilon'_6) + .0045\ A_7 \cos(147^\circ.5 + \varepsilon''_{14} - \varepsilon'_7)$$

In the same manner, by putting $s = 2$ in (19), we get the following:

SEMIDIURNAL COMPONENTS.

$$m_1 = .0225\ A_4 \cos(71^\circ.3 + \varepsilon''_1 - \varepsilon'_4) + .0225\ A_5 \cos(288^\circ.7 + \varepsilon''_1 - \varepsilon'_5) + .0257\ A_8 \cos(110^\circ.5 + \varepsilon''_1 - \varepsilon'_8) + .0257\ A_9 \cos(249^\circ.5 + \varepsilon''_1 - \varepsilon'_9)$$

$$m_2 = .0104\ A_3 \cos(3^\circ.8 + \varepsilon''_2 - \varepsilon'_3) + .0260\ A_4 \cos(251^\circ.5 + \varepsilon''_2 - \varepsilon'_4) + .0078\ A_5 \cos(289^\circ.0 + \varepsilon''_2 - \varepsilon'_5) + .0225\ A_8 \cos(71^\circ.3 + \varepsilon''_2 - \varepsilon'_8)$$

$$m_3 = .0099\ A_2 \cos(356^\circ.4 + \varepsilon''_3 - \varepsilon'_2) + .0219\ A_4 \cos(249^\circ.5 + \varepsilon''_3 - \varepsilon'_4) + .0074\ A_5 \cos(290^\circ.5 + \varepsilon''_3 - \varepsilon'_5) + .0210\ A_8 \cos(287^\circ.0 + \varepsilon''_3 - \varepsilon'_8)$$

$$m_4 = .0093\ A_2 \cos(19^\circ.0 + \varepsilon''_4 - \varepsilon'_2) + .0155\ A_3 \cos(40^\circ.0 + \varepsilon''_4 - \varepsilon'_3) + .1332\ A_8 \cos(226^\circ.8 + \varepsilon''_4 - \varepsilon'_5)$$

$$m_5 = .0062\ A_2 \cos(47^\circ.0 + \varepsilon''_5 - \varepsilon'_2) + .1332\ A_9 \cos(133^\circ.2 + \varepsilon''_5 - \varepsilon'_9) + .0207\ A_{10} \cos(313^\circ.0 + \varepsilon''_5 - \varepsilon'_{10})$$

$m_6 = .0236\ \mathrm{A}_5 \cos (110^\circ.6 + \varepsilon''_6 - \varepsilon'_5)$

$m_7 = .0115\ \mathrm{A}_2 \cos (4^\circ.2 + \varepsilon''_7 - \varepsilon'_2) + .0108\ \mathrm{A}_3 \cos (8.5 + \varepsilon''_7 - \varepsilon'_3) + .0325\ \mathrm{A}_4 \cos (253.7 + \varepsilon''_7 - \varepsilon'_4)$

$m_8 = .0101\ \mathrm{A}_2 \cos (153^\circ.8 + \varepsilon''_8 - \varepsilon'_2) + .0755\ \mathrm{A}_4 \cos (153^\circ.9 + \varepsilon''_8 - \varepsilon'_4) + .0053\ \mathrm{A}_5 \cos (206^\circ.4 + \varepsilon''_8 - \varepsilon'_5)$

$m_9 = .0037\ \mathrm{A}_2 \cos (154.0 + \varepsilon''_9 - \varepsilon'_2) + .0755\ \mathrm{A}_5 \cos (206^\circ.4 + \varepsilon''_9 - \varepsilon'_5)$

$m_{10} = .0260\ \mathrm{A}_5 \cos (108^\circ.6 + \varepsilon''_{10} - \varepsilon_5)$

If we use sin for cos in these expressions and change the sign, it is seen from (19) that we get those of n_{e}.

For terdiurnal, quarter-diurnal, &c., components the corrections depending upon m and n are unimportant, since only the amplitudes of these components, which are very small, enter into the expressions.

It is seen from the smallness mostly of the numerical coefficients, taken in connection with the values of A, that most of the terms in the preceding expressions of m and n are very small. The large values of m_{11} and m_{12} in the diurnal components, arising from the very small value of $(i_{s} - i_{e'})$ in that case, occur where there is no sensible diurnal component, and hence are of no importance, since after applying the corrections the result is approximately 0. By using the values of τ_{e}, which have been given for each class of M_{e} tides, terms containing A_1 of the semidiurnal tide disappear, and as this is much the largest of the amplitudes, the values of m and n are thereby much diminished. Indeed, when these values of τ_{e} are used, all the corrections depending upon m and n may be neglected, except in very nice theoretical investigations, such as determining the moon's mass; and this is especially the case when the results of the observations of a series of years are used, in which case the effects of these small corrections are mostly eliminated from the averages.

16. It yet remains to determine, from averages of the observations, the quantities M_s and N_s in (16). This must be done by means of (12), in which a is the average of all the hourly observations of H during the time $\tau_{e'}$ contained within the limits $\varphi'_{e'}$ and $\varphi''_{e'}$. If we make these limits include 15° of the argument $\varphi_{e'}$, as is usual, we shall then have twenty-four of these averages. If we take the limits so that the middle of the included intervals correspond to $\varphi_{e'} = 0$, $\varphi_{e'} = 15^\circ$, $\varphi_{e'} = 30^\circ$, &c., and denote the corresponding values of the averages by a_0, a_1, a_2, &c., we get from (12), by supplying the implied constant of integration A_0, twenty-four equations, as follows:

$$a_0 = \mathrm{A}_0 + \mathrm{M}_1 + 0 + \mathrm{M}_2 + 0 + \mathrm{M}_3 + 0 + \text{\&c.}$$
$$a_1 = \mathrm{A}_0 + \mathrm{M}_1 \cos 15^\circ + \mathrm{N}_1 \sin 15^\circ + \mathrm{M}_2 \cos 30^\circ + \mathrm{N}_2 \sin 30^\circ + \mathrm{M}_3 \cos 45^\circ + \mathrm{N}_3 \sin 45^\circ + \text{\&c.}$$
$$a_2 = \mathrm{A}_0 + \mathrm{M}_1 \cos 30^\circ + \mathrm{N}_1 \sin 30^\circ + \mathrm{M}_2 \cos 60^\circ + \mathrm{N}_2 \sin 60^\circ + \mathrm{M}_3 \cos 90^\circ + \mathrm{N}_3 \sin 90^\circ + \text{\&c.}$$
$$a_3 = \mathrm{A}_0 + \mathrm{M}_1 \cos 45^\circ + \mathrm{N}_1 \sin 45^\circ + \mathrm{M}_2 \cos 90^\circ + \mathrm{N}_2 \sin 90^\circ + \mathrm{M}_3 \cos 135^\circ + \mathrm{N}_3 \sin 135^\circ + \text{\&c.}$$

and so forth.

The solution of these twenty-four equations by the method of least squares gives us a certain number of normal equations, the number depending upon the number of terms taken into account in the expression of a. These normal equations are entirely independent of one another, and give directly, without any further solution, the values of the constants, A_0, M_1, N_1, M_2, N_2, &c. The following, given in the most convenient form for computation, are all that we shall have occasion to use in the tidal analysis:

$$(22) \qquad 24\mathrm{A}_0 = a_0 + a_1 + a_2 + a_3 \quad . \quad . \quad . \quad . \quad + a_{23}$$

$$(23) \qquad \begin{cases} 12\,\mathrm{M}_1 = \mathrm{A} + (\mathrm{B} + \mathrm{B}') \cos 15^\circ + (\mathrm{C} + \mathrm{C}') \cos 30^\circ + (\mathrm{D} + \mathrm{D}') \cos 45^\circ \\ \qquad\qquad + (\mathrm{E} + \mathrm{E}') \cos 60^\circ + (\mathrm{F} + \mathrm{F}') \cos 75^\circ \\ 12\,\mathrm{N}_1 = \mathrm{A}' + (\mathrm{B} - \mathrm{B}') \sin 15^\circ + (\mathrm{C} - \mathrm{C}') \sin 30^\circ + (\mathrm{D} - \mathrm{D}') \sin 45^\circ \\ \qquad\qquad + (\mathrm{E} - \mathrm{E}') \sin 60^\circ + (\mathrm{F} - \mathrm{F}') \sin 75^\circ \end{cases}$$

in which

$$\begin{array}{ll} A = (a_0 - a_{12}) & A' = (a_6 - a_{18}) \\ B = (a_1 - a_{13}) & B' = (a_{23} - a_{11}) \\ C = (a_2 - a_{14}) & C' = (a_{22} - a_{10}) \\ D = (a_3 - a_{15}) & D' = (a_{21} - a_9) \\ E = (a_4 - a_{16}) & E' = (a_{20} - a_8) \\ F = (a_5 - a_{17}) & F' = (a_{19} - a_7) \end{array}$$

$$(24) \quad \begin{cases} 12\,M_2 = A_2 + (B_2 + B'_2) \cos 30^\circ + (C_2 + C'_2) \cos 60^\circ \\ 12\,N_2 = A'_2 + (B_2 - B'_2) \sin 30^\circ + (C_2 - C'_2) \sin 60^\circ \end{cases}$$

in which

$$\begin{array}{ll} A_2 = (a_0 - a_6) + (a_{12} - a_{18}) & A'_2 = (a_7 - a_9) + (a_{15} - a_{21}) \\ B_2 = (a_1 - a_7) + (a_{13} - a_{19}) & B'_2 = (a_{11} - a_{17}) + (a_{23} - a_5) \\ C_2 = (a_2 - a_8) + (a_{14} - a_{20}) & C'_2 = (a_{10} - a_{16}) + (a_{22} - a_4) \end{array}$$

$$(25) \quad \begin{cases} 12\,M_3 = A_3 + (B_3 + B'_3) \cos 45^\circ \\ 12\,N_3 = A'_3 + (B_3 - B'_3) \sin 45^\circ \end{cases}$$

in which

$$\begin{array}{ll} A_3 = (a_0 - a_4) + (a_8 - a_{12}) + (a_{16} - a_{20}) & A'_3 = (a_2 - a_6) + (a_{10} - a_{14}) + (a_{18} - a_{22}) \\ B_3 = (a_1 - a_5) + (a_9 - a_{13}) + (a_{17} - a_{21}) & B'_3 = (a_7 - a_{11}) + (a_{15} - a_{19}) + (a_{23} - a_3) \end{array}$$

$$(26) \quad \begin{cases} 12\,M_4 = A_4 + (B_4 - B'_4) \cos 60^\circ \\ 12\,N_4 = 0 \quad + (B_4 - B'_4) \sin 60^\circ \end{cases}$$

in which

$$\begin{aligned} A_4 &= (a_0 - a_3) + (a_6 - a_9) + (a_{12} - a_{15}) + (a_{18} - a_{21}) \\ B_4 &= (a_1 - a_4) + (a_7 - a_{10}) + (a_{13} - a_{16}) + (a_{19} - a_{22}) \\ B'_4 &= (a_5 - a_8) + (a_{11} - a_{14}) + (a_{17} - a_{20}) + (a_{23} - a_2) \end{aligned}$$

$$(27) \quad \begin{cases} 12\,M_6 = (a_0 - a_2) + (a_4 - a_6) + (a_8 - a_{10}) + (a_{12} - a_{14}) + (a_{16} - a_{18}) + (a_{20} - a_{22}) \\ 12\,N_6 = (a_1 - a_3) + (a_5 - a_7) + (a_9 - a_{11}) + (a_{13} - a_{15}) + (a_{17} - a_{19}) + (a_{21} - a_{23}) \end{cases}$$

$$(28) \quad \begin{cases} 12\,M_8 = (a_0 + a_3 + a_6 + a_9 + a_{12} + a_{15} + a_{18} + a_{21}) + (a_1 + a_2 + a_4 + a_5 + a_7 + a_8 \\ \qquad + a_{10} + a_{11} + a_{13} + a_{14} + a_{16} + a_{17} + a_{19} + a_{20} + a_{22} + a_{23}) \cos 120^\circ \\ 12\,N_8 = ((a_1 - a_2) + (a_4 - a_5) + (a_7 - a_8) + (a_{10} - a_{11}) + (a_{13} - a_{14}) + (a_{16} - a_{17}) \\ \qquad + (a_{19} - a_{20}) + (a_{22} - a_{23})) \sin 120^\circ \end{cases}$$

From these, with the values of a_0, a_1, a_2, &c., obtained directly from the observations, we get A_s, M_s, and N_s, and then by means of these we get from (16) the true amplitudes and epochs of the components, or by neglecting m and n we get the uncorrected amplitudes and epochs, and then from these, by (17) and (20), we get the corrected ones.

The values of $\frac{1}{k_s}$ in (16) and (20), when the intervals are $\varphi'' - \varphi = 15^\circ$, are by (14)—

$$(29) \quad \frac{1}{k_1} = 1.0028, \ \frac{1}{k_2} = 1.0115, \ \frac{1}{k_3} = 1.0262, \ \frac{1}{k_4} = 1.0472, \ \frac{1}{k_6} = 1.1106, \ \frac{1}{k_8} = 1.2092$$

Since the values of M and N, obtained from each of the preceding normal equations, and consequently the values of A and ε are entirely independent of the other equations, we can get the amplitude and epoch of any one or more of the components in the expression of a, (12), and neglect the rest, so that it is not necessary to exhaust the residuals; but it is best to take in all sensible terms in the expression, for the residuals then furnish a check generally, within very narrow limits, of the accuracy of the computations. We can, moreover, use these residuals then in determining the probable errors of M and N, and hence of A and ε.

If we put—

$v =$ the residuals,

$e =$ the probable error of the average a, regarded as a single observation,

$r =$ the probable errors of the unknown quantities A_o, M, and N,

$m =$ the number of the unknown quantities,

we shall have—

$$e = 0.6745 \sqrt{\frac{\Sigma v^2}{24 - m}} \tag{30}$$

The weight of each of the constants deduced from the normal equations is 12, except that of A_o, which is 24, and hence we have for the probable error of A_o—

$$r = \frac{e}{\sqrt{24}} = 0.2041\, e \tag{31}$$

and for the probable error of M and N, and consequently of the amplitude A—

$$r = \frac{e}{\sqrt{12}} = 0.2882\, e \tag{32}$$

The probable error of the epochs, expressed in degrees, will be—

$$r' = \frac{e}{\sqrt{12}\,\mathrm{A}} \times 57^\circ.3 = 16^\circ.51 \times \frac{e}{\mathrm{A}} \tag{33}$$

17. The observations of H in (3), from which the averages or values of a are obtained for each interval of 15° of the arguments φ_s are given for each solar hour, that is, for each of the values of φ_2 equal 0, 1, 2, 3, &c., hours expressed in degrees. Hence, for the solar components we simply group together all the observations throughout the time τ, approximately one year, belonging to each of the solar hours, and thus obtain twenty-four averages from which the amplitudes and epochs are obtained by the method already given. But when the observations are grouped with reference to the other arguments, it is necessary to know the value of the argument φ_s in relation to that of φ_2. This is given by the equation—

$$\varphi_s = \varphi_2 + (i_s - i_2)\, t \tag{34}$$

in which t is the time of the observation expressed in hours from the epoch, or time $t = 0$. It is therefore necessary to have a table giving the values of each of the arguments φ_s for each hour of every day in the year. Table I, computed from (34), with the values of i_s given in Schedule I, is such a table of the principal arguments expressed in hours of 15° each, and the multiples of 24 hours being neglected. It is given for the first and last days of the year only, merely as a sample, the complete table in manuscript having been used. The solar hour is found in the first column under φ_2, and the corresponding values of the other arguments φ_s, given to the nearest integer only, are found under their appropriate heads. Since all the observations within the limits of a half hour preceding and following each hour come in the same group, it is only necessary to have in the tables the hour belonging to the middle of the group, or, in other words, the values of φ_s in hours to the nearest integer. The averages then correspond to the even hours of φ_s.

18. The epochs obtained directly from the analysis by the preceding method are those of ε' in (8), in which the time, $t = 0$, is the same for all the components. In the expression of H (3), the angle $i_s t + c_s$ is the hour-angle of the M_s tide, and the epoch ε_s is that which belongs to the case in which t is reckoned from the passage of the fictitious moon M_s over the meridian. The former, ε', varies very much generally from year to year; the latter, ε, is theoretically the same for each year. After the reduction of ε' to ε we can compare the several values of ε belonging to each of a series of years and estimate the probable error. This reduction is made by the last of equations (9), by adding sc_s to the epoch ε'_n. It is necessary, therefore, to have the value of c_s for the assumed time of $t = 0$ at the beginning of each year. The values of c_s for each class of M_s tides is obtained from the expressions of c_s in Schedule I, in which the values of λ, λ', v, &c., at the beginning of each year must be used. With the data contained in the introduction to Peirce's Tables of the Moon,

these values of c_e have been thus computed for a series of years extending back and forward from the present time, and given in Table II. This table can be readily extended further each way by means of the differences. In this table t in (3) is supposed to be reckoned from midnight, and hence 180° have been added to the values given by the expressions in the schedule. By means of this table the reductions above are readily made for any year contained in the table. In this table are also contained the values of $\tilde{\omega}$, ω, and $\tilde{\omega}'$, which are often needed.

In the reductions of the epochs of any of the shallow-water components which do not fall within any of the fourteen classes of components given in Schedule I, the values of c', given by its expression in the several schedules of the shallow-water components, must be used. After such reductions the epochs of these components for different years should also be the same.

19. The amplitudes and epochs obtained directly from the analysis require still another reduction in a few cases, to make the results of different years comparable one with another. These are the cases in which the components are affected by other and smaller components of so nearly the same period that they cannot be separated by the analysis. We have two cases of this sort in the diurnal and two in the semidiurnal components, as is seen from Schedule I. In the diurnal components the M_3 component is affected by the M'_3 component of very nearly the same period, as is seen from the corresponding values of i, and the M_6 component by the N'_6 of nearly the same period.

If we let A and ε be the amplitude and epoch of the principal component, and A′ and ε' be the same as affected by the smaller component of nearly the same period, and also let a be the amplitude of the smaller component, we shall have—

$$\frac{A_3}{A'_3} = \frac{1}{\sqrt{1 + e_3^2 + 2\, e_3 \cos \omega}}$$

$$\tan \Delta \varepsilon'_6 = \frac{-\, e_3 \sin \omega}{1 + e_3 \cos \omega}$$

$$\frac{A_6}{A'_6} = \frac{1}{\sqrt{1 + e_6^2 + 2\, e_6 \cos \omega}}$$

$$\tan \Delta \varepsilon'_6 = \frac{e_6 \sin \omega}{1 + e_6 \cos \omega}$$

in which $e = \frac{a}{A}$ and $\Delta \varepsilon' = \varepsilon - \varepsilon'$ is the correction to be added to ε' to reduce it to ε. The preceding expressions are the same as those of § 97, Tidal Researches, in a different form and with a different notation. With the values given by the preceding expressions, we make the reductions by putting

$$(35) \qquad \left\{ \begin{aligned} A &= A' \times \frac{A}{A'} \\ \varepsilon &= \varepsilon' + \Delta \varepsilon' \end{aligned} \right.$$

The values of e in the preceding expressions are known upon the hypothesis that the amplitudes of components having the same period, or nearly, are proportional to the coefficients in the variable terms in the forces preceding them, which is true for deep waters without friction, but may not be strictly so in other cases. The reductions, however, having been made upon this hypothesis, the comparisons of the yearly results with one another for a considerable series of years will show whether this hypothesis is correct, for if it is not the amplitudes and epochs will have a variable term with a period equal to that of ω, the longitude of the lunar node.

Upon the hypothesis that the amplitudes are proportional to the forces we get, with the values of P_n, in Schedule I:

$$e_3 = \frac{a_3}{A_3} = \frac{.071}{.5306 - 13.1\delta\mu}$$

$$e_6 = \frac{a_6}{A_6} = \frac{.084}{.3813}$$

With these values of e_3 and e_6, the values of the reducing factors and corrections of the epochs given by the preceding expressions have been computed for every tenth degree of ω, as given in Tables III and IV. The values should be taken from the tables with the argument ω taken from Table II for the middle of the year. With these values the corrected amplitudes and epochs A and ε are readily obtained from the uncorrected ones A′ and ε' by means of (35).

In the semidiurnal components the M_1 component is affected by the M'_1 component of nearly the same period, and the M_3 component by the M'_3 component. Adopting the same hypothesis and the same general notation as in the similar cases of diurnal components, we have in these cases—

$$\left.\begin{aligned} \frac{A_1}{A'_1} &= \frac{1}{\sqrt{1+e_1^2+2\,e_1\cos\omega}} \\ \tan\Delta\varepsilon'_1 &= \frac{e_1\sin\omega}{1+e\cos\omega} \end{aligned}\right\}\text{ in which } e_1=\frac{a_1}{A_1}=\frac{.0359}{1.000}$$

$$\left.\begin{aligned} \frac{A_3}{A'_3} &= \frac{1}{\sqrt{1+e_3^2+2\,e_3\cos\omega}} \\ \tan\Delta\varepsilon'_3 &= \frac{-e_3\sin\omega}{1+e_3\cos\omega} \end{aligned}\right\}\text{ in which } e_3=\frac{a_3}{A_3}=\frac{.0359}{.1256-3.2\,\delta\mu}$$

From these expressions the reducing factor and $\Delta\varepsilon'$ have been computed for every tenth degree of ω, and given in Tables V and VI. With these quantities, taken from the tables with ω as an argument, the reductions are made by (35). The expressions above are the same, in a different form and with a different notation, as those in Tidal Researches, § 101.

20. In determining the amplitudes and epochs of the components of long period, we can use the daily averages instead of the hourly observations, and still adopt the same method as in the case of the short-period components. The averages of H in this case, taken for equal spaces of the long-period arguments, may likewise be included in the expression of a (12), in which $\varphi_{e'}$ then becomes one of the long-period arguments, and the corresponding value of $si_{e'}$ becomes $u_{e'}$, the value of u in (4) belonging to the long-period argument with reference to which the observations are grouped. We shall, therefore, have in place of (10) $\varphi_e=\varphi_{e'}+(i_e-u_{e'})\,t$, in which all the observations falling within given limits of the argument $\varphi_{e'}$ may likewise be regarded as being evenly distributed through the argument φ_e, whether one of long or short period, as in the case of (10), although we use the averages of the daily averages. The formulæ, therefore, of either (13), (15), and (16), or of (18), (19), and (20) can be used, as in the case of short-period components, by putting $u_{e'}$ for $i_{e'}$ in the expressions of m and n. On account of the large value of $(i_e-u_{e'})\,\tau$ in the denominator in the case of the short-period components, the terms in m and n depending upon the short-period components, even when A_e is large, are of no importance, and this is also the case when the terms in m and n depend upon the long-period components, since the values of A_e in these are always very small. We may therefore neglect m and n in (16) and (19) without material error.

II.—ANALYSIS OF THE TIDES OF PULPIT COVE.

21. It now only remains under this head to apply the methods, principles, formulæ, and tables, given under the preceding head, to the observations on hand. This requires a great amount of work, of which the details, of course, cannot be given here, but, for the most part, the results merely. We shall, however, give one example, somewhat in detail, to show how the work has been done throughout. Since the first year's observations are not quite complete, the series having been commenced on the 21st of January, and therefore requires to be treated a little differently in some of the details, we shall select as our example the lunar or M_1 tide of 1871. All the hourly observations throughout the year, falling within the limits of each of the twenty-four equal divisions of 15° of the argument φ_1, were included in separate groups and the averages taken, and denoted by a_0, a_1 a_2, &c., a_{23}. The group in which each hourly observation is to be included is found by comparing the day and the hour in the first column of Table I with the corresponding number in the

column under M_1, which is the argument or value of φ_1 for that date. The same is done for any other of the M_e tides. In this manner were obtained the twenty-four averages, or values of a, in feet, in the following table:

a		1		2		3		4		6		8		v
a_0	5.502	(a_0-a_{12})	+.033	(a_0-a_6)	−9.146	(a_0-a_4)	−6.206	(a_0-a_3)	−3.603	(a_0-a_2)	−1.333	(a_0-a_1)		−.008
1	5.626	(1−13)	46	(1−7)	8.912	(1−5)	8.128	(1−4)	6.082	(1−3)	3.479	(1−2)	−1.209	+ 5
2	6.835	(2−14)	31	(2−8)	6.481	(2−6)	7.813	(2−5)	6.919	(2−4)	4.873			− 4
3	9.105	(3−15)	+ 11	(3−9)	−1.954	(3−7)	5.433	(3−6)	5.543	(3−5)	4.649			+ 12
4	11.708	(4−16)	− 24	(4−10)	+3.217	(4−8)	−1.608	(4−7)	−2.830	(4−6)	2.940	(4−5)	−2.046	− 21
5	13.754	(5−17)	+ 3	(5−11)	7.328	(5−9)	+2.695	(5−8)	+0.438	(5−7)	−0.784			+ 21
6	14.648	(6−18)	− 29	(6−12)	9.179	(6−10)	6.157	(6+9)	3.589	(6−8)	+1.332			− 8
7	14.538	(7−19)	59	(7−13)	8.958	(7−11)	8.112	(7−10)	6.047	(7−9)	3.479	(7−8)	+1.222	− 4
8	13.316	(8−20)	40	(8−14)	6.512	(8−12)	7.847	(8−11)	6.890	(8−10)	4.825			+ 5
9	11.059	(9−21)	52	(9−15)	+1.965	(9−13)	5.479	(9−12)	5.590	(9−11)	4.633			− 2
10	8.491	(10−22)	62	(10−16)	−3.241	(10−14)	+1.687	(10−13)	+2.911	(10−12)	3.022	(10−11)	+2.065	+ 9
11	6.426	(11−23)	86	(11−17)	7.325	(11−15)	−2.668	(11−14)	−0.378	(11−13)	+0.846			− 20
12	5.469	(12−0)	33	(12−18)	9.208	(12−16)	6.263	(12−15)	3.625	(12−14)	−1.335			+ 11
13	5.580	(13−1)	46	(13−19)	9.017	(13−17)	8.171	(13−16)	6.152	(13−15)	3.514	(13−14)	−1.224	− 3
14	6.804	(14−2)	31	(14−20)	6.552	(14−18)	7.873	(14−17)	6.947	(14−16)	4.928			− 9
15	9.094	(15−3)	− 11	(15−21)	−2.017	(15−19)	5.503	(15−18)	5.583	(15−17)	4.657			+ 11
16	11.732	(16−4)	+ 24	(16−22)	+3.179	(16−20)	−1.624	(16−19)	−2.865	(16−18)	2.945	(16−17)	−2.019	− 1
17	13.751	(17−5)	− 3	(17−23)	7.239	(17−21)	+2.640	(17−20)	+0.395	(17−19)	−0.846			+ 2
18	14.677	(18−6)	+ 29	(18−0)	9.175	(18−22)	6.124	(18−21)	3.566	(18−20)	+1.321			− 9
19	14.597	(19−7)	59	(19−1)	8.971	(19−23)	8.085	(19−22)	6.044	(19−21)	3.486	(19−20	+1.241	+ 15
20	13.356	(20−8)	40	(20−2)	6.521	(20−0)	7.854	(20−23)	6.844	(20−22)	4.803			− 5
21	11.111	(21−9)	52	(21−3)	+2.006	(21−1)	5.485	(21−0)	5.609	(21−23)	4.599			− 10
22	8.553	(22−10)	62	(22−4)	−3.155	(22−2)	+1.718	(22−1)	+2.927	(22−0)	3.051	(22−23)	+2.041	+ 9
23	6.512	(23−11)	+ 86	(23−5)	−7.242	(23−3)	−2.593	(23−2)	−0.323	(23−1)	+0.886			+ 4

The columns in this table headed 1, 2, 3, &c., contain the differences of the values of a which enter into the expressions of M_1, N_1, M_2, N_2, &c., in (23,) (24), &c. After the first the subscript numbers of a only are written, as (1 — 13) for $(a_1 - a_{13})$, (1 — 7) for $(a_1 - a_7)$, &c. It is seen that these subscripts in the same column all have a common difference, so that by writing the values of a on a separate slip and placing it by the side of the column of values in the table, these differences are all readily taken. With these differences the values of A, B, C, &c.; A′, B′, C′, &c.; A_2, B_2, C_2; A'_2, B'_2, C'_2; A_3, B_3; A'_3, B'_3; A_4, B_4, B'_4 are found.

If we assume that the averages, or values of a, may be expressed by (12), or as written out in detail for this case, by

$$a = A_0 + A_1 \cos(\varphi_1 - \varepsilon_1) + A_2 \cos(2\varphi_1 - \varepsilon_2) + A_3 \cos(3\varphi_1 - \varepsilon_3) + A_4 \cos(4\varphi_1 - \varepsilon_4) + A_6 \cos(6\varphi_1 - \varepsilon_6) + A_8 \cos(8\varphi_1 - \varepsilon_8)$$

we get from (22), (23), (24), &c., the following values of A_0, M_3, and N_3; and then from (16) by neglecting m and n and putting $k = 1$, we get the corresponding values of A_3 and ε_3:

$A_0 = 10.0518$

$M_1 = +\ .0257$,	$N_1 = -\ .0160$,	$A_1 = .0303$,	$\varepsilon_1 = 328^\circ.1$
$M_2 = -\ 4.6843$,	$N_2 = -\ 0.9500$,	$A_2 = 4.7798$,	$\varepsilon_2 = 191^\circ.47$
$M_3 = +\ .0010$,	$N_3 = -\ .0017$,	$A_3 = .0020$,	$\varepsilon_3 = 300^\circ.1$
$M_4 = -\ .0065$,	$N_4 = -\ .0199$,	$A_4 = .0208$,	$\varepsilon_4 = 254^\circ.1$
$M_6 = +\ .0896$,	$N_6 = +\ .0507$,	$A_6 = .1035$,	$\varepsilon_6 = 29^\circ.5$
$M_8 = -\ .0104$,	$N_8 = +\ .0051$,	$A_8 = .0115$,	$\varepsilon_8 = 154^\circ.3$

With these values of the amplitudes and epochs, A and ε, the preceding expression of a represents the values of a in the preceding table with the residuals v in the last column. It is seen from the residuals that there are no other sensible terms in the expression of a above, and the amplitude A_3 of the terdiurnal component is so small that it cannot be regarded as being a real

component. These residuals, moreover, furnish almost a perfect check against any errors in the work, either of taking the averages of the groups of observations or of computing the amplitudes and epochs from these averages. Each of the 24 values of a above is the average of about 365 observations, and any error in taking these averages shows itself in the residuals, so that if any of these residuals are unusually large, it is necessary to revise the work until the residuals fall within the usual limits. Any error, likewise, of any consequence in the work of obtaining the amplitudes and epochs from these averages is shown by the residuals, since any such error not only causes the residuals to be unusually large, but also causes a preponderance of plus or minus signs in different parts of the column of residuals. It is better, therefore, to obtain the uncorrected amplitudes and epochs of the tidal components, which are the true ones belonging to the expression of a, so that the accuracy of the work may be tested by means of the residuals, and then to correct these amplitudes and epochs, than it is to correct the co-ordinates M and N in order to obtain directly the corrected ones.

In the manner which has just been explained sixteen tides for each of the six years, making ninety-six in all, have been treated, and the work checked by means of the residuals. In most of them, however, it was found necessary to use only two terms in the preceding expression of a, which always left the residuals so small as to indicate that if they contained yet any other real terms, they were too small to be of any importance. It is seen from the formulæ for obtaining the constants in any tidal component that they are entirely independent of those of any of the others, so that we can obtain those of any one of them and neglect the balance if we wish, but the residuals then do not furnish a check for the accuracy of the work.

22. If the averages were those of the true tide merely, unaffected by the abnormal disturbances of winds, ocean currents, and changes of barometric pressure, and all the real terms in the preceding expression of a were taken into account, we should have no residuals. These residuals, then, are the parts of these disturbances which are not eliminated from the averages by the number of observations used, together with the effects of very small terms neglected in the expression of a. It is important, therefore, to estimate the probable errors in the results arising from these causes.

From the residuals in the preceding table we get $\Sigma v^2 = .002586$. With this value, putting $m = 13$, since there are 13 unknown quantities, we get from (30) $e = .0103$ of a foot, or about $\frac{1}{8}$ of an inch, for the probable error of the averages of one year's observations, upon the hypothesis that the residuals contain no real neglected terms in the expression of a. With this value of e we likewise get from (32) $r = .0030$ of a foot, or about $\frac{1}{28}$ of an inch, for the probable error of the amplitudes A, arising from uneliminated abnormal disturbances. From (33) we get $r' = 0^\circ.03$ for the probable error of the epochs ε. If the residuals are still affected by any real neglected terms in the expression of a, then the probable errors are still less than these estimates. The example in hand is about an average one, the residuals in some cases giving larger and in others smaller probable errors, without much range of variation.

In the course of so much work there are of course some small errors which escape detection by means of an inspection of the residuals. The effects of such errors are also included in these probable errors. As the residuals must be due mostly to uneliminated abnormal disturbances, and only in a comparatively few cases the result of error, the part of the probable error due to the latter cause must be very small, so that we have a complete check against errors of any importance.

23. Having obtained the amplitudes and epochs in the expression of a preceding, the next step is to correct them, so as to reduce them to the true amplitudes and epochs of (8). This is done by means of (17) and (20) after the values of m and n have been computed by means of the formulæ of § 15. These formulæ give in this case, for the diurnal component, $m_1 = -.0028$; $n_1 = .0022$; and for the semidiurnal, $m_1 = -.0185$ and $n_1 = -.0030$. With these values of m and n and the values of $\frac{1}{k_1}$, $\frac{1}{k_2}$, &c., in (29), we get from (20) and (17), in which these uncorrected amplitudes and epochs are distinguished by A′ and ε'', the following corrected values—

$$A_1 = .0332 \qquad \varepsilon'_1 = 324.3 \qquad A_2 = 4.8533 \qquad \varepsilon'_2 = 191^\circ.50$$

It still remains to reduce all the epochs to their values in (2) and (3), and this is done by means of the last of (9) with the values of c_s taken from Table II. From this table we get, for the year

1871, $c_1 = 64^{\circ}.9$, $2\,c_1 = 129^{\circ}.92$, $3\,c_1 = 194.88$, $4\,c_1 = 259^{\circ}.84$, $6\,c_1 = 29^{\circ}.76$, $8\,c_1 = 159.68$. These values, added to the values of the epochs ε' already obtained, give the epochs of (3).

24. The following are the true amplitudes and epochs of (3) for the sixteen M_e tides, as obtained from the averages of the observations of each of the six years taken separately, and corrected and reduced as in the example of the preceding section. The subscript numbers of A and ε denote that they belong to the diurnal, semidiurnal, &c., components. The amplitudes are given in feet.

M_1 or M.

	1870.	1871.	1872.	1873.	1874.	1875.
$A_1=$	.0424	.0332	.0221	.0395	.0352	.0229
$\epsilon_1=$	6°.9	29°.2	235°.8	218°.2	197°.1	175°.6
$A_2=$	4.9406	4.8533	4.8558	4.8037	4.7369	4.7648
$\epsilon_2=$	322°.07	321°.42	322°.12	321°.55	320°.63	320°.40
$A_3=$	.0122	.0020	.0092	.0120	.0058	.0019
$\epsilon_3=$	263°.0	135°.0	161°.0	123°.0	279°.0	229°.0
$A_4=$	.0386	.0214	.0194	.0281	.0198	.0219
$\epsilon_4=$	159°.6	153°.9	173°.0	114°.5	126°.6	121°.2
$A_6=$	.1176	.1148	.1213	.1253	.1222	.1192
$\epsilon_6=$	60°.9	59°.5	64°.6	60°.7	60°.3	57°.8
$A_8=$	.0172	.0140	.0090	.0176	.0137	.0156
$\epsilon_8=$	336°.0	314°.0	354°.0	336°.0	326°.0	320°.0

M_2 or S.

	1870.	1871.	1872.	1873.	1874.	1875.	Average.
$A_1=$	.0676	.0154	.0215	.0195	.0195	.0019	.0242±.0043
$\epsilon_1=$	128°.6	78°.0	67°.2	123°.3	73°.0	25°.2	65°.9±10°.8
$A_2=$	.8245	.7345	.7755	.7965	.7456	.7460	.7706±.0069
$\epsilon_2=$	350°.0	356°.0	357°.2	353°.7	353°.5	357°.8	354°.70±9°.80
$A_3=$	.0096	.0093	.0904	.0027	.0072	.0018	
$\epsilon_3=$	31°.0	63°.0	300°.0	297°.0	13°.0	59°.0	
$A_4=$	.0078	.0042	.0032	.0058	.0051	.0037	
$\epsilon_4=$	113°.0	73°.0	346°.0	222°.0	350°.0	29°.0	

M_3 or K.

	1870.	1871.	1872.	1873.	1874.	1875.
$A_1=$	.4376	.4626	.4744	.4915	.5120	.4973
$\epsilon_1=$	136°.2	137°.5	138°.8	134°.9	132°.7	130°.0
$A_2=$	.2406	.2381	.2540	.2331	.2936	.3040
$\epsilon_2=$	22°.0	6°.8	12°.4	12°.6	11°.0	355°.3

M_4 or L.

	1870.	1871.	1872.	1873.	1874.	1875.	Average.
$A_1=$	.0162	.0312	.0368	.0097	.0254	.0091	
$\epsilon_1=$	314°.0	346°.0	255°.0	152°.0	45°.0	318°.0	
$A_2=$	.3362	.1724	.1945	.2847	.2230	.2093	.2367±.0167
$\epsilon_2=$	189°.5	187°.1	155°.9	192°.8	218°.5	208°.9	192°.1±6°.4

M_5 or N.

	1870.	1871.	1872.	1873.	1874.	1875.	Average.
$A_1=$	.0042	.0059	.0326	.0267	.0119	.0171	
$\epsilon_1=$	258°.0	226°.0	339°.0	117°.0	127°.0	311°.0	
$A_2=$	1.0459	1.1359	0.9860	0.9286	0.9909	1.0268	1.0190±.0192
$\epsilon_2=$	295°.4	291°.3	286°.5	289°.1	291°.2	288°.5	290.3±0°.8

M_6 or O.

	1870.	1871.	1872.	1873.	1874.	1875.
$A_1=$	.3440	.3594	.3974	.4062	.4217	.4441
$\epsilon_1=$	100°.8	96°.6	102°.7	100°.1	106°.0	107°.9
$A_2=$	.1831	.0937	.1727	.0614	.0981	.0960
$\epsilon_2=$	343°.7	298°.4	225°.6	171°.4	301°.1	305°.5

M_7 or P.

	1870.	1871.	1872.	1873.	1874.	1875.	Average.
$A_1=$	.1524	.1511	.1484	.1516	.1548	.1601	.1530±.0011
$\epsilon_1=$	127°.3	132°.6	136°.7	132°.2	123°.6	130°.5	130°.5±1°.2
$A_2=$	.1168	.0865	.0482	.0521	.0490	.0628	
$\epsilon_2=$	98°.8	306°.0	230°.1	98°.9	68°.2	240°.8	

M_8 or λ.

	1870.	1871.	1872.	1873.	1874.	1875.	Average.
$A_1=$	.0037	.0107	.0152	.0027	.0156	.0146	
$\epsilon_1=$	118°.0	273°.0	264°.0	161°.0	212°.0	115°.0	
$A_2=$	.0833	.1320	.0427	.1196	.1559	.1770	.0637
$\epsilon_2=$	161°.5	256°.1	146°.0	256°.3	124°.5	233°.9	196°.3

M_9 or ν.

	1870.	1871.	1872.	1873.	1874.	1875.	Average.
$A_1=$	.0016	.0112	.0040	.0055	.0026	.0031	
$\epsilon_1=$	123°.0	191°.0	95°.0	151°.0	168°.0	37°.0	
$A_2=$	.1618	.3348	.0933	.3197	.2152	.2888	.2735
$\epsilon_2=$	301°.0	316°.7	306°.3	316°.8	263°.3	323°.3	308°.0

M_{10} or μ.

	1870.	1871.	1872.	1873.	1874.	1875.	Average.
$A_1=$	.0252	.0190	.0148	.0152	.0122	.0250	
$\epsilon_1=$	208°.0	339°.0	93°.0	131°.0	2°.0	146°.0	
$A_2=$	.0433	.0336	.0150	.0313	.0480	.0352	.0322
$\epsilon_2=$	237°.2	194°.2	175°.9	240°.6	219°.0	201°.7	216°.2

M_{11} or R.

	1870.	1871.	1872.	1873.	1874.	1875.
$A_1=$	.0310	.0251	.0216	.0225	.0133	.0088
$\epsilon_1=$	266°.0	17°.0	358°.0	283°.0	354°.0	290°.0
$A_2=$	.0680	.0260	.0554	.0502	.0348	.0619
$\epsilon_2=$	227°.0	26°.0	92°.0	352°.0	182°.0	8°.0

M_{12} or T.

	1870.	1871.	1872.	1873.	1874.	1875.	Average.
$A_1=$	.0118	.0079	.0097	.0196	.0156	.0067	
$\epsilon_1=$	31°.0	45°.0	82°.0	20°.0	87°.0	78°.0	
$A_2=$	.1887	.1035	.1901	.2332	.1555	.0871	.0220
$\epsilon_2=$	239°.0	138°.8	50°.2	331°.2	243°.1	122°.6	287°.7

M_{13} or J.

	1870.	1871.	1872.	1873.	1874.	1875.	Average.
$A_1=$	.0245	.0140	.0258	.0308	.0194	.0092	.0196
$\epsilon_1=$	340°.5	266°.2	320°.2	314°.5	323°.1	292°.4	315°.4
$A_2=$	.0036	.0156	.0044	.0252	.0088	.0139	
$\epsilon_2=$	162°.0	219°.0	287°.0	94°.0	143°.0	150°.0	

M_{14} or Q.J

	1870.	1871.	1872.	1873.	1874.	1875.	Average.
$A_1=$	.0595	.0577	.0727	.0578	.0770	.0726	.0662±.0024
$\epsilon_1=$	244°.8	271°.2	258°.7	246°.3	272°.0	283°.8	262°.8±3°.7
$A_2=$	.0153	.0146	.0112	.0252	.0190	.0220	
$\epsilon_2=$	357°.0	100°.0	150°.0	189°.0	75°.0	76°.0	

$M_2 S_2$ or M S.

	1870.	1871.	1872.	1873.	1874.	1875.
$A_4=$	.0357	.0157	.0252	.0095	.0284	.0104
$\epsilon_4=$	308°.0	210°.0	21°.0	299°.0	311°.0	75°.0

M_2 (S_2 S_2) or $_2$S M.

	1870.	1871.	1872.	1873.	1874.	1875.
$A_2=$	.0388	.0142	.0212	.0246	.0553	.0246
$\epsilon_2=$	297°.0	135°.0	318°.0	270°.0	327°.0	41°.0

In all cases in which the amplitudes and epochs above should be the same for the several successive years, the most probable values, as deduced from the several results of the six years, have been given, and in most cases the probable errors. Where the epochs differ but little from the means, these have been obtained from the averages simply, but where the epochs, from the various disturbing causes, differ so much from the means that the cosines of the differences from the mean differ sensibly from unity, the components for the several years have been resolved into two co-ordinates, having the same relations each year to the disturbing forces, and then from the averages of these co-ordinates, new amplitudes and epochs have been deduced for the average results.

25. Where the tide-components vary from year to year from the effects of smaller components of very nearly the same period, the amplitudes and epochs must be purified from the effects of these before we can obtain the most probable values from the averages of the results of the six years. This is done by the method given in § 19.

In the diurnal components we have two cases of this kind, M_3 and M_6. The amplitudes and epochs of the M_3 diurnal tide are corrected in the following manner:

Year.	ω	A′	$\frac{A}{A'}$	A	Residual.	ϵ'	$\delta\epsilon$	ϵ	Residual.
	°				°	°	°	°	°
1870	110.00	.4376	1.0393+0.7$\delta\mu$	.4548+0.3$\delta\mu$	+020	136.2	−7.5	128.7	−0.9
1871	90.34	.4626	0.9923 - 0.5	.4590−0.2	+62	137.5	−7.6	129.9	+0.3
1872	70.96	.4744	0.9520−1.4	.4517−0.7	−11	138.8	−6.9	131.9	+2.8
1873	51.63	.4915	0.9200−2.1	.4522−1.0	−6	134.9	−5.5	129.4	−0.2
1874	32.31	.5120	0.8974−2.3	.4594−1.1	+66	132.7	−3.7	129.0	−0.6
1875	12.98	.4973	0.8843− 2.5	.4398 1.2	−130	130.0	−1.5	128.5	−1.1
Average............	....	....	....	.4528±.0019 - 0.65$\delta\mu$	....	....	....	129.6±0.35	....

The column under the head of ω contains the longitude of the moon's node for the middle of each year, taken from Table II. The columns under A′ and ϵ' are uncorrected amplitudes and epochs of the M_3 diurnal tide in the preceding section, distinguished here from the corrected ones by an accent, and $\frac{A}{A'}$ and $\delta\epsilon$ are taken from Table III with ω as an argument. The corrections are then made by (35). These corrections leave the final results extremely satisfactory, the residuals and the probable errors being very small. Besides, the signs of the residuals do not indicate that they are affected by any term of long period, such as that of the period of the moon's node, and hence the corrections for the effects of the terms depending upon the moon's node, seem to have been accurately made. The probable error of the epoch, reduced to lineal measure is $\frac{0°.35}{57°.3}\times.4528=.0028$, or about one-thirtieth of an inch, which is a very little more than that of the amplitude.

In the correction of the epochs above, the small effect depending upon the correction of the moon's mass has been neglected, and the residuals and probable errors in both the amplitudes and epochs are upon the hypothesis that the assumed mass of one-eightieth of that of the earth needs

no correction. But it would require a very large change in the moon's mass to affect the results much. A change of one-tenth part of the moon's mass would give $\delta\mu = .00125$, with which value the corrections of the amplitudes above, due to this cause, would be extremely small.

The M_6 diurnal tide is corrected in the same manner as in the preceding case by using Table IV instead of Table III. The following values of A′ and ε' are taken from the values given in the preceding section and distinguished here from the corrected ones by an accent. The factor $\frac{A}{A'}$, and the correction $\delta\varepsilon$, are taken from Table IV with ω as an argument.

Years.	ω	A′	$\frac{A}{A'}$	A	Residual.	ε'	$\delta\varepsilon$	ε	Residual.
	°	*Feet.*		*Feet.*	°	°		°	°
1870	110.00	.3440	1.0554	.3631	+45	100.8	+12.6	113.4	+2.3
1871	90.34	.3594	0.9778	.3514	−72	96.6	+12.4	109.0	−1.1
1872	70.96	.3974	0.9160	.3640	+54	102.7	+11.0	113.7	+2.6
1873	51.63	.4062	0.8700	.3534	−52	100.1	+ 8.6	108.7	−2.4
1874	32.31	.4217	0.8392	.3539	−47	106.0	+ 5.7	111.7	+0.6
1875	12.98	.4441	0.8230	.3656	+70	107.9	+ 2.3	110.2	−0.9
Average............	....	....	....	.3586 ± .0015	....	....	....	291.1 ± 0.55	....

In this, as in the preceding case, the corrected results are very satisfactory, the residuals and the probable errors being quite small, and the signs of the residuals indicate that the corrections for the effects of the term depending upon the moon's node are correctly made. The probable error of the amplitude here, in the average of the six years, is a little less, and that of the epochs a little greater, than in the preceding case.

26. The two semidiurnal components which need to be corrected for the effect of the two terms of nearly the same periods depending upon the moon's node, are those of M_1 and M_3, the mean lunar and declinational components.

The following values of A′ and ε' are taken from the semidiurnal component of the M_1 tide in § 24, and distinguished here by an accent, as in the preceding cases. The values of $\frac{A}{A'}$, and $\delta\varepsilon$ are taken from Table V with ω as an argument.

Years.	ω	A′	$\frac{A}{A'}$	A	Residual.	ε'	$\delta\varepsilon$	ε	Residual.
		Feet.		*Feet.*		°	°	°	°
1870	110.00	4.9406	0.98735	4.8781	−168	322.07	−1.92	320.15	+0.33
1871	90.34	4.8533	0.99915	4.8494	−455	321.34	−2.07	319.27	−0.55
1872	70.96	4.8558	1.01120	4.9102	+153	322.12	−1.96	320.16	+0.34
1873	51.63	4.8037	1.02228	4.9107	+158	321.55	−1.65	319.90	+0.08
1874	32.31	4.7369	1.03105	4.8841	−108	320.63	−1.13	319.50	−0.32
1875	12.98	4.7648	1.03609	4.9368	+419	320.40	−0.47	319.93	+0.11
Average............	....	....	....	4.8949 ± .0084	....	....	....	319.82 ± 0.10	....

Here the probable error of the average amplitude is much greater than that of the preceding cases, and the probable error of the epoch, reduced to lineal measure, is about the same as that of the amplitude. In the case of the amplitudes it is seen, also, that the minus signs rather prevail in the first half of the series and the plus signs in the latter half. The average of the first three is .0157 feet, or nearly $\frac{1}{5}$ of an inch, while that of the last three is the same with the contrary sign. This result has an important bearing upon the principle by which the corrections for the lunar nodal term have been made, which is, that larger and smaller tides are proportional to the forces upon which they depend. This principle is theoretically correct for deep-water tides without friction, but I have not found it to be strictly correct in any case which I have examined. In the tides of Brest, Liverpool, Kurrachee, San Francisco, and others, and notably so in the Boston tides, I have found that the larger semidiurnal tides are smaller in proportion to the forces than the smaller

ones. The same is shown by the residuals above to be true in the tides of Pulpit Cove; for these residuals show that the larger amplitudes of the first part of the series, corrected upon the principle of proportionality to the forces, are more than $\frac{1}{3}$ of an inch smaller than the corrected smaller amplitudes of the last half of the series. Having a series of only six years on hand, which is less than one-third of the period of the moon's node, this matter cannot be so clearly brought out here, but if we had a series of 19 years, it would be interesting to trace the residuals, given by the corrected amplitudes as above, around the whole period; and they would undoubtedly have contrary signs in the two opposite parts of the period corresponding to the largest and the smallest amplitudes, as I have found in the case of the Liverpool tides. (Tidal Researches, § 206).

The following uncorrected amplitudes and epochs, A′ and ε', of the declinational semidiurnal component, are taken from § 24 under M_3, and the reducing factors and the corrections of the epochs are taken from Table VI.

Years.	ω	A′	$\frac{A}{A'}$	A	Residual.	ε'	$\delta\varepsilon$	ε	Residual.
	°	*Feet.*		*Feet.*		°	°	°	°
1870	110.00	.2406	1.0624 + 2.3$\delta\mu$	.2556 + 0.6$\delta\mu$	+260	22.0	−16.6	5.4	+6.6
1871	90.34	.2381	0.9631 0.0	.2293 0.0	− 3	6.8	16.0	350.8	−8.0
1872	70.96	.2540	0.8882 − 2.0	.2256 −0.5	− 40	12.4	13.9	358.5	−0.3
1873	51.63	.2331	0.8348 − 3.1	.1946 −0.8	−350	12.6	10.8	1.8	+3.0
1874	32.31	.2936	0.8010 − 3.9	.2352 −1.1	+ 56	11.0	7.0	4.0	+5.2
1875	12.98	.3040	0.7814 − 4.3	.2375 −1.3	+ 79	355.3	− 2.9	352.4	−6.4
Average				.2296 ± .0055 − 0.5$\delta\mu$				358.8 ± 1.7	

The probable errors here are smaller than in the preceding case. The probable error of the epoch, reduced to lineal measure, is ± .0069, a little greater than that of the amplitudes. The signs of the residuals in both the amplitudes and epochs are entirely satisfactory, and the correction for the effect of the small term of nearly the same period, depending upon the moon's node, seems to be accurately made in this case.

27. The amplitudes and epochs of five components of long period have been obtained from the analysis of the observations by the method explained in § 20. All the daily averages of observations throughout the year contained within each equal division of 15° of each argument were put into one group and the averages taken, from which were obtained, as in the case of the short period components, twenty-four averages for each argument, and from these the amplitudes and epochs were obtained as in the short-period tides. For the purpose of arranging the daily averages in their proper groups a table of the values of each argument for the middle of each day throughout the year was made, so that it would be readily seen from the table in which group each daily average was to be put.

The corrections for m and n (19) become of no importance in long-period tides, on account of the large value of the denominator $s\,(i^1_e - i_{e'})\,\tau$. Putting for si_e its value in the case of the lunar semidiurnal tide, which is 28°.98410, and for $si_{e'}$ the value of $u_{e'}$, as directed in § 20, and taking the daily value of u_2 in § 4 as an example, we should have for the hourly value of u_2 0°.5444, and hence $(si_e - u_2) = 28°.4397$. With this value, putting $\tau = 8760$, the number of hours in a year, the denominator becomes 4265, and the coefficient of A_e for the maximum value of the numerator, would be only $\frac{1}{2132}$. This in the case of the tides in hand, in which A for the lunar tide is less than 5 feet, would give a value for m of only about .0023 of a foot, or $\frac{1}{36}$ of an inch. For all the other short period components the value would be much less.

The errors arising from the unequal distribution of the observations through the arguments may amount to much more than these, but they cannot amount to anything of much importance, as will be shown by the following results.

As the arguments all commence with $t = 0$, for the beginning of the year, the epochs immediately obtained had to be corrected by adding the values of k_e given in § 4, in order to make the epochs of the several years comparable with one another. A correction also for the amount of

change in the arguments in twelve hours, had to be applied, since by using the daily averages the time of $t = 0$ was noon instead of midnight.

The following are the results obtained in the manner just explained:

Lunar elliptic component.

	1870.	1871.	1872.	1873.	1874.	1875.
A =	.082	.069	.024	.074	.057	.014
ϵ =	96°.3	35°.0	34°.2	295°.9	279°.3	109°.5

Lunar declinational component.

	1870.	1871.	1872.	1873.	1874.	1875.
A =	.012	.037	.048	.047	.073	.040
ϵ =	1°.1	208°.3	10°.7	211°.8	4°.4	94°.8

Solar elliptic and declinational components.

	1870.	1871.	1872.	1873.	1874.	1875.
A_1 =	.080	.176	....	.120	.177	.262
ϵ_1 =	119°.9	162°.3	....	167°.1	187°.5	225°.8
A_2 =	.090	.093	....	.097	.026	.152
ϵ_2 =	47°.6	57°.1	....	111°.1	145°.6	74°.3

$M_2\ S_2$ (shallow-water component).

	1870.	1871.	1872.	1873.	1874.	1875.	Average.
A =	.190	.157	.163	.229	.180	.123	.174 + .010
ϵ =	160°.5	150°.7	152°.3	161°.3	159°.3	158°.5	157.1 + 1°.2

The varying values of A, and the scattering values of ϵ, falling somewhat in all parts of the circumference of the circle, indicate that there is no sensible lunar elliptic or declinational component, which, it will be shown, is what theory requires. The epochs ϵ_1, of the solar elliptic, and those of ϵ_2, of the solar declinational component, falling somewhat in the same part of the circumference of the circle, indicate real components; but since the period of the argument is one year in this case, they are undoubtedly due to meteorological causes, such as changes of winds and of barometric pressure, and likewise of ocean currents, all of which may have a variation with an annual period. The amplitudes and epochs of these components for 1872 could not be obtained on account of a change in the zero of the tide-gauge during this year.

The shallow-water component is the $M_2\ S_2$ long-period component of Schedule IV. The small range in the values of the epochs for the several years, as well as the largeness of the amplitudes, shows that this is a real tide-component with an amplitude of more than two inches. The small probable errors of both the amplitude and epoch, the former being only one-eighth of an inch, show that the principle which has been adopted of using the daily averages cannot lead to much error, for the greater part of these probable errors is no doubt due to the accidental uneliminated disturbances of winds, barometric pressure, &c., since they are nearly of the same order as those in the short-period tides.

The height of the mean level of the sea above the zero plane of the tide-gauge was found to be in feet as follows:

	1870	1871	1872	1873	1874	1875
A_0 =	10.1885	10.0768	8.5042	10.2312	10.1286	10.2786

It is seen that there was a great change for some reason in the zero plane of the tide-gauge in 1872. The series of observations is too short to decide the interesting question in geology, whether the continent is rising or falling, unless the rate of change were considerable, and we could be certain that the zero of the tide-gauge was kept on the same level plane throughout.

III.—COMPARISONS OF OBSERVATION WITH THEORY.

28. It is proposed under this head to ascertain how nearly the preceding results from observation satisfy the various theoretical relations between the amplitudes, epochs, and tidal forces, first published in my Tidal Researches. This is important, since if these relations or laws can be shown from comparisons with observation to hold generally for all tide-stations, then these relations need no longer be sought by a laborious analysis of tide observations.

Let us consider, first, the mean lunar tide, as obtained from analysis, and given in § 24. At least five of the six components given there are real tides. The first, of which the amplitude is denoted by A_1, is a very small diurnal tide depending upon corresponding forces. It is the resultant of two components arising from the two terms in Schedule I, denoted by m_1 and n_1, in which the periods, as denoted by the values of i_e, are very nearly the same as that of the mean moon. Put-

ting A and ϵ for the amplitude and epoch of the larger, and A′ and ϵ' for those of the smaller tide components, they and their resultant may be put into the following form:

$$A \cos (i_1 t + \tilde{\omega} - \tfrac{1}{2}\pi - \epsilon) + A' \cos (i_1 t - \tilde{\omega} - \tfrac{1}{2}\pi - \epsilon') = A'' \cos (i_1 t + \tilde{\omega} - \tfrac{1}{2}\pi - \delta\epsilon)$$

in which
$$A'' = \sqrt{A^2 + A'^2 - 2AA' \cos 2\tilde{\omega}} = A\sqrt{1 + \left(\frac{11}{52}\right)^2 - \frac{22}{52}\cos 2\tilde{\omega}}$$

$$\tan \delta\epsilon = \frac{A' \sin 2\tilde{\omega}}{A - A' \cos 2\tilde{\omega}} = \frac{11 \sin 2\tilde{\omega}}{52 - 11 \cos 2\tilde{\omega}}$$

In these expressions $\tilde{\omega}$ is the longitude of the moon's perigee. Since the periods are so nearly the same the amplitudes of the tidal components must be proportional to the coefficients of the terms in the forces giving rise to them, which are .052 and .011, and upon this principle we get the second forms of expression of A″ and tan $\delta\epsilon$.

It is seen from the resultant expression of the two components that the amplitudes and epochs are slightly variable, the variation depending upon the value of $\tilde{\omega}$, and also that the epochs obtained from observation, which are equal to $\epsilon - \tilde{\omega} - \delta\epsilon$, must diminish as $\tilde{\omega}$ increases, and also be affected by the inequality $\delta\epsilon$. The uneliminated abnormal disturbances are so great in comparison with this small tide, with an amplitude of only about one-third of an inch, that we cannot make a comparison of these inequalities with theory, but it is seen that the observed epochs ϵ decrease on the average about 40° per year, the rate at which $\tilde{\omega}$ increases, and it will be shown further on that the average observed epoch is that which theory requires.

Theory gives no relations between the forces, and the amplitudes and epochs of the tides from which we can determine the latter, but it only establishes certain relations between the amplitudes and epochs of the different components of approximately the same periods. We have to depend, therefore, upon observation for the amplitude and epoch of one or more of the larger components of each class, and then from these theory determines the rest. We have to depend upon observation for the amplitudes and epochs of the mean lunar semidiurnal tide, and can make no direct comparison of them with theory.

The quarter-diurnal lunar tide, of which the amplitude is denoted by A_4, is the shallow-water component of Schedule IV, designated by $M_2 M_2$. The small range of the values of the epoch ϵ_4 for the several years show that this is a real component, though quite small, the average of the observed amplitudes being only about $\frac{1}{4}$ of an inch. This component in the Liverpool tides has an amplitude of nearly nine inches, which indicates that the shallow-water components there, in general, must be very much larger than at Pulpit Cove. The only theoretical relation which we have in this tide for comparison with observation is that its amplitudes for different years should be in proportion to the squares of the amplitudes of the mean lunar semidiurnal component, as is seen from the value of R in Schedule IV. This relation holds in this case, the amplitudes of both components decreasing with the series, but the probable errors in the very small amplitudes of A_4 are proportionately so large that this agreement with theory must be regarded as being most likely accidental. In the Liverpool tides, where the amplitude of this component is very much larger, this theoretical law is satisfied by observation very accurately.

The one-sixth diurnal lunar, or M_1 tide, of which the amplitude is denoted by A_6, is the shallow-water component of Schedule V, designated by $M_2 (M_2 M_2)$. This, although a component of the second order, is much larger than the quarter-diurnal component. It is surprising to see how all the abnormal and other disturbances are eliminated in this case, as shown by the nice agreement of the results for the several years. This arises no doubt from the fact that the neglected corrections depending upon m and n, (19), in the diurnal and semidiurnal tides, are entirely insensible in this case, for reasons which have been already given, and that only the abnormal accidental disturbances have to be eliminated.

The one-eighth diurnal lunar component is that of Schedule VI, designated by $M_2 M_6$. Although the amplitudes of this component obtained by the analysis is extremely small, yet the values of the epochs all falling within a range of 40° shows that there is a real component, having a period of only three hours with an amplitude of about one sixth of an inch. It is surprising to see how completely

so many large inequalities with various periods, together with all the abnormal disturbances, can be eliminated, and so small a component brought out with so much certainty.

29. Of the solar components given under the head of M_2 or S, § 24, the first is a diurnal tide having the same period as that of the mean sun, or at least sensibly so, and arises in the same manner as the corresponding one in the lunar components. The amplitudes and epochs in this case, however, should be sensibly the same from year to year, on account of the very slow motion of the sun's perigee. The values of the epochs, as brought out by the analysis of the observations, although somewhat irregular on account of the uneliminated abnormal disturbances, are as regular as can be expected in so small a component and indicate a real term. The amplitude of this component, according to theory, should be to that of the corresponding lunar component very nearly in the proportion of the solar and lunar forces, and hence less than half as great. If we except the first value, that of 1870, which for some reason is much larger than the rest, the averages of the balance give about this ratio.

The amplitudes of the mean solar semidiurnal component should be the same from year to year. The differences given by the analysis are due to the abnormal uneliminated disturbances and to the small neglected corrections due to *m* and *n* (19), except for the principal terms. The average of the six years, however, gives a value with a probable error of only about $\frac{1}{12}$ of an inch.

The scattering values of the epochs, ϵ_3, of the terdiurnal component indicate that there is no real component of this period. The values also of the epochs, ϵ_4, indicate that there is no real sensible quarter-diurnal solar component, and this confirms the theoretical relations given in Schedule IV, in which this component is designated by S_2 S_2. From the values of R there given, it is seen that the ratio of this component to the corresponding lunar component is as A_2^2 to A_1^2, A_1 and A_2, being the amplitudes respectively of the lunar and solar semidiurnal components, and hence the amplitude of the solar quarter-diurnal component should be only about $\frac{1}{40}$ of that of the lunar. But this latter was found to be only about $\frac{1}{4}$ of an inch, and hence the amplitude of the solar quarter-diurnal component would be by theory only about $\frac{1}{160}$ of an inch, and hence the component is too small to be brought out in the analysis of the observations.

The one-sixth-diurnal solar component, designated by S_2 (S_2 S_2) in Schedule V, should have an amplitude by theory, as seen from the values of R in the schedule, which is to that of the corresponding lunar component, as A_2^3 to A_1^3, that is, only about $\frac{1}{250}$ of the latter, and hence entirely insensible.

30. In all the other M_e tides only two components, the diurnal and semidiurnal, were brought out in the analysis, the residuals showing that if there were any other sensible components of a lower order they were too small to be of any importance. Of the diurnal components only those belonging to the M_3, M_6, M_7, M_{13}, and M_{14} tides depend upon terms in the tidal forces. These we shall consider further on. The scattering values of the epochs, ϵ_1, of the others, as well as the smallness of the amplitudes in most cases as brought out by the analysis, show that there are no real diurnal components belonging to these classes of tides, which is in accordance with theory, for there are not only no terms in the tidal forces to give rise directly to such components, but there are none in Schedule III of the shallow-water diurnal components having the same periods respectively.

The quarter-diurnal shallow-water component M_2 S_2 of Schedule IV, from the results of the analysis given in § 24, does not seem to be a sensible component at Pulpit Cove, though at Liverpool it has an amplitude of about five inches. The small amplitudes given by the analysis are simply the effects of the uneliminated irregularities, as the irregular values of the epochs indicate. According to the theoretical relations given by the values of R in Schedule IV, the amplitude of this component should be to that of the lunar quarter-diurnal component as A_2 to $\frac{1}{2}A_1$, that is, as .7706 to $\frac{1}{2} \times 4.895$, or 1 to 3 nearly. But as the latter was found to be only about $\frac{1}{4}$ of an inch, the amplitude of the former should be only about $\frac{1}{12}$ of an inch, and hence the component is too small to be shown by the analysis of the observations. This, together with the example given in the last section, furnishes two cases in which the theoretical relations between the amplitudes in Schedule IV are confirmed, inasmuch as these relations give no sensible components when none are found by the analysis. In the Liverpool tides, where the amplitudes are so large that very accurate comparisons could be made, I found that these relations were accurately satisfied by the results

obtained from observation. If these theoretical relations, therefore, can be regarded as being sufficiently confirmed by observations, none of the other quarter-diurnal components of Schedule IV are sensible in the tides of Pulpit Cove, and need not be sought after by analysis of the observations.

The results of the analysis for the M_2 (S_2 S_2) semidiurnal shallow-water component of Schedule V show, by the irregularities of the values of the epochs for the several years, that if this component is not entirely insensible, it is at least very small. The principal terms in this class of components, it is seen from the values of u, have the same periods as components depending directly upon the forces, and hence cannot be separated from them in the analysis. Although the component M_2 (S_2 S_2) is not sensible in the tides of Pulpit Cove, yet the theoretical relations of the amplitudes given by the expressions of R are such that some of the other components may be very considerable. These relations make the amplitude of the M_2 (M_2 M_2) component, which is combined with the principal semidiurnal lunar component, nearly 40 times as great as that of the M_2 (S_2 S_2) component, so that, although the latter is insensible, yet the former might have an amplitude of several inches.

31. The amplitudes of the five diurnal components depending directly upon the forces, if they are not sensibly affected by shallow-water components, should satisfy the five equations (167), Tidal Researches. The left-hand members of these equations are, in the notation used in analyzing the tides, A_1 of the M_3, M_6, M_7, M_{13}, and M_{14} tides, respectively. With the values of the latter three taken from § 24, and of the first two from § 25, these equations become—

$$\begin{aligned}
.4528 - 0.65\,\delta\mu &= (.5306 - 13.1\,\delta\mu)\,(1 + .230\,\mathrm{E})\,A_0 \\
.3586 &= .3813\,(1 - .230\,\mathrm{E})\,A_0 \\
.1530 &= (.1730 - 13.6\,\delta\mu)\,(1 + .196\,\mathrm{E})\,A_0 \\
.0196 &= .011\ \ (1 + .458\,\mathrm{E})\,A_0 \\
.0662 &= .052\ \ (1 - .458\,\mathrm{E})\,A_0
\end{aligned}$$

The unknown quantities in these equations are A_0, E, and $\delta\mu$, but the terms depending upon the latter vanish if we regard the assumed mass of the moon .0125 as correct, and we then have only the two former.

These equations might be solved by the method of least squares and the most probable values of the unknown quantities obtained in this way, but as the last two do not have much weight, on account of the smallness of the amplitudes of the components upon which they depend, we shall determine them from the solution of the first three. This solution gives—

$$A_0 = .8915 \qquad \mathrm{E} = -.234 \qquad \delta\mu = -.00050 \pm .00012$$

The probable error has been obtained from the 18 residuals in satisfying the first three of the preceding equations with the values of the unknown quantities just obtained, and with the values of the amplitudes given by each separate year in §§ 24 and 25 substituted in the first members. These residuals are for each of the six years in the three classes of equations as follows:

+ .0016	+ .0045	− .0006
+ 69	− 72	− 19
− 10	+ 54	− 46
− 4	− 52	− 14
+ 68	− 47	+ 18
− 127	+ 70	+ 71

The weight for the unknown quantity $\delta\mu$ obtained from the solution is 1118. With this weight these residuals give the probable error above. With the correction of μ above we get for the moon's mass—

$$\mu = .0125 - (.00050 \pm .00012) = \frac{1}{83.3 \pm 0.77}$$

This mass obtained is very satisfactory, considering the shortness of the series of observations and the smallness of the diurnal tide in the Atlantic, and the probable error is small. But it must

be understood here that this is the error only arising from the uneliminated effects from the averages of the accidental disturbances, and does not include theoretical errors which may have the same effect from year to year. We need, therefore, some test of the theory, and this is had, in some measure, in trying whether the values of A_0 and E obtained from the first three of the preceding equations also satisfy the other two. These values give for the first members of these equations .0088 and .0515, leaving residuals of + .0108 and + .0147. These residuals are small, the larger being only $\frac{1}{6}$ of an inch, still they are larger than the probable errors of these amplitudes given in § 24, and indicate that there are errors in the theory which are greater than the probable errors arising from uneliminated effects of accidental disturbances. If the five equations had been treated by the method of least squares, these residuals would have been divided in some measure among the other equations, and consequently would have been diminished a little, but this would have changed the value of $\delta\mu$ but little on account of the small weight of the last two equations.

The theory upon which these equations are based is that of deep-water tides, and if they do not precisely satisfy the results obtained by the analysis from the observations, the explanation is to be found in the effects of shallow-water components, which are not taken into account in these equations. If we turn back to Schedule III and examine the shallow-water diurnal components given by theory, we find from the value of $u = i_3$ that there are three which have the same period as the component depending directly upon the tidal forces, of which the amplitude is the first member of the first equation, and one affecting the amplitudes, or first members, of each of the other four equations in the same manner. Now, since the effect of these shallow-water components cannot be separated from the others by analysis, we do not know whether they have a sensible effect upon the preceding equations, but since the five equations cannot be completely satisfied with any values of the unknown quantities, and we have found from the analysis that there are sensible shallow-water components among some of the other classes, it is most probable that these equations are slightly affected by sensible diurnal shallow-water components, and that the correction of the moon's mass, which we have obtained, may likewise be slightly affected by them.

The effect of these disturbing components of Schedule III might be determined by obtaining from the analysis of the observations the components $M_2 P_1$ and $S_2 O_1$, or both, since these are relatively considerable components, and then from the theoretical relations given under the heads of R and q, the amplitudes and epochs of the disturbing components could be determined. This has not been done in the preceding analysis. The best way perhaps to test the preceding theory, and to obtain a true value of the moon's mass, would be to analyze the observations of some station so near to deep-sea water that all the disturbing shallow-water components would disappear. The small probable error given above is interesting in showing with what great accuracy the moon's mass could be obtained from the observations by means of these new equations, based upon the relations between the several components of the diurnal tide, even in the case of so small diurnal tides as those of the Atlantic, if we could either correct accurately or get rid of the disturbing shallow-water components, so that the probable error would depend merely upon uneliminated accidental disturbances.

Having now obtained the value of A_0, this multiplied into .052, the coefficient of the term m_1, in Schedule I, gives .0460 for the average amplitude of the M_1 diurnal component, which is comparatively much greater than the average of those given in § 24, obtained from the analysis of the observations. The absolute difference, however, is only about $\frac{1}{8}$ of an inch.

32. The first three equations of (168), Tidal Researches, together with two others depending upon the M_{13} and M_{14} components of Schedule I, deduced in the same manner, should likewise be satisfied by the results of the preceding analysis, if there are no shallow-water disturbing components. These equations, neglecting the quantities of a third order depending upon G′, and reducing the coefficients of G expressed in terms of the radius to degrees, and putting the values of ϵ given in §§ 24 and 25, in the first members, are:

	Residuals.
$309^\circ.6 = L_0 + 13^\circ.18\,G$	0.0
$291^\circ.1 = L_0 - 13^\circ.18\,G$	0.0
$310^\circ.5 = L_0 + 11^\circ.22\,G$	$+\ 2^\circ.3$
$315^\circ.4 = L_0 + 26^\circ.25\,G$	$-\ 3^\circ.2$
$262^\circ.8 = L_0 - 26^\circ.25\,G$	$-18^\circ.9$

The first two of these equations are based upon the two principal diurnal components, and the other three should have comparatively small weight on account of the smallness of the components upon which they depend. We shall therefore determine the unknown quantities from the first two. These give $L_0 = 300^\circ.3$ and $G = .701$. These values satisfy the equations with the annexed residuals. The first two are not much larger than the probable errors of observation, but the last one is much greater than the probable error, $\pm 3^\circ.7$, of the epoch $262^\circ.8$ from observation, which indicates that the theory is not completely satisfied by observation. This is no doubt on account of the disturbing effect of small shallow-water components not taken into account in the theory. The amplitude of this component is only 0.8 of an inch, so that a disturbing component with an amplitude of only $\frac{1}{3}$ of an inch, with the proper epoch, would be sufficient to change the epoch of the affected component by the amount of the residual, and make observation agree with theory.

If the preceding equations had been treated by the method of least squares, the residuals would have been distributed among all the equations and somewhat diminished, and the values of the constants obtained would have been slightly different.

The value of G above is the time in days by which the maximum of any inequality in the diurnal tide follows the time of the maximum in the forces, as for instance, the time that the maximum lunar diurnal tide follows the moon's greatest declination, or the time that the minimum follows the time of its crossing the equator, for the diurnal tide does not absolutely vanish at this time, as is usually supposed, unless the conditions should be such as to make $E = 0$, in the tidal expressions of the preceding section, as in the case of the equilibrium theory.

33. The amplitudes of the several semidiurnal components obtained by analysis of the observations should satisfy the following three sets of equations, the second members being common to all:

Liverpool.		Kurrachee.		Pulpit Cove.		Residuals.	Tide component.
.3162	or	.3733	or	.1574	$= (.4582 - 36.2\,\delta\mu)\,(1 + .4255\,E)$	0	M_2
.0256	or	.0233	or	.0066	$= .0240\,(1 - .4255\,E)$	+ .0279	M_{10}
.0916	or	.0978	or	.0469	$= (.1256 - 3.2\,\delta\mu)\,(1 + .4599\,E)$	0	M_3
.0544	or	.0266	or	.0484	$= .0286\,(1 + .228\,E)$	− .0372	M_4
.1913	or	.2396	or	.2082	$= .1922\,(1 - 228\,E)$	0	M_5
.0266	or	.0190	or	.0130	$= .0078\,(1 + 1975\,E)$	− .0092	M_8
.0620	or	.0480	or	.0558	$= .0375\,(1 - 1975\,E)$	− .0189	M_9
.0185	or	.0410	or	.0045	$= .0266\,(1 + .4083\,E)$	+ .0088	M_{12}

The first five of these equations are taken from Tidal Researches (171), neglecting the terms of a third order depending upon ϵ', and putting for the first members of the equations their numerical values obtained from the analysis of the observations, and the others are additional equations obtained in the same manner and based upon other still smaller tide components. The components included in the equations are denoted by M_2, M_{10}, M_3, &c. The preceding are the equations which belong to deep-water tides without friction varying from the law of proportionality to the first power of the velocities, and with the two unknown quantities determined from observation should represent the tides in every part of the world. In addition to the amplitudes of the several tide components of Pulpit Cove in the first members of these equations, those for Liverpool, and Kurrachee, India, are also given for the sake of convenient comparison. These are not the absolute values of the amplitudes, but the values in terms of that of the mean lunar component of each station. In tides of shallow waters, the effects of the small shallow-water terms are not included in the relations expressed by these equations.

By the equilibrium theory the first members of these equations should be equal to the second members with $E = 0$, and hence should be exactly the same at all tide stations in every part of the world. It is seen, however, from a mere inspection of the equations, how very imperfectly this theory represents the tides at all of the three stations, and especially the tides of Pulpit Cove, and how different must be the value of E at the several stations. It is seen from the first equation that the amplitudes of the solar semidiurnal tide at Liverpool and Kurrachee are less than what

they should be by the equilibrium theory by about $\frac{1}{3}$ and $\frac{1}{6}$ respectively, and that at Pulpit Cove it is only about $\frac{1}{3}$ of what it should be.

It is readily seen from an inspection of these equations that they can be satisfied only very imperfectly for Pulpit Cove, within any determined values of $\delta\mu$ and E, and that they can be much better satisfied by multiplying the first members of the equations by an unknown constant. This constant is introduced upon the hypothesis that the tidal components are diminished by the effect of friction which is as a higher power than the first power of the velocity, as I have at various times explained. Upon this hypothesis large tides are diminished by friction more than small ones in proportion to their amplitudes, and hence where there is one large component, as the mean lunar, and a number of much smaller ones, since the amplitudes of the latter are obtained by analysis from the differences between the larger and smaller resultant tides, the smaller components are diminished more than the larger ones in proportion to their magnitudes, unless friction is as the first power of the velocity. If we take the first, third, and fifth of the preceding equations for Pulpit Cove, and introduce a constant factor c, we have—

$$.1574\, c = (.4582 - 36.2\, \delta\mu)\ (1 + .4255\, \mathrm{E})$$
$$.0469\, c = (.1256 - \ \ 3.2\, \delta\mu)\ (1 + .4599\, \mathrm{E})$$
$$.2082\, c = \ .1922\, (1 - .228\, \mathrm{E})$$

The solution of these equations gives—

$$\delta\mu = .00263 \qquad \mathrm{E} = -1.164 \qquad c = 1.166$$

The solution of all the equations by the method of least squares would give values for these constants differing but little from those above on account of the smallness of the amplitudes in the neglected equations, which gives them little weight. The value of $\delta\mu$ above gives for the moon's mass $\mu = \frac{1}{66}$, which is much too large, as is usually the case where the relations differ much from those of the equilibrium theory. The equations for Liverpool give $\mu = \frac{1}{70}$, and for Kurrachee, where the relations approximate more nearly to those of the equilibrium theory, $\mu = \frac{1}{78.6}$, which is perhaps not very much in error.

The values of the constants above satisfy the preceding eight equations with the residuals annexed, which are in terms of the amplitude of the mean semidiurnal tide, which is nearly 5 feet. The fourth residual, therefore, is more than 2 inches. Without the introduction of the constant c these residuals would be much larger. The value of this constant indicates that the amplitudes of all the smaller components relatively to that of the mean lunar, are diminished in the ratio of 1.166 to 1, or about $\frac{1}{7}$ part; of this we have already had an indication, as shown in § 26. By the introduction of the constant c, or one equivalent to it, I have in all cases found that the observations are better represented by theory, and a better mass of the moon is obtained, which indicates that there is an effect due to friction or some other cause which diminishes the amplitude of the tides.

Upon the large value of E, required to satisfy the preceding equations, depends the peculiar type of tides at Pulpit Cove, in which the solar and declinational components are very small, and the amplitude of the M_5 or principal elliptic component is very large, even a fourth larger than the solar component, although the tidal force in the former case to that of the latter is about as .192 to .458. This peculiar type of tide, so different from that of European tides and of tides generally, seems to extend all along our coasts, and is indicated by a difference in the values of E, so far as yet obtained, for our tides and for those of other parts of the world. These values are, as shown above: for Pulpit Cove, E = − 1.164; for Boston Harbor, E = − 1.412; for New York Harbor, E = − .963. In European tides we have: for Liverpool, E = − .288; Brest, E = − .323; and at Kurrachee, India, E = − .440. The value of the constant c for Boston Harbor is 1.350, and for New York Harbor, 1.245, both considerably greater than that found above for Pulpit Cove. The value of c for Brest is 1.050, which is very nearly unity, as it should be by the equilibrium theory.

According to the value of c above for Pulpit Cove, the correction applied to A′, § 26, to reduce it to A, was too great by $\frac{1}{7}$ part of the correction, since that correction was made upon the

hypothesis that large and small tides are exactly proportional to the forces. If we decrease the correction there applied by $\frac{1}{7}$ of the whole, we get—

Year.	A.	Residuals.
	Ft.	
1870.......	4.8871	+ 21
1871.......	4.8500	— 350
1872.......	4.9024	+ 174
1873.......	4.8954	+ 104
1874.......	4.8631	— 219
1875.......	4.9122	+ 272
Average ..	4.8850 ± .0075	

This correction makes the residuals more satisfactory with regard to the signs, and gives a smaller probable error. It therefore more fully establishes the principle that large tides are smaller in proportion to the tidal forces than small ones.

34. The epochs obtained from the analysis of the tide-observations should satisfy the following equations, obtained from (172) Tidal Researches, by neglecting the small terms of the third order depending upon G′, and reducing the coefficients of G to degrees, if there are no perturbations from the shallow-water terms:

Epochs.	Residuals.	Tide-components.
$^\circ$ $\quad$ $^\circ$	$^\circ$	
$319.8 = L_0$	0.0	M_1
$354.7 = L_0 + 24.4\,G$	— 4.6	M_2
$216.2 = L_0 - 24.4\,G$	+ 39.5	M_{10}
$358.8 = L_0 + 26.3\,G$	— 36	M_3
$12.1 = L_0 + 13.1\,G$	+ 31.1	M_4
$290.3 = L_0 - 13.1\,G$	— 8.3	M_5
$16.3 = L_0 + 11.3\,G$	+ 38.3	M_8
$308.0 = L_0 - 11.3\,G$	+ 7.5	M_9
$287.7 = L_0 + 23.4\,G$	— 69.1	M_{12}

The epochs in the first members are obtained from § 24 and § 26; the tide-components to which the epochs belong are given in the last column. Since the M_1 or mean lunar component is very much larger than any of the others it should have proportionate weight. We shall therefore let it determine $L_0 = 319^\circ.8$. The next most important components are M_2, M_3, and M_5. Letting the equations of these determine G, we get $G = 1.62$. The value of G is the time in days by which the maximum of the varying tide follows the maximum of the varying tidal force producing it, and it is the mean time on the average by which the greatest tides follow the conjunction or opposition of the moon and sun. This differs but little from the values found for Boston Harbor, Liverpool, and Brest.

The value of L_0 divided by $2 \times 14^\circ.492$ gives $11^h\,2^m$ for the mean lunitidal interval. This for Boston Harbor was found to be $11^h\,26^m$.

If all the preceding equations had been used in determining the probable values of the constants, giving to each equation its proper weight, the result would have differed but little from what we have obtained, since the components upon which the other equations are based are so small that the equations would have been entitled to little weight. The large residuals in the epochs of these small components do not indicate very large absolute errors, since the amplitudes being very small, a very slight perturbation changes the epochs very much. The residuals, however, are so much larger than the probable errors of the epochs, as obtained from observations by analysis, that they, as well as the residuals in the amplitudes in the preceding section, indicate considerable derangements from the shallow-water terms. If we now turn to Schedule II, it will be seen that there is a shallow-water component affecting each of the semidiurnal components M_1, M_2 and M_3, and from Schedule V it is seen that there are two such components affecting each one of the semi-

diurnal components. These components do not seem to be very large in general at Pulpit Cove, where they have been obtained from the analysis, but they are, no doubt, sufficient to so disturb the relations as to prevent a nice agreement between observation and the theory of deep-water tides, and hence to render unreliable the determination of the moon's mass from these relations.

From the preceding investigation of the shallow-water tides, I think that we can now see clearly why it is that satisfactory and consistent values of the moon's mass have not in general been obtained from the relations of the semidiurnal tides; for these relations are disturbed by the various shallow-water components, which do not enter into the theory of deep-water tides, which has been used in determining the moon's mass. The perfection of the tidal theory, so as to represent accurately the results of observation at all tide stations, and give a correct mass of the moon, depends now mainly upon the study of the shallow-water terms.

With regard to the determination of the moon's mass, from the results so far as obtained the relations of the diurnal tides promise better success in the future than those of the semidiurnal tides. The diurnal tides are not affected by so many of the shallow-water components, and it is probable that these can be determined from the analysis of the observations, since there are two comparatively quite large components with periods differing from those of any others, and hence can be determined by analysis of the observations; and then from the theoretical relations given in Schedule III the others can be, at least approximately, determined, and the components of deep-water tides which they affect can be corrected for their effect. The relations of these corrected results, obtained from the analysis of the observations, should then agree with the theoretical relatives, and give a correct mass of the moon.

35. The expression of the long-period tides, (4) in the case of a sea covering the whole earth, in which case a becomes unity, gives for the amplitudes, $0^{ft}.230(1 - 3\cos^2\theta)\,H_c$, in which θ is the polar distance, which, for Pulpit Cove, is 45° 51′. This for the declinational component, in which $H = .312$, we get .033 feet for the amplitude, or only a little more than $\frac{1}{3}$ of an inch. The effect of the constants might increase or diminish this some; we cannot say how much. The results in § 27 do not indicate that this largest term has been brought out in the analysis, and this was hardly to be expected, but the analysis at least shows that they are very small, as theory requires they should be. The shallow-water component of long period has a considerable amplitude, and is very clearly brought out by the analysis, but there are no theoretical relations with which to compare it.

IV.—PRACTICAL APPLICATIONS.

36. For the purpose of computing the hourly co-ordinates or heights of the tides at any future time equations (3) can be used, in which A and ε are given as the results of the preceding analysis of the observations and c_e are contained for the beginning of each year in Table II. The values of i_e are also contained in Schedule I. These depend upon the astronomical elements of the moon and sun and have been obtained from the developments of the tidal forces.

To facilitate the computations there should be made a table of the values of $i_e\,t$ for each day and hour of the year, exactly similar to Table I, except that the values should be given accurately in degrees. With the known values, then, of c_e and $\varepsilon_{(e,s)}$ the angle $(si_e\,t + sc_e - \varepsilon_{(e,s)})$ would be known for each M_e tide for any day and hour, and then with these values and the values of $A_{(e,s)}$, the part of H belonging to each tide-component can be readily computed for any assumed time or value of t, and the sum of all these, including the constant A_0, will be the value of H, or height of the tide, at the assumed time, above the assumed zero plane. This plane may be that of mean low-water, or it may be still lower, so as to have the signs of all the co-ordinates positive.

Such a table as the one just described could be used for any tide-station whatever, but the computations for any particular station would be still much more facilitated by making out a table of the values of each M_e tide, including all the sensible diurnal, semidiurnal, &c., components of that tide, for each degree, or each tenth degree when the tide is small, of the argument $i_e\,t + c_e$; for then with the values of the arguments, $i_e\,t$, obtained from the table of the preceding form, for any day and hour of the year, with the values of c_e for that year added, we have the arguments for each of the M_e tides with which to enter the tables of these tides and get their values without any computation. The sums of all these individual tide values added together, then give the height of the resultant tide.

If there are any sensible terms of long period they can be included in the formula of equation (3) by putting for these terms $si_c = u_c$ in equation (4), reducing the values of u_c there given to hourly values. But it is more convenient to compute these terms separately for each day, or in some cases for each month only; for the values for each hour are then readily obtained by interpolation. When there are tables formed for each tide-station, these long-period terms can be given for each tenth degree of the argument, and the arguments are obtained from the values of u_c and k_c of equation (4) in the same manner as those of the short-period components of equation (3) from the values of i_c and e_c.

When the tides are computed, the uncorrected amplitudes and epochs of the components which are affected by the lunar nodal components of nearly the same period should be used. These can be obtained from the mean or corrected ones by means of Tables III to VI by reversing the process by which the corrected ones were obtained from the uncorrected ones, that is by dividing instead of multiplying by $\frac{A}{A'}$ and subtracting $\Delta\varepsilon$ instead of adding; or the mean values of the amplitudes and epochs can be used, and the four lunar nodal components be added to the series. The amplitudes of these components, which are denoted by a in § 19, can be obtained from the expressions of e in that section. The epochs will be sensibly the same as those of the larger terms of nearly the same period. When tables are made for each tide-station the effects of these terms can be given for each tenth degree of the argument ω, having a period of nearly 19 years.

37. But the Coast and Geodetic Survey will probably never undertake to furnish hourly co-ordinates of the heights of the tide at any port, but merely the times and heights of high and low water, or of high water, only. For such a purpose such tables as the preceding could not be used with convenience. One objection in my mind to adopting the harmonic analysis of the tides was that it did not give the results in a form convenient for computing the heights and times of high and low water simply. I have, however, succeeded in putting the results into a form by which I think these latter can be computed with as much or even more facility than they can be by the old forms of expressions of high water and lunitidal intervals, especially where the diurnal tide is large, as on the Pacific coast. By the formula of Tidal Researches (22) we can put equation (8) into the following form:

$$H = H_0 + R\cos(2\,i_1\,t - \varepsilon'_{(1,2)} + \beta)$$

in which

$$R = \sqrt{M^2 + N^2} = \frac{M}{\cos\beta}$$

$$\tan\beta = \frac{N}{M}$$

$$M = \Sigma_n A_n \cos\left((i_n - 2\,i_1)\,t - \varepsilon'_n + \varepsilon'_{(1,2)}\right)$$

$$N = \Sigma_n A_n \sin\left((i_n - 2\,i_1)\,t - (\varepsilon'_n + \varepsilon'_{(1,2)})\right)$$

In this expression of H we have $\varepsilon'_{(1,2)}$, which is the epoch of the mean lunar semidiurnal tide, divided by $2\,i_1$, equal to the mean lunitidal interval, and we shall have for the variable lunitidal interval, as affected by the other components—

$$L = \frac{\varepsilon'_{(1,2)} - \beta}{2\,i_1}$$

in which $\Delta\varepsilon$ must have its value belonging to the time of high water. This assumes that the time of high water occurs when the angle $2\,i_1\,t - \varepsilon_{(1,2)} + \beta$ is eqval 0. This is not strictly correct in any case, especially when the diurnal components are large, but is sufficiently correct for the tides of Pulpit Cove, where these diurnal components are very small.

If we therefore have the value of M and N for the time of high water, we readily obtain R and β for this time, the former of which added to the constant H_0, is the height of high water, and the latter divided by $2\,i$ is the variable part of the lunitidal interval to be added to the mean value.

As the values of the angles in the expressions of M and N are required at the time of high water, which is one of the quantities sought, the two quantities sought can only be obtained by approximations, but this is readily done, and a second approximation is generally sufficient. The

first approximation is that which we obtain by assuming the time of water to be the high water of the mean lunar semidiurnal component. This, from the expression of H, is—

$$T = \frac{\varepsilon'_{(1,2)}}{2\,i_1} + \frac{2\,n\pi}{2\,i} = \frac{\varepsilon'_{(1,2)}}{2\,i_1} + n \text{ lunar days}$$

in which n is any integral number, and the days are reckoned from the beginning of the year. If we then have a table of the time of the passage of the mean moon over the meridian throughout the year, which is readily obtained by successive additions, the first part of the expression of T gives the time to be added to the time of the transit of the mean moon over the meridian in order to obtain the mean time of high water. The value of ε is constant, and that of c_1 is constant through the year, but changes every year.

Putting this value of T for the value of t in the expressions of M and N, we get the first approximate value of β. Adding this to the value of $\varepsilon'_{1,2}$ in the expression of T above, this will give a second approximation of T, and then putting t equal this value of T in the expressions of M and N, we generally get the values of R and β with sufficient accuracy, and then the preceding expression of L gives the lunitidal interval.

The computations would be very much facilitated by having the values of the angles $(i_n - 2i_1)\,t$ for each day throughout the year for all the sensible terms in the expressions of M and N given in tables, for then their values could be readily obtained for any other time by interpolation, and such a table would serve for every tide station.

The computations would also be very much more facilitated in the case of any particular tide station to have the values of each of the terms in the expressions of M and N given in tables for every tenth degree of the arguments, from which, with the arguments obtained also from the tables mentioned above, the values of each term could be readily obtained. Such tables, however, could only be used for the particular tide station.

38. It has been found that the unknown constants E and G, in the tidal expressions, as determined from the observations of Pulpit Cove, differ very little from those of the tides of Boston Harbor. This is a result of great practical importance, since it shows that the type of the tide varies very little from Boston to Pulpit Cove, and it is therefore probable that it varies very little from Cape Cod to Eastport. The same tidal tables and the same computations of the tides for any one station, as Boston or Pulpit Cove, could probably be used for every station from Cape Cod to Eastport, without any error of any practical importance, by simply changing two constants, that of the amplitude and epoch of the mean lunar tide. The former would simply require the amplitude of the resultant tide at any one station to be multiplied by a certain constant to reduce it to that of any other station, and the other would require that the time or lunitidal interval should be changed by a constant quantity. Such an arrangement would save a great amount of labor in computing the tides for a number of places along the coast.

39. Another result of great practical importance is the probable errors of the amplitudes of the several tide components as obtained from one year's observations. From comparing the results of each of the six years with one another this has been found in the maximum case to be only one-fourth of an inch, and on the average only about one-sixth of an inch. If we wish results, therefore, of which the probable error is not more, say, than one-fourth of an inch, one year's observations are sufficient, and for all practical purposes one-fourth of an inch is of no consequence whatever. For the tidal predictions, the most accurate formulæ do not admit of any accurate comparisons with individual observations, since the abnormal meteorological disturbances may at any time be a foot, or even much more.

TABLES REFERRED TO IN THE TEXT.

TABLE I.—*Containing the values of* $i_e t$, *equation* (3), *for each day and hour of the year.*

M_2 Solar time.	M_1.	M_3.	M_4.	M_5.	M_6.	M_7.	M_8.	M_9.	M_{10}.	M_{11}.	M_{12}.	M_{13}.	M_{14}.	M_2S_2.	$M_2(S_2S_2)$.
d. h.															
Jan. 1 0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
4	4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
5	5	5	5	5	5	5	5	5	5	5	5	5	4	5	5
6	6	6	6	6	6	6	6	6	6	6	6	6	5	6	6
7	7	7	7	7	7	7	7	7	7	7	7	7	6	7	7
8	8	8	8	8	7	8	8	8	7	8	8	8	7	8	8
9	9	9	9	9	8	9	9	9	8	9	9	9	8	9	9
10	10	10	10	9	9	10	10	10	9	10	10	10	9	10	10
11	11	11	11	10	10	11	11	10	10	11	11	11	10	11	11
12	12	12	12	11	11	12	12	11	11	12	12	12	11	12	12
13	13	13	13	12	12	13	13	12	12	13	13	14	12	13	13
14	14	14	14	13	13	14	14	13	13	14	14	15	13	14	14
15	14	15	15	14	14	15	15	14	14	15	15	16	13	15	16
16	15	16	16	15	15	16	16	15	15	16	16	17	14	16	17
17	16	17	17	16	16	17	17	16	16	17	17	18	15	17	18
18	17	18	18	17	17	18	18	17	17	18	18	19	16	18	19
19	18	19	19	18	18	19	19	18	18	19	19	20	17	19	20
20	19	20	20	19	19	20	20	19	19	20	20	21	18	20	21
21	20	21	21	20	20	21	21	20	20	21	21	22	19	21	22
22	21	22	22	21	20	22	22	21	21	22	22	23	20	22	23
23	22	23	23	22	21	23	23	22	21	23	23	0	21	23	0
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.	.	.	.	.	.	.	.	.	.	.	.	.	.	.	.
Dec. 31 0	16	0	7	2	8	0	9	23	8	12	12	5	3	20	8
1	17	1	8	3	9	1	10	0	9	13	13	6	4	21	9
2	18	2	9	4	10	2	11	1	10	14	14	7	5	22	10
3	19	3	10	4	11	3	12	2	11	15	15	8	6	23	11
4	20	4	11	5	12	4	13	3	12	16	16	9	7	0	12
5	21	5	12	6	13	5	14	4	13	17	17	10	8	1	13
6	22	6	13	7	14	6	15	5	14	18	18	11	9	2	14
7	23	7	14	8	15	7	16	6	15	19	19	12	10	3	15
8	0	8	15	9	16	8	17	6	16	20	20	13	11	4	16
9	1	9	16	10	17	9	18	7	17	21	21	14	11	5	17
10	2	10	17	11	18	10	19	8	18	22	22	15	12	6	18
11	3	11	18	12	19	11	20	9	19	23	23	16	13	6	19
12	4	12	19	13	20	12	21	10	20	0	0	17	14	7	20
13	5	13	19	14	21	13	22	11	20	1	1	18	15	8	21
14	6	14	20	15	21	14	23	12	21	2	2	20	16	9	22
15	7	15	21	16	22	15	0	13	22	3	3	21	17	10	23
16	8	16	22	17	23	16	1	14	23	4	4	22	18	11	0
17	9	17	23	18	0	17	2	15	0	5	5	23	19	12	1
18	10	18	0	19	1	18	3	16	1	6	6	0	19	13	2
19	11	19	1	20	2	19	4	17	2	7	7	1	20	14	3
20	11	20	2	21	3	20	5	18	3	8	8	2	21	15	5
21	12	21	3	22	4	21	6	19	4	9	9	3	22	16	6
22	13	22	4	23	5	22	7	20	5	10	10	4	23	17	7
23	14	23	5	23	6	23	8	21	6	11	11	5	0	18	8

NOTE.—In the use of this table for leap year, January 1 in the table corresponds to January 2; January 2 in the table to January 3, and s on until February 29th.

TABLE II.—*Containing the values of the constants c_e, and also of $\tilde{\omega}$, $\tilde{\omega}$, and ω for midnight preceding the first (leap year the second) of January of each year from* 1860 *to* 1890.

Year.	c_1.	c_3.	c_4.	c_5.	c_6.	c_7.	c_8.	c_9.	c_{10}.	c_{11}.	c_{12}.	c_{13}.	c_{14}.	$\tilde{\omega}$.	$\tilde{\omega}'$.	ω.
	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°	°
1860	75.20	101.07	14.68	315.72	139.34	248.93	240.52	89.89	330.41	0.27	179.73	250.03	260.38	146.91	280.53	312.72
1861	305.58	100.83	289.42	141.74	240.33	249.16	196.16	235.00	71.16	0.14	179.86	338.51	272.65	187.57	280.55	293.40
1862	175.96	100.59	204.16	327.75	341.32	249.41	151.80	20.11	171.91	0.01	179.99	67.00	284.92	228.23	280.57	274.07
1863	46.33	100.35	118.90	153.77	82.31	249.65	107.44	165.23	272.66	359.88	180.12	155.48	297.19	268.89	280.59	254.74
1864	264.52	101.10	27.97	321.06	157.94	248.90	56.54	292.49	349.04	0.25	179.75	258.01	271.02	309.67	280.60	235.36
1865	134.89	100.86	302.71	147.08	258.93	249.14	1.18	77.61	89.79	0.12	179.88	346.50	283.29	350.33	280.62	216.03
1866	5.27	100.62	217.45	333.09	359.92	249.38	327.82	222.72	190.54	359.99	180.01	74.98	295.56	30.99	280.64	196.70
1867	235.65	100.38	132.19	159.11	100.91	249.62	283.46	7.83	291.29	359.86	180.14	163.46	307.83	71.66	280.65	177.37
1868	93.83	101.13	41.27	326.40	176.54	248.87	232.57	135.10	7.67	0.23	179.77	266.00	281.67	112.43	280.67	157.99
1869	324.21	100.89	316.00	152.42	277.53	249.11	188.21	280.21	108.42	0.10	179.90	354.48	293.94	153.09	280.69	138.66
1870	194.59	100.65	230.74	338.43	18.52	249.35	143.84	65.33	209.17	359.97	180.03	82.96	306.21	193.76	280.71	119.34
1871	64.96	100.41	145.48	164.45	119.51	249.59	99.48	210.44	309.93	359.85	180.15	171.45	318.48	234.42	280.72	100.01
1872	283.15	101.16	54.56	331.74	195.14	248.84	48.59	337.71	26.30	0.21	179.79	273.98	202.32	275.19	280.74	80.63
1873	153.52	100.92	329.30	157.75	296.13	249.08	4.23	122.82	127.05	0.08	179.92	2.46	304.59	315.86	280.76	61.30
1874	23.90	100.68	244.03	343.77	37.12	249.32	319.87	267.93	227.80	359.95	180.04	90.95	316.86	356.52	280.77	41.97
1875	254.28	100.45	158.77	169.79	138.11	249.55	275.51	53.05	328.56	359.83	180.17	179.43	329.13	37.18	280.79	22.64
1876	112.46	101.19	67.85	337.68	213.73	248.81	224.61	180.31	44.93	0.19	179.81	281.96	302.96	77.96	280.81	3.26
1877	342.84	100.95	342.59	163.09	314.73	249.05	180.25	325.43	145.68	0.06	179.94	10.45	315.23	118.62	280.83	343.93
1878	213.22	100.71	257.32	349.11	55.72	249.29	135.89	110.54	246.43	359.94	180.06	98.93	327.50	159.28	280.84	324.61
1879	83.59	100.48	172.06	175.12	156.71	249.52	91.53	255.65	347.19	359.81	180.19	187.41	339.77	199.94	280.86	305.28
1880	301.78	101.22	81.14	342.42	232.33	248.78	40.64	22.92	63.56	0.17	179.83	289.95	313.61	240.72	280.88	285.90
1881	172.15	100.98	355.88	168.43	333.33	249.02	356.28	168.03	164.31	0.04	179.95	18.43	325.88	281.38	280.89	266.57
1882	42.53	100.74	270.62	354.45	74.32	249.25	311.92	313.15	265.06	359.92	180.08	106.91	338.15	322.04	280.91	247.24
1883	272.91	100.51	185.45	180.46	175.31	249.49	267.55	98.26	5.82	359.79	180.21	195.40	350.42	2.71	280.93	227.91
1884	131.09	101.25	94.43	347.75	250.93	248.75	216.66	225.53	82.19	0.15	179.85	297.93	324.26	43.48	280.95	208.53
1885	1.47	101.01	9.17	173.77	351.93	248.99	172.30	10.64	182.94	0.03	179.97	26.42	336.53	84.14	280.96	189.20
1886	231.85	100.78	283.91	359.79	92.92	249.22	127.94	155.77	283.69	359.90	180.10	114.90	348.79	124.81	280.98	169.87
1887	102.22	100.54	198.64	185.80	193.91	249.46	83.58	300.87	24.45	359.77	180.23	203.38	1.06	165.47	281.00	150.55
1888	320.41	101.28	107.72	353.09	269.53	248.72	32.68	68.13	100.82	0.13	179.86	305.92	334.90	206.24	281.01	131.17
1889	190.79	101.05	22.46	179.11	10.53	248.95	348.32	213.25	201.57	0.01	179.99	34.40	347.17	246.91	281.03	111.84
1890	61.16	100.81	297.20	5.12	111.52	249.19	303.96	358.36	302.32	359.88	180.12	122.88	359.44	287.57	281.05	92.51

TABLE III.

ω	A_3 / A'_3 No.	A_3 / A'_3 Log.	Δe_3	ω
°			°	°
0	0.8816 − 2.5 $\delta\mu$	9.9453	± 0.00	360
10	0.8831 2.5	9.9460	1.18	350
20	0.8878 2.4	9.9483	2.33	340
30	0.8953 2.3	9.9520	3.43	330
40	0.9052 2.2	9.9567	4.47	320
50	0.9176 2.1	9.9626	5.40	310
60	0.9326 1.8	9.9607	6.20	300
70	0.9500 1.4	9.9777	6.83	290
80	0.9696 1.0	9.9866	7.33	280
90	0.9915 0.5	9.9963	7.62	270
100	1.0149 + 0.1	0.0064	7.68	260
110	1.0393 0.7	0.0167	7.50	250
120	1.0638 1.4	0.0269	7.07	240
130	1.0873 2.1	0.0363	6.39	230
140	1.1089 2.8	0.0449	5.48	220
150	1.1275 3.5	0.0522	4.36	210
160	1.1421 4.0	0.0577	3.05	200
170	1.1514 4.2	0.0612	1.57	190
180	1.1546 + 4.3	0.0624	± 0.00	180

TABLE IV.

ω	A_6 / A'_6 No.	A_6 / A'_6 Log.	Δe_6	ω
°			°	°
0	0.8194	9.9135	± 0.00	360
10	0.8213	9.9145	1.80	350
20	0.8268	9.9174	3.58	340
30	0.8362	9.9223	5.28	330
40	0.8494	9.9291	6.90	320
50	0.8666	9.9378	8.41	310
60	0.8878	9.9483	9.76	300
70	0.9132	9.9605	10.90	290
80	0.9430	9.9745	11.80	280
90	0.9768	9.9898	12.43	270
100	1.0144	0.0062	12.72	260
110	1.0554	0.0234	12.62	250
120	1.0990	0.0410	12.10	240
130	1.1429	0.0580	11.12	230
140	1.1858	0.0740	9.67	220
150	1.2246	0.0880	7.75	210
160	1.2555	0.0988	5.43	200
170	1.2756	0.1057	2.80	190
180	1.2826	0.1081	± 0.00	180

TABLE V.

ω	A_1 / A'_1 No.	A_1 / A'_1 Log.	Δe_1	ω
°			°	°
0	1.03724	0.01588	± 0.00	360
10	1.03662	0.01562	0.37	350
20	1.03486	0.01488	0.73	340
30	1.03195	0.01366	1.07	330
40	1.02800	0.01199	1.37	320
50	1.02320	0.00966	1.62	310
60	1.01774	0.00764	1.82	300
70	1.01181	0.00510	1.95	290
80	1.00563	0.00244	2.03	280
90	0.99936	9.99972	2.06	270
100	0.99318	9.99703	2.02	260
110	0.98735	9.99447	1.92	250
120	0.98195	9.99209	1.75	240
130	0.97711	9.98994	1.53	230
140	0.97297	9.98810	1.28	220
150	0.96966	9.98662	1.00	210
160	0.96727	9.98555	0.68	200
170	0.96583	9.98490	0.35	190
180	0.96534	9.98468	± 0.00	180

TABLE VI.

ω	A_3 / A'_3 No.	A_3 / A'_3 Log.	Δe_3	ω
°			°	°
0	0.7777 − 4.4 $\delta\mu$	9.8908	± 0.00	360
10	0.7798 4.4	9.8920	2.22	350
20	0.7860 4.2	9.8954	4.40	340
30	0.7965 4.0	9.9012	6.53	330
40	0.8113 3.6	9.9092	8.57	320
50	0.8306 3.2	9.9194	10.48	310
60	0.8551 2.7	9.9320	12.22	300
70	0.8849 2.0	9.9469	13.75	290
80	0.9203 − 1.1	9.9639	15.02	280
90	0.9616 0.0	9.9830	15.95	270
100	1.0090 + 1.3	0.0039	16.50	260
110	1.0624 2.3	0.0263	16.58	250
120	1.1210 4.6	0.0496	16.13	240
130	1.1833 6.6	0.0731	15.02	230
140	1.2462 8.7	0.0956	13.23	220
150	1.3056 10.8	0.1158	10.75	210
160	1.3552 12.6	0.1320	7.62	100
170	1.3880 13.8	0.1424	3.95	190
180	1.4002 14.3	0.1462	± 0.00	180

NOTE.—Where in these tables there is a double sign the upper one must be used when the argument is found on the left and the lower one when it is found on the right.

LIST OF SKETCHES.

PROGRESS SKETCHES.

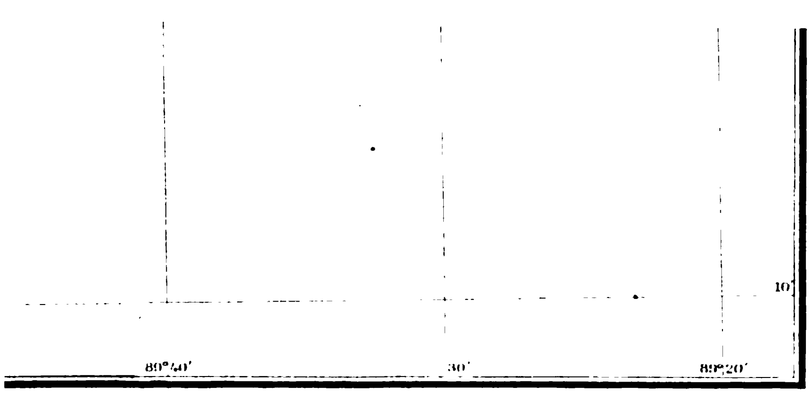
ILLUSTRATIONS.

1 A

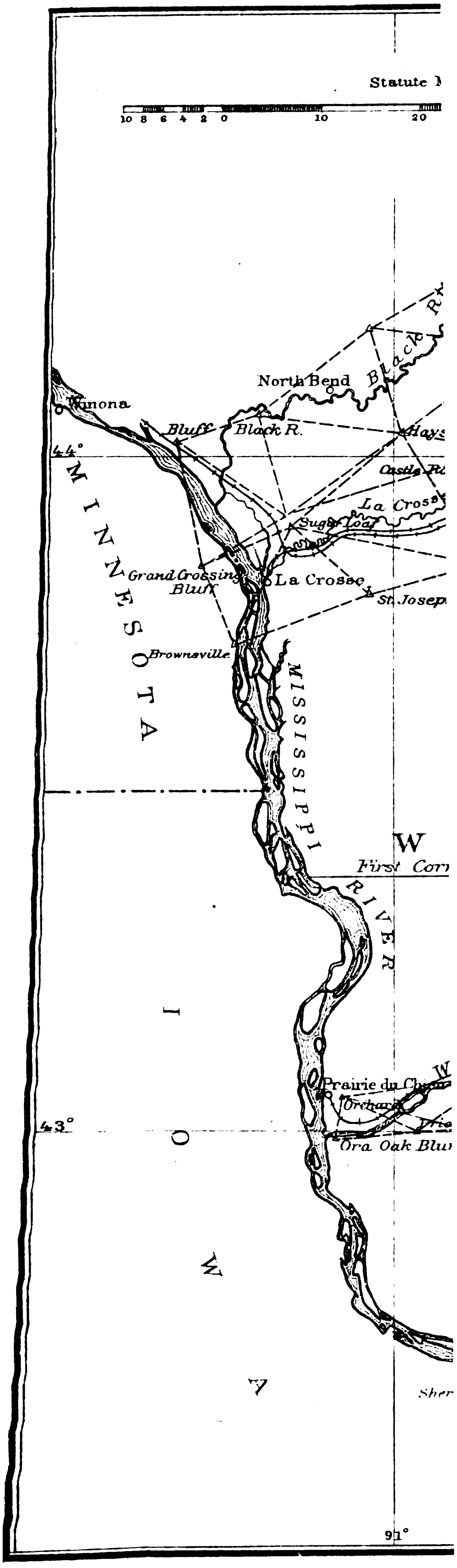
1

14

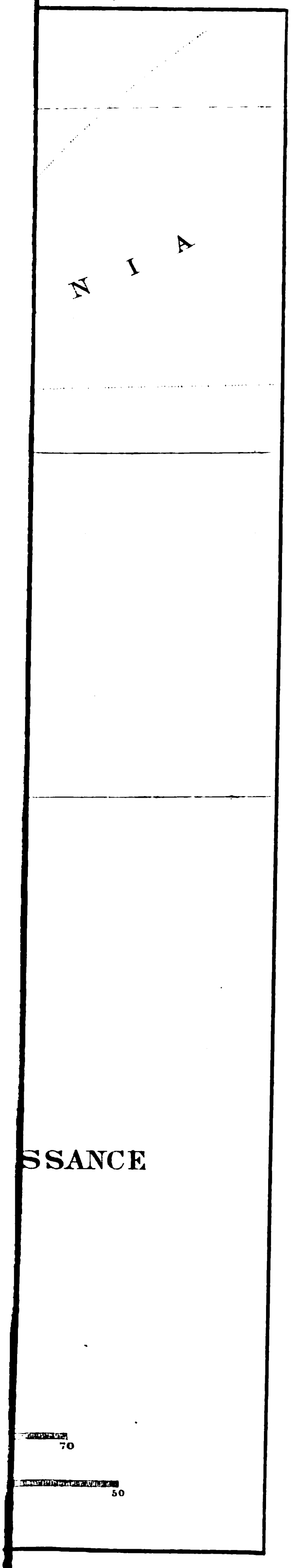
37 30'


10'
89°40'
30'
89°20'

No 22


Statute M
10 8 6 4 2 0 10 20
North Bend
Black R
Winona
Bluff
Black R.
Hays
44°
Castle R
La Cross
Sugar Loaf
MINNESOTA
Grand Crossing Bluff
La Crosse
St. Josep
Brownsville
MISSISSIPPI RIVER
W
First Corr
IOWA
Prairie du Chien
Orchard
43°
Ora Oak Blu
Sher
91°

detic Survey Report for 1878.



26